Theory of Numbers in Jaina *Agama*

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by

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DECLARATION

I **Samani Vinay Prajna** hereby declare that the research work incorporated in the present thesis entitled **"Theory of Numbers in Jaina *Āgama*"** is my own work and is original carried out under the guidance of my research supervisor. This work (in part or in full) has not been submitted to any Institute for the award of a Degree or a Diploma. I have properly acknowledged the sources and the material collected from secondary sources wherever required. I solely own the responsibility for the originality of the entire content.

I have also completed the relevant requirement of Ordinance in force.

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This is to certify that **"Theory of Numbers in Jaina *Āgama*"** is a original work of Samani Vinay Prajna, which has been prepared under my supervision for the award of Ph.D. degree in the Department of Jainology and Comparative Religion & Philosophy. To my knowledge, this thesis has not been submitted in any Indian and foreign University for the award of any degree.

I recommend the thesis to be submitted for the award of Ph.D. degree of Jain Vishva Bharati Institute, Ladnun, Rajasthan, India.

(Prof. Samani Riju Prajna)

INTRODUCTION

India has always dominated the world in the field of arithmetic, and indeed the art of numbers plays a leading role in Indian culture.

As Ifrah remarks, *It is precisely this skill in the field of mathematics which led Indian scholars, at very early stage, to develop their astonishing arithmetical speculations which could involve numbers comprising hundreds of decimal places, whilst maintaining a clear idea of the order of the ascending powers of ten within a nomenclature which contained a highly diversified terminology, based as it was upon both specific etymologies and improvised term born out of a highly creative symbolical imagination.*

Jainism is one of the oldest streams of philosophy and religion in India. Jaina canonical literatures or *Āgama* cover wide range of subjects. Originally, in spite of being metaphysical and spiritual in nature, it provides the knowledge of many branches of science and non-science subjects like biology, physics, mathematics, astronomy, psychology, geography, history, health science etc. Today, the research is being carried out in many of the subjects, yet a lot is to be done.

Being, a student of Mathematics and having keen interest in *Āgama,* I chose to merge both the fields and the outcome is this research work entitled as ***Theory of Numbers in Jaina Āgama,*** which is a tiny effort to unearth the hidden seeds of mathematics in Jaina *Āgama*.

The objectives of this research work are:

1. **Number Theory** is the backbone of Mathematics. Therefore, to study the history of '**Theory of numbers**' in Jaina canonical literature and the texts.
2. To explain the various aspects of theory of numbers explained in Jaina *Āgama* in modern mathematical notation.
3. To study the application of **Number Theory** in various Jaina philosophical concepts.

When I began my studies on topic of this thesis the first problem arose as to the selection of period. I should concentrate on them since the *Āgama* were last edited and put in written

form at Vallabhī council (according to *Śvetāmbara* tradition). Many scholars argue that state or stage of mathematics there in should be regarded as available on middle of 5th century (450 century A.D.)

But since the *aṅga*-s were first composed during the time of Bhagawāna Mahāvīra or immediately thereafter some relics of 6th century B.C. were also found in that. Hence I have chosen the period of 500 B.C. – 500 C. A.D. Thus this thesis may be regarded as evolution of mathematical concepts in that period and lot of examples of the same have been given.

The grand edifice of mathematics rests on the **Number Theory.** During my studies I found that Jaina scriptures are replete with numbers and concepts related to numbers because of their spiritual, philosophical and metaphysical requirements. There are numbers, finite and infinite. As well as infinitesimal, similarly, while describing a large number of aspects, it prompted me then to study the **Theory of Numbers** as is available in our oldest scriptures i.e. *Āgama*.

Jaina mathematics is important historically as crucial link between the mathematics of the Vedic period (before 500 BC) and that of the *classical period* (after 500 A.D.), known as *Dark Age*. Tremendous work is being done by Jaina saint-scholars during that period. A significant historical contribution of Jaina mathematicians is by making free Indian mathematics from its religious and ritualistic constraints.

In this study I included *Āgama* and non-*āgamic* scriptures composed over span of 1000 years. i.e. 500 BC - 500 A.D. Since *Āgama* were last edited in 5th century A.D. I have decided to include some non-*āgamic* works till then or probably a century later. That's why I expanded the scope of my study to Ācārya Sarvanandi, Jinabhadragaṇi Kṣamāśramaṇa, Ācārya Yativṛṣabha, Ācārya Umāsvāti etc. There are some works equally ancient and treated at par with *Āgama* but not included in *Āgama* are also considered. E.g. *Jyotiṣkaraṇḍaka*, *Jīvasamāsa* (c. 4/5 C. A.D.), etc. In this work the Digambara tradition recognizes *Ṣaṭkhaṇḍāgama*) (c. 2/3 C. A.D.) as equivalent to *Āgama*. Therefore it is also included in my studies.

For real understanding of Number Theory in Jaina sources I had to rule out the possibility of its coverage by other sources. Hence I started study the world scenario from alternate sources. The outcome of this is **Chapter One**, **A Global Synopsis of Mathematics.** The

question is how the **number theory** ideas hold in Jainism and whether they were originated and evolved from mathematical development in other cultures. Therefore it was necessary to study the mathematical development specially the **number theory** in other ancient cultures. The following civilizations have been studied and their mathematical attainment chronologically- Harappan, Egyptian, Babylonian, Vedic, Greek, China, and Mayan and there uniqueness viz. unit fraction of Egypt, sexagesimal system of Babylonia, mathematical logic of Greek, computational skills of Chinese, vegisimal system with accurate observation of Mayan and of course decimal system of Vedic and large number of Buddhists from our land.

Hence, chapter 1 is totally devoted to mathematics of the Indian and non- Indian cultures. In that process I have highlighted the unique features and strength of mathematics of those lands and cultures.

The **Chapter Tw**o is **Mathematics in Jaina Heritages.** After finishing this preliminary study I had to go through oceanic extent of the Jaina Mathematics. I then shifted naturally to an overview of **number theory**, related topics, discussed in *Āgama* and few non-*āgamic* works as mentioned earlier. This chapter deals with ancient Jaina mathematics, purpose for mathematical initiatives and actual sources which were of course limited to *āgamic* source-*aṅga*, *upāṅga* etc. and very few contemporary non-*āgamic* sources.

However, I found that only certain *āgamic* works were rich in mathematical contents. And hence I concentrated on them. These *āgamic* works *Bhagavaī, Thāṇaṁ, Samavāo, Jambūdvīpaprajñapti, Sūrya-Candraprajñapti Jīvājīvābhigama, Anuyogadvāra Sūtra, Prajñāpanā Sūtra*, *Uttarādhyayana*, *Ṣaṭkhaṇḍāgama*. Besides these the non- *āgamic* work considered by me are *Jyotiṣkaraṇḍaka*, *Jīvasamāsa*, *Tiloyapaṇṇatti*, *Lokavibhāga, Sabhāṣyatattvārthādhigama Sūtra, Viśeṣāvaśyaka Bhāṣya*, *Bṛhatkṣetra Samāsa etc*.

Karma theory is full of calculus. So, I have not touched it. In Chapter 2, I have taken an extensive survey of the mathematical content specially that pertaining to number theory.

The greatest gift of India to the world of mathematical is recognized as decimal system and zero with place value. I therefore began with the decimal system including zero, with its role in positional value in **Chapter Three** entitled as **Number Theory (1): Decimal System**.

I tried to reason out for evolution of numbers. Although, no such symbols are employed in the scriptures, but the internal evidence was so strong that one can conclude that such huge calculation can be possible only through the implements of numerals and writing material. I found, to my surprise that the earliest evidence of this points towards the Jaina sources including *āgamic* works. Hence, chapter 3 elaborately deals with it, where the evolution of decimal systems, numerals, place value, zero etc. I then projected the richness of Jaina scriptures in these aspects in detail. The direct proof of the earliest evidences of use of zero for positional value from Jaina sources is given which is earliest known so far.

This chapter also highlights the fascination of Jaina *ācārya*-s with large numbers and as to how these numbers outstripped those encountered in other cultures. About 2000 years back, Jaina *ācārya*-s studied the number field (z^+) in depth. They adopted then prevailing decimal system but went to unimaginable heights. Beginning with 1,000,000,000,000,000,000,000… *koḍākoḍākoḍī* (10^{21}), they reached a monstrous number *śīrṣaprahelikā* = $(84 \times 10^5)^{28}$ =75826325307301024115797356997569640621896684808018329 6×10^{140} (a 194 digit number).

The comparative list of the largest countable number *śīrṣaprahelikā* found in *āgamic* and non-*āgamic* works is also given.

The **Chapter Four** is **Number Theory (2): Transfinite Numbers** which shows that the Jaina scholars were not satisfied with mere large numbers. They started imagining numbers beyond this and struck upon the theory of number field classifying it in 3 parts: *asaṅkhyāta* (numerable), *asaṅkhyāta* (innumerable) and *ananta* (infinity). This theory bears great similarity with the modern theory of transfinite numbers propounded by the great German mathematician George Cantor (1874). I was dismayed to see that the theory was rejected by giant contemporary mathematician and as a result Cantor had to lose the mental balance. That similar ideas could be thought of about 1500 years earlier was astounding. Therefore, chapter 4 deals with in rather great detail with this theory and providing many complicated calculations. A very complicated circuitous and recursive process is employed for arriving at the estimate of A (*asaṅkhyāta*), which is far greater than all the large numbers of modern mathematics except probably the Graham's number.

This chapter also explains in some detail as to how Jaina saint- scholars explored the world of very small (infinitesimal) equally proficiently as that of very big – infinity. To broad analysis

In **Chapter Five**, **Number Theory (3): Mathematical Solutions of Metaphysical Problems,** various metaphysical concepts discussed in *Āgama* etc. related to Number Theory, have been dealt with and some of them (occupancy problem, partition theory etc.) have great application in modern science. It is also shown that how the *āgamic* works are full of application of permutations and combinations and combinatorics, which has a term found as *bhaṅga samutkirtana* in *āgamic* works. It is also shown as to how Jaina scholars were able to deal with fractions, extraction of square roots of large numbers etc. While dealing with a large number of aspects of certain things, *Āgama* were able to pin-point the exact size etc. of those things and giving exact measures of their relative size. *Āgama* are full of these aspects fondly called by them *appa bahuya* (*alpabahutva*, maxima-minima).

In **Chapter Six**, **Conclusion** I have listed the things explored and culled from the great ocean of *Āgama*.

I felt myself fortunate for selecting the topic of **Theory of Numbers in Jaina Āgama** as my doctoral work because I could get the opportunity to delve in the ocean of *Āgama*, if not very deep. This research work is the baby steps to find the gems of Theory of Numbers in the ocean of *Āgama.* Giant steps are yet to be taken. Since the topic I chose is very vast, I had to limit myself to broad analysis. By taking just one scripture like *Bhagavaī, Jambūdvīpaprajñapti, Anuyoyadvārasūtra, Prajñāpanā* etc. research can be done in depth. For readers convenience glossary of the technical words is provided at the end of this thesis.

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(Samaṇī Vinaya Prajñā)

ABBREVIATION

Texts :

ADS	*Anuyogadvāra Sūtra*
BKS	*Bṛhatkṣetrasamāsa*
BS	*Bhagavatī Sūtra*
GSS	*Gaṇitasārasaṅgraha*
JDP	*Jambūdvīpaprajñapti*
JJ	*Jīvajīvābhigama Sūtra*
JK	*Jyotiṣkaraṇḍaka*
JS	*Jīvasamāsa*
KP	*Kaṣāya Pāhuḍa*
LP	*Lokaprakāśa*
LV	*Lokavibhāga*
PS	*Prajñāpanā Sūtra*
RV	*Rājavārtika*
SAM	*Samavāo*
SCP	*Sūryaprajñapti Candraprajñapti*
SK	*Ṣaṭkhaṇḍāgama*
TSB	*Sabhāṣyatattvārthādhigamasūtra*
TS	*Thāṇaṁ*
TP	*Tiloyapaṇṇatti*
US	*Uttarādhyayana Sūtra*
VB	*Viśeṣāvaśyaka Bhāṣya*

Others :

JD	*Jambūdvīpa*
LS	*Lavaṇasamudra*
DK	*Dhātakīkhaṇḍa*

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Chapter-1

A Global Synopsis of the Mathematics

Mathematics knows no races or geographic boundaries;
for mathematics, the cultural world is one country.

~ Hilbert, David (1862-1943) ~

Entire world is in the lap of numbers. It can be said that no mathematics means no existence. This chapter highlights the brief ideas of mathematics around the globe, how it was developed, how significant is its contribution in various fields and what role was played by great mathematicians. This is necessary to understand the role played by Jaina Mathematics, especially its uniqueness during the period 500 BC to 500 A.D., the period under this study.

1.1 Mathematics: The Breath of Human Life

Mathematics is the breath of human life, society and culture. Right from childhood one thinks of it naturally. It is the need in every walk of life. Ācārya Mahāvīra (850 A.D.), the great Jaina mathematician, in *Gaṇitasārasangraha* (*gāṭhā* 1.9-11) has appreciated mathematics by describing the importance of maths in human life as:

"In all transactions which relate to worldly, Vedic or other similar religious affairs counting is of use. In the science of love, in the science of wealth, in music and in drama, in the art of cooking, in medicine, in architecture, in prosody, in poetics and poetry, in logic and grammar and such other things, and in relation to all that constitutes the peculiar value of arts, the science of counting (*gaṇita*) is held in high esteem. What is the good of saying much? Whatever there is in all the three worlds which are possessed of moving and non-moving beings cannot exist as apart from *gaṇita* (measurement and calculation)."[1]

[1] *Gaṇitasārasaṅgraha* of Mahāvīrācārya, ed. by L. C. Jain, Jain Sanskriti Sanrakṣaka Sangha, Solapur, 1962, p. 2.
cf. Datta, Bibhutibhushan, Singh, Avadesh Narayan, *History of Hindu Mathematics*, Bharatiya Kala Prakashan, Delhi, Vol. I, 2nd edn., 2004, p. 5.

Gaṇita, therefore, has been the iconic part of every culture. Of course, the structure of the existing knowledge of mathematics is the outcome of the gradual development of the civilizations over centuries. Each culture has contributed to give it a concrete shape.

1.2 History of Mathematics

"When and where did mathematics originate" is a difficult question to answer. Since most of the sources are damaged or lost in the course of time the roots are difficult to find. Systematic and logical development of course can be credited to various civilizations. Exciting discoveries of ancient mathematics on papyri in Egypt and clay tablets in Mesopotamia pushed back the origins of written mathematical records by at least fifteen hundred years. Egyptians also played an important role in measurements and counting and they were aware of computations with fractions, arithmetic operations and playing with the numbers for their religious, social and trade purposes.[1]

Greek mathematics greatly refined the methods (especially through the introduction of deductive reasoning and mathematical rigor in proofs) and expanded the subject matter of mathematics. Chinese mathematics made early contributions, including a place value system. The Hindu-Arabic numeral system and the rules for the use of its operations, in use throughout the world today, likely evolved over the course of the early part of first half millennium AD in India and were transmitted to the west via Islamic mathematics.

Islamic world contributed in bringing together the technique of measurement from Egypt and the technique of computation originated in India [Joseph, 2011. pp.10-11].

India has always dominated the world in the field of arithmetic, and indeed the art of numbers plays a leading role in Indian culture.[2] Indian science was born out of a mystical and religious culture and the etymology of the Sanskrit words used to describe the numbers and the science of numbers bear witness to this fact. The term for the 'science of numbers' in Sanskrit is *saṅkhyāna,* which means arithmetic. The word is frequently used in this sense in Jaina literature where the science of number was considered to be one of the fundamental requirements for the full development of the monk; likewise in Buddhist

[1] Jain, L.C., *Basic Mathematics (Exact Sciences from Jaina Sources)*, Rajasthan Prakrit Bharati Sansthan, Jaipur, Vol. I, 1982, p. 15.

[2] Georges Ifrah, *The Universal History of Numbers: From Prehistory to the Invention of the Computer*, Wiley, New York, 2000, p. 440.

literature it was considered to be the most noble of the arts. The word 'number' itself is *saṅkhyā* or *saṅkhyeya*. *saṅkhyā* is also the term used for one of the six orthodox systems of the Hindu philosophy of the six *darśana*.[1]

According to Jaina legend Lord Ṛṣabha taught the *Brāhmī lipi* (script) to his daughter Brāhmī and *gaṇita* (arithmetic) to Sundarī. Tradition believes that Lord Ṛṣabha was the first *tīrthaṅkara* in the prehistoric period[2] and in the historic age mathematics for explaining the philosophical concepts was delivered by Lord Mahāvīra and learned by *gaṇaḍhara* which was further transmitted to the successive *ācārya*-s by oral tradition. Mathematical concepts and their philosophical implication, ascribed to Bhagawāna Mahāvīra (600 BC), are mentioned in the various Jaina canons (*āgama*-s) documented about 1500 to 2500 years ago.

In Buddhist literature arithmetic is regarded as the first and the noblest of arts.[3] They classified *gaṇita* in three sections: *mudrā* (finger arithmetic), *gaṇanā* (mental arithmetic) and *saṅkhyāna* (higher arithmetic).[4] Buddhist literature also demonstrates an awareness of innumerable and infinite numbers.

In the Vedic period, records of mathematical activity are mostly to be found in Vedic texts associated with ritual activities. However, as in many other early agricultural civilizations, the study of arithmetic and geometry was also impelled by secular considerations. Examples of geometric knowledge (*rekhā-gaṇita*) are to be found in the *Śulba-sūtras* of *Bauddhyāyana* (800 BC) and *Apasthmaba* (600 BC) which describe techniques for the construction of ritual altars in use during the Vedic era.

Jaina canonical text *Sthānāṅga Sūtra* (300 BC) in chapter ten mentions about 10 types of finer things and out of them last two are - *gaṇita* (mathematics) and *bhaṅga*[5] (permutations and combnations). In *Samavāo*[6] (*sūtra* 72.7) out of 72 arts first three mentioned are *lekha*

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[1] Georges Ifrah, *The Universal History of Numbers: From Prehistory to the Invention of the Computer*, p. 440.

[2] *Jambūdvīpaprajñapti Sūtra*, ed. by Yuvācārya Miśrīmal ji Madhukar, with Hindi translation and appendices, Agam Prakashan Samiti, Beawar, 1st edn., 1994, p. 55, (*sūtra* 2.35).

[3] *History of Hindu Mathematics*, Vol. I, p. 4.

[4] *Ibid*. p. 7.

[5] *Ṭhāṇaṁ*, Chief Synod Ācārya Tulsī, ed. by Yuvācārya Mahāprajña. with text, Sanskrit translation and commentary in Hindi, Jain Vishva Bharati, Ladnun, 1976, p. 907.

[6] *Samavāo*, Chief Synod Ācārya Tulsī, ed. and commentary by Ācārya Mahāprajña, Jain Vishva Bharati, Ladnun, 1984, p. 246.

(*lehaṁ*), *gaṇita* (*gaṇaṇā*) and *rūpa* (*rūvaṁ*). *Nāyādhammakahāo* (1.1.85) and *Rājapraśnīya* (805) also mention these three. In *Arthaśāstra of Kauṭilya* (400 BC) it is mentioned that having undergone the ceremony of tonsure, the student shall learn the alphabets (*lipi*) and arithmetic (*saṅkhyāna*). *Hāthīgumphā* inscription (2nd C. A.D.) mentions that king Khārawela spent nine years in learning *lekha* (alphabets), *rūpa* (geometry) and *gaṇanā* (arithmetic).[1]

1.3 Development of Mathematics

In *The Crest of The Peacock* (chapter 1, p. 13) Joseph writes that ancient Egypt and Mesopotamia, Greece (and the Hellenistic world), India, China, the Islamic world and Europe played an important role in the development of Mathematics on the basis of two criteria: 1) Existing evidence. 2) The nature and the direction of transmission of mathematical knowledge.

The earliest records of mathematics show it arising in response to practical needs in agriculture, business, and industry. In Egypt and Mesopotamia, there existed well developed number system as early as the 3rd millennium BC (Joseph 2000, p. 16). It was used for surveying and mensuration; estimates of the value of π (pi) are found in both locations. The creation of special algorithm for the basic arithmetic operation was the outcome of Egyptian hieroglyphic.

To Aristotle (c. 350 BC) Egypt was the cradle of Mathematics. His teacher, Eudoxus, one of the notable mathematicians of the time, had studied in Egypt before teaching in Greece. Even earlier Thales (d. 546 BC) the legendry founder of mathematics and Pythagoras (c. 500 BC) one of the earliest of greatest Greek mathematicians, were reported to have travelled widely in Egypt and Mesopotamia and learned much of their mathematics from these areas. Some sources even credit Pythagoras with having travelled as far as India in search of knowledge, which could explain some of the parallels between Indian and Pythagorean philosophy and religion.[2]

The civilization which cultivated around the Twin Rivers, the Tigris and the Euphrates in Mesopotamia contributed to mathematics by establishing the place-value system of sexagesimal, which is one of the most significant developments in the history of mathematics. There are strong evidences as Friberg (2007) also mentions that there was

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1 *History of Hindu Mathematics*, Vol. I, p. 6.

2 Joseph, George Gheverghese, *The Crest of The Peacock: The Non-European Roots of Mathematics*, Princeton University Press, New Jersey, 2011, p. 6.

link between the mathematical traditions of three civilizations Egypt, Mesopotamia and Greece. There is some evidence of developments in India and China during this same period, but few records have survived.

During the first half of the first millennium after Christian era, the Central American Mayan civilization attained great heights in many fields like art, sculpture, mathematics, astronomy etc. They developed the numeral system with base twenty.

By the second half of the first millennium AD the contacts between Islamic world and India played the most important role in spreading and developing the mathematics around the world. They spread Indian numerals, associated algorithms, trigonometry (especially the sine function) and the solution of the general and indeterminate equations.

This early mathematics is generally empirical, arrived at by trial and error as the best available means for obtaining results, with no proofs given.

1.4 Mathematics in Prehistoric Period

No one knows how counting began. The primitive people, perhaps hunters or others, kept the record of each animal by just drawing the vertical line for each count. It is very common even today in the rural areas and small villages.

The very oldest counting tools that archaeologists have yet dug up are the numerous animals bones found in Africa and Western Europe and marked with one or more sets of notches. These tally sticks are between twenty and thirty-five thousand years old.[1]

A four-thousand-year-old clay, tablet kept in Berlin museum, gives the value of $n^2 + n^3$ for n= 1, 2, 3,…10, 20, 30, 40, 50 from which it has been surmised that the Mesopotamians may have used these values in solving cubic equations after reducing them to the form $x^3 + x^2 = c$.[2]

The Ishango bone, found near the headwaters of the Nile river (north-eastern Congo) shown in the figure below, consists of a series of tally marks carved in three columns running the length of the bone. Common interpretations are that the Ishango bone shows either the earliest known demonstration of sequences of prime numbers[3] or a six-month lunar calendar.

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1 *The Universal History of Numbers*, p. xix (Introduction).

2 *The Crest of The Peacock*, p. 6.

3 Williams, Scott W., *The Oldest Mathematical Object is in Swaziland*, The Mathematics Department of The State University of New York, Buffalo, 2005 (*www.math.buffalo.edu*).

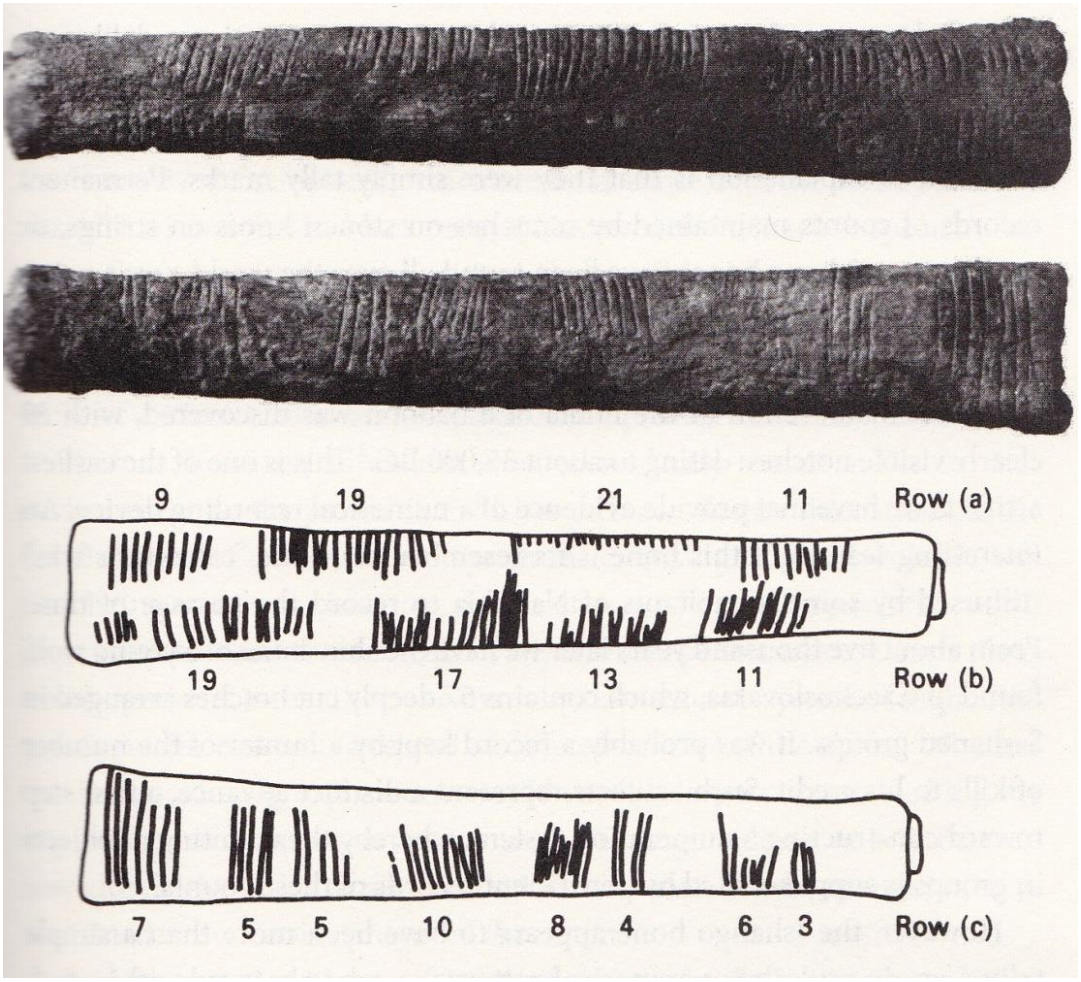

Fig 1.1: Ishango Bone (courtesy of Dr. J. de Heinzelin)

1.5 Egyptian Mathematics

In Egypt problems for agricultural requirement like irrigation, drainage and flood control, distributing the land among peasantry and the storage house, gave rise to mathematics.

Three types of source materials on Egyptian mathematics can be distinguished: table text, problem text and administrative.[1] The texts are written on the papyri which are more durable than palm leaves, bark or bamboos used by Indian and Chinese. The major source is the Ahmes (or Ahmose) papyrus copied by a scribe around 1650 BC from and older document, which also known as Rhind papyrus is brought to British Museum. The second source is Moscow papyrus written in about 1850 BC.[2]

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1 *The Crest of The Peacock*, p. 83.

2 *Ibid*, p. 82.

The Rhind papyrus (in the British Museum) is a copy made in the 17th century BC of a text two centuries older still. In it is found a long table of fractional parts to help with division, followed by the solutions of 84 specific problems in arithmetic and geometry. The Golenishchev papyrus (in the Moscow Museum of Fine Arts) dating from the 19th century BC, presents 25 problems of a similar type. These problems reflect well the functions the scribes would perform, for they deal with how to distribute beer and bread as wages and how to measure the areas of fields as well as the volumes of pyramids and other solids.[1] The geometric problems in the papyri seek measurement of figures like rectangles and triangles of given base and height by means of suitable arithmetic operations.

Written evidence of the use of mathematics dates back to at least 3000 BC with the ivory labels found in Tomb U-j at Abydos. These labels appear to have been used as tags for grave goods and some are inscribed with numbers.[2] Further evidence of the use of the base 10 number system can be found on the Narmer Macehead which depicts offerings of 400,000 oxen, 1,422,000 goats and 120,000 prisoners.[3]

The numeral system and arithmetic operations: The Egyptians like the Romans after them, expressed numbers according to decimal scheme, using separate symbols for 1, 10, 100, 1000 and so on. Each symbol appeared in the expression for a number as many times as the value it represented occurred in the number itself. For example, 𓎆𓎆𓏺𓏺𓏺𓏺 stood for 24. This rather cumbersome notation was used within the Hieroglyphic writing found in stone inscription and other formal texts.[4] For larger numbers this procedure can be improved by considering multiples of one of the factors by 10, 20 or even by higher orders magnitude (100, 1000…) as necessary (in the Egyptian decimal notation, these multiples are easy to workout). Thus one can find the product of 28 by 27 by setting out the multiples of 28 by 1, 2, 4, 8, 10 and 20. Since the entries 1, 2, 4 and 20 add up to 27, one has only to add up the corresponding multiples to find the answer.

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1 *Internet Encyclopædia Britannica, Mathematical-astronomy,* by Menso Folkerts, Encyclopædia Britannica Inc., 2015. (*www.britannica.com/topic/mathematics/Mathematical-astronomy).*

2 Imhausen, Annette, *Ancient Egyptian Mathematics: New Perspectives on Old Sources*, The Mathematical Intelligencer, Vol. 28, Nr. 1, 2006, pp. 19–27.

3 *The Universal History of Numbers*, p. 165.

4 *Internet Encyclopædia Britannica,* 2015, (*www.britannica.com/topic/mathematics/The-numeral-system-and-arithmetic-operations*).

A remarkable result is the rule for the volume of the truncated pyramid (Golenishchev papyrus, problem 14). The scribe assumes the height to be 6, the base to be a square of side 4, and the top a square of side 2. He multiplies one-third the height times 28, finding the volume to be 56; here 28 is computed from $2 \times 2 + 2 \times 4 + 4 \times 4$. Since this is correct, it can be assumed that the scribe also knew the general rule: $A = (h/3)\ (a^2 + ab + b^2)$ for the volume of a pyramid: one-third the height times the area of the base.[1]

Unit fractions

An interesting and one of the most puzzling episodes in the history of human thought is the 2000-year reign of ancient Egyptian mathematics, the use of unit fractions. The Egyptians used some special notation for fractions such as 1/2, 1/3 and 2/3 and in some texts for 3/4, but other fractions were all written as unit fractions of the form 1/n or sums of such unit fractions.

The Rhind Mathematical Papyrus, an ancient Egyptian mathematical work, includes a mathematical table for converting rational numbers of the form 2/*n* into Egyptian fractions (sums of distinct unit fractions), the form the Egyptians used to write fractional numbers. The text describes the representation of 51 rational numbers. It was written in approximately 1650 BCE by Ahmes.

The ancient Egyptians operated with fractions almost exclusively in terms of *unit* fractions. For example, instead of saying 2/5 of my land was flooded, they would say 1/3 + 1/15 of my land was flooded. The representation of rational numbers as sums of unit fractions dates back to the time of ancient Egypt.

[1] *Internet Encyclopædia Britannica,* 2015, (*www.britannica.com/topic/mathematics/Geometry)*.

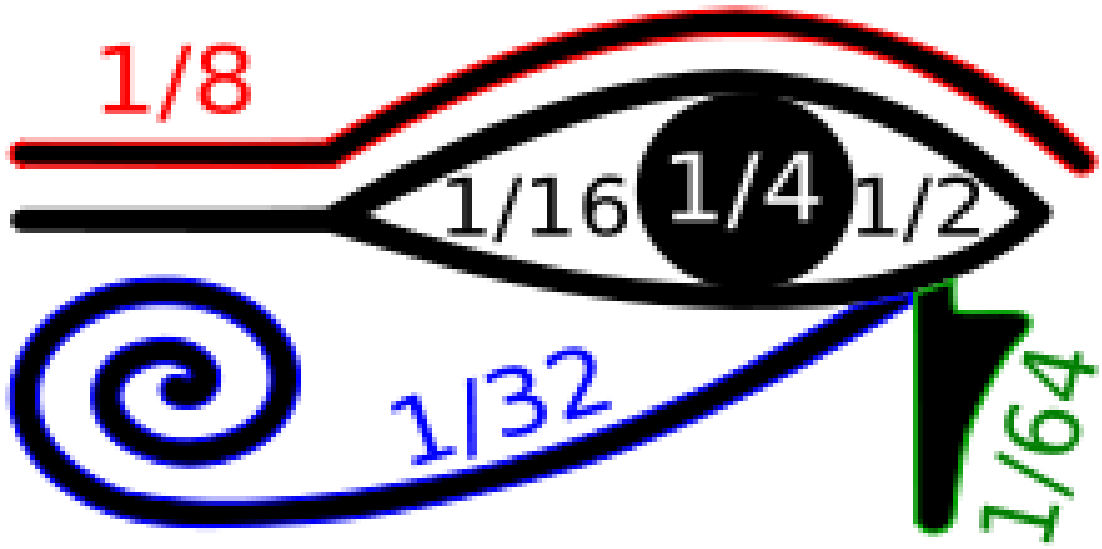

Fig: 1.2 Unit Fraction

Operating with unit fraction is a singular feature of Egyptian mathematics and is absent from almost every other mathematical tradition.[1]

As we can see a few examples of unit fractions[2] :

$$2/7 = 1/4 + 1/28$$

$$2/59 = 1/36 + 1/236 + 1/531$$

Today this subject survives mainly as a source of mathematical puzzles and problems in abstract number theory.

1.6 Babylonian mathematics

Babylonian mathematics refers to mathematics of the people of Mesopotamia (modern Iraq) from the days of the early Sumerians through the Hellenistic period almost to the dawn of Christianity.[3] It is named Babylonian mathematics due to the central role of Babylon as a place of study. Later under the Arab Empire, Mesopotamia, especially Baghdad, once again became an important centre of study for Islamic mathematics.

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1 *The Crest of The Peacock*, p. 92.

2 *Ibid*, p. 95.

3 Boyer, C. B., *A History of Mathematics*, John Wiley & Sons Inc., New York, 1991, p. 24.

Out of half a million inscribed tablets (cuneiform) more than four hundred are directly related with mathematics which are now safe in the museum of Berlin, London, Paris, and Strasburg and the universities of Yale, Columbia, Chicago and Pennsylvania. Two types of mathematical tablets are generally found, *table-texts* and *problem texts*. Table-texts are for some purpose, such as multiplication tables, weights and measures tables, reciprocal tables, and the like. The second class of tablets are concerned with the solutions or methods of solution to algebraic or geometrical problems. A very large portion of the texts belong to a period beginning of the new Babylonian period around 600 BC is found consisting of considerable accomplishment in astronomy.[1]

The Babylonian system of mathematics was sexa-gesimal (base 60) numeral system. From this we derive the modern day usage of 60 seconds in a minute, 60 minutes in an hour, and 360 degrees in a circle. The Babylonians were able to make great advances in mathematics for two reasons. Firstly, the number 60 is a superior highly composite number, having factors of 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 (including those that are themselves composite), facilitating calculations with fractions. Additionally, unlike the Egyptians and Romans, the Babylonians had a true place-value system, where digits written in the left column represented larger values much as in the modern decimal system, although of course using base 60 instead of base 10. Thus, 𒐗 𒐗 𒐗 in the Babylonian system is represented as, $1 \times 60^2 + 1 \times 60 + 1$, or 3,661. It may, however, represent $1 \times 60^3 + 1 \times 60 + 1$ as concept of place value and zero was not fully developed then. Also, to represent the numbers 1 - 59 within each place value, two distinct symbols were used, a unit symbol (𒐗) and a ten symbol (𒌋) which were combined in a similar way to the familiar system of Roman numerals (e.g. 23 would be shown as 𒌋𒌋𒐗𒐗𒐗). Sumerians wrote multiplication tables on clay tablets and dealt with geometrical exercises and division problems. The Babylonians used pre-calculated tables to assist with arithmetic.

A complete set of sexagesimal multiplication table was available for many numbers. Moreover the table of reciprocal was available for the 'regular' integers up to 81 that are multiples of 2,3 or 5. They would know the value of $\sqrt{2}$ which is verified from the tablet at Yale university. For example, two tablets found at Senkerah on the Euphrates in 1854, dating from 2000 BC, give lists of the squares of numbers up to 59 and the cubes of

[1] *The Crest of The Peacock*, pp. 133-134.

numbers up to 32. The Babylonians used the lists of squares together with the formulae to simplify multiplication.

$$ab = \frac{(a+b)^2 - a^2 - b^2}{2}$$

$$ab = \frac{(a+b)^2 - (a-b)^2}{4}$$

The Babylonians did not have an algorithm for long division.[1] Instead they based their method on the fact that $\frac{a}{b} = a \times 1/b$, together with a table of reciprocals. Numbers whose only prime factors are 2, 3 or 5 (known as 5-smooth or regular numbers) have finite reciprocals in sexagesimal notation, and tables with extensive lists of these reciprocals have been found. They used the Rule of position for fractions whose denominator was a power of 60. To compute 1/13 or to divide a number by 13 the Babylonians would use an approximation such as[2]

$$\frac{1}{13} = \frac{7}{91} = 7 \times \frac{1}{91} \approx 7 \times \frac{1}{90} = 7 \times \frac{40}{60^2} = \frac{280}{60^2} = \frac{4}{60} + \frac{40}{60^2}$$

As well as arithmetical calculations, Babylonian mathematicians also developed algebraic methods of solving equations. Once again, these were based on pre-calculated tables.

Babylonians knew the common rules for measuring volumes and areas. The ancient Babylonians had known of theorems on the ratios of the sides of similar triangles for many centuries, but they lacked the concept of an angle measure and consequently, studied the sides of triangles instead.[3] The Babylonians used geometric shapes in their buildings and design and in dice for the leisure games which were so popular in their society, such as the ancient game of backgammon. Their geometry extended to the calculation of the areas of rectangles, triangles and trapezoids, as well as the volumes of simple shapes such as bricks and cylinders (although not pyramids).

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1 O'Connor, J. J. and Robertson, E. F., *An overview of Babylonian mathematics*, 2000. (*www-history.mcs.st-andrews.ac.uk/HistTopics/Babylonian_mathematics.html*)

2 https://en.wikipedia.org/wiki/Babylonian_mathematics

3 *A History of Mathematics*, 1991, pp. 158–159.

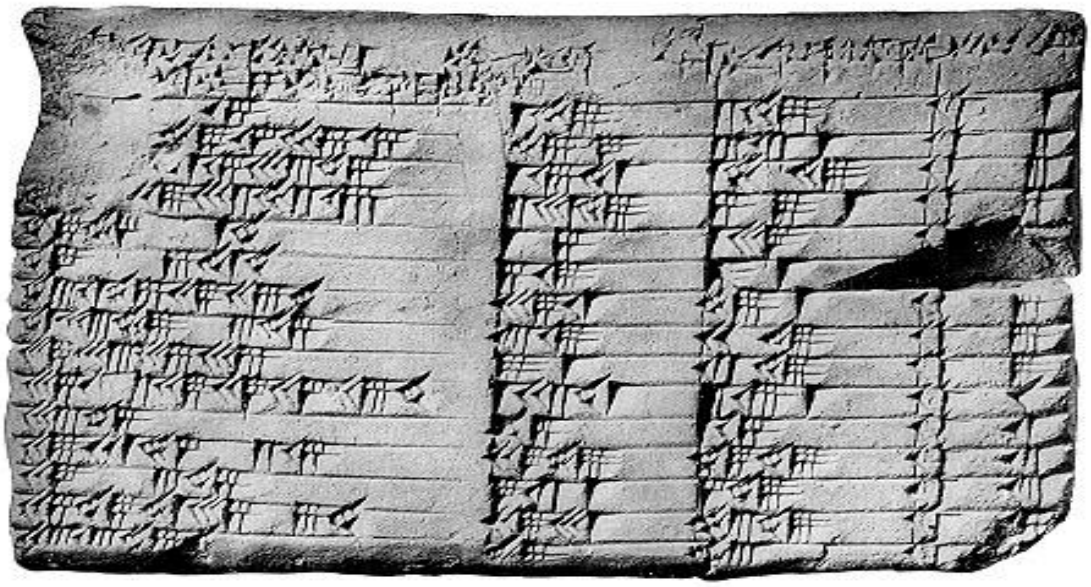

Fig 1.3: Plimpton 322

The famous 322 clay tablet, believed to date from around 1800 B.C., suggests that the Babylonians may well have known the secret of right-angled triangles (that the square of the hypotenuse equals the sum of the square of the other two sides) many centuries before the Greek Pythagoras.[1]

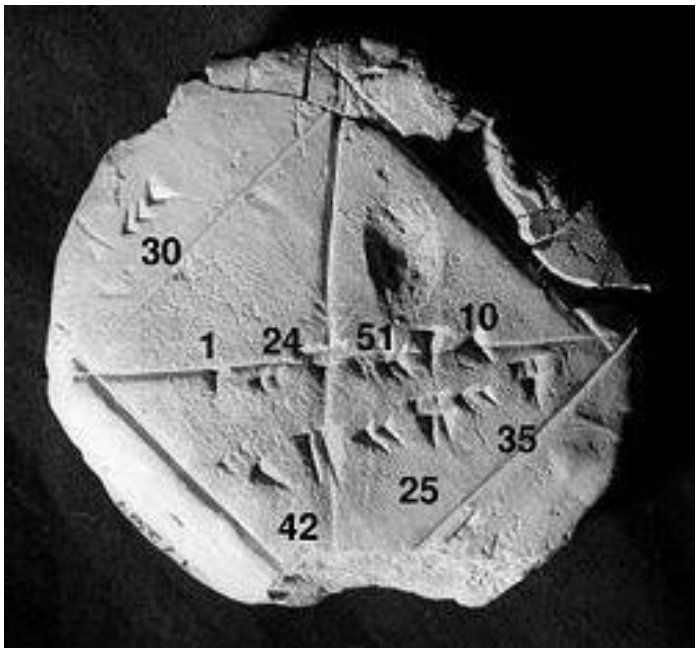

Fig 1.4: Square with both diagonals

[1] Ore, Oyestin, *Number Theory and Its History*, Dover Publications, 1988, p. 176.

An interesting tablet in the collection of Yale University shows a square with its diagonals; on one side is written "30," under one diagonal "42 25 35," and right along the same diagonal "1 24 51 10" (i.e. $1 + 24/60 + 51/60^2 + 10/60^3$). This third number is the correct value of $\sqrt{2}$ to four sexagesimal places (equivalent in the decimal system to 1.414213…, which is too low by only 1 in the seventh place), while the second number is the product of the third number and the first and so gives the length of the diagonal when the side is 30.[1]

In sum, Babylonian mathematics, like that of the Egyptians, is mostly practical but apparently more advanced.

1.7 Greek Mathematics

The origin of Greek mathematics is not documented. Though no direct evidence is available, it is generally thought that the neighbouring Babylonian and Egyptian civilizations had an influence on the younger Greek tradition.[2] The Greek divided the mathematics into two – arithmetic (the study of magnitude or discrete quantity) and geometry (that of magnitude, continuous quantity) and considered both to have originated in practical activities.

Similarly, arithmetic started with the commerce and trade of Phoenician merchants.[3] Inscriptions on stone, for example, reveal use of a numeral system the same in principle as the familiar Roman numerals.

Historians traditionally place the beginning of Greek mathematics proper to the age of Thales of Miletus (ca. 624 - 548 BC). So **Greek mathematics**, developed from the 7th century BC to the 4th century AD around the shores of the Eastern Mediterranean. The word "mathematics" itself derives from the ancient Greek *μάθημα* (*mathema*), meaning "subject of instruction". But the origins of Greek mathematics are not easily documented.[2] The Greek geometer, Thales of Miletus, is credited for giving some logical reasoning (rather than by intuition and experimentation) for several elementary results involving circles and angles of triangles. The next major Greek mathematician is Pythagoras (born ca. 572 B.C.) of Samos.

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1 *Internet Encyclopedia Britannica*, *History of Mathematics* (*www.britannica.com/topic/mathematics/The-numeral-system-and-arithmetic-operations-65970*).

2 Hodgkin, Luke, *A History of Mathematics: From Mesopotamia to Modernity*, Oxford University Press, 2005.

3 *Internet Encyclopedia Britannica*, (*www.britannica.com/topic/mathematics/Geometry*).

Greek mathematics was much more sophisticated than the mathematics that had been developed by earlier cultures. All surviving records of pre-Greek mathematics show the use of inductive reasoning, that is, repeated observations used to establish rules of thumb. Greek mathematicians, by contrast, used deductive reasoning. The Greeks used logic to derive conclusions from definitions and axioms, and used mathematical rigor to prove them.[1] Greek mathematics constitutes a major period in the history of mathematics, fundamental in respect of geometry and the idea of formal proof. Greek mathematics also contributed importantly to ideas on number theory, mathematical analysis, applied mathematics, and, at times, approached close to integral calculus. **Thales** is also credited by tradition with having made the first proof of two geometric theorems - The two earliest mathematical theorems, Thales' theorem and Intercept theorem.

In geometry, **Thales' theorem** states that if A, B and C are points on a circle where the line AC is a diameter of the circle, then the angle ∠ABC is a right angle. It is believed that Thales learned that an angle inscribed in a semicircle is a right angle during his travels to Babylon.[2]

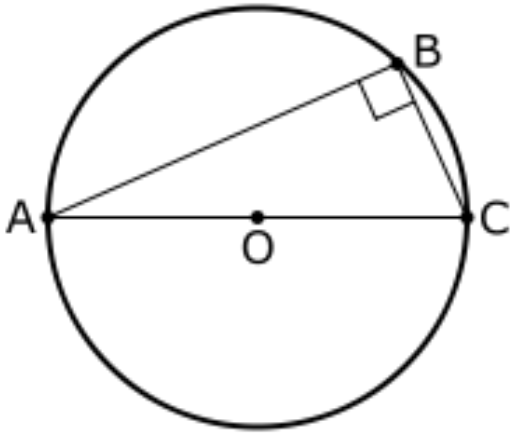

Fig 1.5: Thales Theorem

From the Greeks came a proof of a general rule for finding Pythagorean triplets: if one takes any whole numbers p and q, both being even or both odd, then $a = (p^2 - q^2)/2$, $b = pq$, and $c = (p^2 + q^2)/2$, as Euclid proves in Book X of the *Elements*.

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1 Martin Bernal, *Animadversions on the Origins of Western Science*, pp. 72–83 in Michael H. Shank, ed., *The Scientific Enterprise in Antiquity and the Middle Ages*, University of Chicago Press, Chicago, 2000, p. 75.

2 *A History of Mathematics*, p. 41.

Another important figure in the development of Greek mathematics is **Pythagoras of Samos** (ca. 580 - 500 BC). Pythagoras founded the Pythagorean School of Mathematics in Cortona, a Greek seaport in Southern Italy. Pythagoras is widely credited with recognizing the mathematical basis of musical harmony and, according to Proclus' commentary on Euclid he discovered the theory of proportional and constructed regular solids. Pythagoras believed that numbers govern the universe.

The Pythagoreans also established the foundations of number theory, with their investigations of triangular, square and also perfect numbers (numbers that are the sum of their divisors). They discovered several new properties of square numbers, such as that the square of a number *n* is equal to the sum of the first *n* odd numbers (e.g. $4^2 = 16 = 1 + 3 + 5 + 7$). They also discovered at least the first pair of amicable numbers, 220 and 284 (amicable numbers are pairs of numbers for which the sum of the divisors of one number equals the other number, e.g. the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, of which the sum is 284; and the proper divisors of 284 are 1, 2, 4, 71, and 142, of which the sum is 220). Pythagoreans preached that all numbers could be expressed as the ratio of integers. They discovered that the diagonal of a square is incommensurable with its side, or in modern language, that the square root of two is irrational. It was shocking for them to discover that square root of 2 was not a rational number. According to legend, Hippasus was murdered for divulging it.

Democritus of Abdera (ca. 460-370 B.C.) should also be included with the Eleatic. Democritus was also a pioneer of mathematics and geometry in particular. We only know this through citations of his works (titled On Numbers, On Geometrics, On Tangencies, On Mapping, and On Irrationals) in other writings, since most of Democritus' body of work did not survive the middle Ages. He discovered that the volumes of a cone and a pyramid are 1/3 the volumes of the respective cylinder and prism.

At this time many efforts were made to solve the three great problems of antiquity: doubling the cube, squaring the circle, and trisecting an angle with just a straight edge and compass. **Hippias of Elis**[1] (ca. 460 B.C.) discovered the trisectrix, which he showed could be used to trisect any angle.

According to Proclus, **Menaechmus** (415-369 B.C.) was one of those who ‘made the whole of geometry more perfect’. Menaechmus is remembered by mathematicians for his

[1] http://www.math.tamu.edu/~dallen/history/greekorg/footnode.html#107

discovery of the conic sections and his solution to the problem of doubling the cube.[1] Menaechmus likely discovered the conic sections, that is, the ellipse, the parabola, and the hyperbola, as a by-product of his search for the solution to the Delian problem.[2]

Eudoxus (c. 408 B.C.) was the most famous of all the classical Greek mathematicians and second only to Archimedes. He developed the theory of proportion, partly to account for and study the *incommensurables* (irrationals). He produced many theorems in plane geometry and furthered the logical organization of proof and introduced the notion of *magnitude*.

He gave the first rigorous proof on the quadrature of the circle. He is credited with the development of method of exhaustion which is regarded in precursor of modern integral calculus.

Euclid, fl. 300 BC: His *Elements* is one of the most influential works in the history of mathematics, serving as the main textbook for teaching mathematics (especially geometry) from the time of its publication until the late 19th or early 20th century.[3] Book 7 deals strictly with elementary number theory: divisibility, prime numbers, Euclid's algorithm for finding the greatest common divisor, least common multiple.

The Elements begins with plane geometry, still taught in secondary school as the first axiomatic system and the first examples of formal proof. It goes on to the solid geometry of three dimensions. Much of the Elements states results of what are now called algebra and number theory, explained in geometrical language.[4]

Archimedes (c. 287 B.C. – c. 212 B.C.), regarded as greatest mathematician of antiquity, anticipated modern calculus and analysis by applying concepts of infinitesimals and the method of exhaustion to derive and rigorously prove a range of geometrical theorems, including the area of a circle, the surface area and volume of a sphere, the area under a parabola[5] and he employed it to approximate the value of $\underline{\pi}$ (Pi). In *The Quadrature of the*

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1 Cooke, Roger. *The History of Mathematics : A Brief Course*. New York: Wiley.1997. p. 103.

2 *A History of Mathematics*.

3 Macardle, et al, *Scientists: Extraordinary People Who Altered the Course of History*, Metro Books, New York, 2008, g. 12.

4 Eves, Howard, *A Survey of Geometry*, Allyn and Bacon, Vol. I, 1963, p. 19.

5 O'Connor, J.J. and Robertson, E.F., *Calculus*, University. Archived from the original, 2007.

Parabola, Archimedes proved that the area enclosed by a parabola and its chord is 4/3 times the area of a triangle with equal base and height. He expressed the solution to the problem as an infinite geometric series, whose sum was 4/3. In *The Sand Reckoner*, Archimedes set out to calculate the number of grains of sand that the universe could contain.

Eratosthenes of Cyrene (c. 276 BC-195/194 BC) was a Greek mathematician, geographer, poet, astronomer, and music theorist. He introduced the *sieve of Eratosthenes*, an efficient method of identifying prime numbers.[1]

Apollonius of Perga (262-190 BC) was known as 'The Great Geometer'. The most characteristic product of Greek mathematics may be the theory of conic sections, by Apollonius largely developed in the Hellenistic period (4th century BC). His work *Conics* was written in eight books but only the first four have survived in Greek. It was Apollonius who gave the ellipse, the parabola, and the hyperbola the names by which we know them.

1.8 Mathematics in China

Chinese mathematics in ancient time is defined as the 'art of calculation' (*suan chu*) and covers a vast range of practices and currents of thoughts that grew up in China between the first millennium B.C. and the fall of Manchu Dynasty in 1911.[2] The history of Chinese math and mathematicians was mostly lost or destroyed over the centuries.

In China mathematics was part of the study for bureaucracy. Liu Hui, an inhabitant of the Wei kingdom (one of the three kingdoms of split China around 220 A.D.) and the influential commentator on the *Jiu Zhang, advocated that* mathematics be one of the six subjects taught to recruits for the state bureaucracy.[3] Liu Hui was the greatest mathematician of ancient China who elaborated several mathematical theorems.

The oldest existent work on geometry in China comes from the philosophical Mohist canon of c. 330 BC, compiled by the followers of Mozi (470–390 BC). *Mo Jing* stated that a point is the smallest unit, and cannot be cut in half, since 'nothing' cannot be halved.

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1 El'natanov, B. A. *A brief outline of the history of the development of the sieve of Eratosthenes, Istor.-Mat. ssled. (in Russian).* **27**: 1983, pp. 238–259 (*https://en.wikipedia.org/wiki/Eratosthenes*)

2 *From Five Fingers To Infinity,* ed. by Frank J. Swetz, Open Court, Illinois, 1994, p. 313.

3 *The Crest of The Peacock,* p. 194

The focus of Chinese was more on arithmetic and advanced algebra for astronomical uses. The abacus was first mentioned in the second century BC, alongside 'calculation with rods' (*suan zi*) in which small bamboo sticks are placed in successive squares of a checkerboard.

Some of the mathematical works in China are:

1. *Zhoubi suanjing (The Arithmetical Classic of the Gnomon and the Circular Paths of Heaven)* (500 BC -200 BC).The *Zhou Bi Suan Jing* contains an in-depth proof of the *Gougu Theorem* (a special case of the Pythagorean Theorem) but focuses more on astronomical calculations (Joseph, p.190). It contains various modern mathematical principles such as simple arithmetic operations, working with fractions using a common denominator, and proofs of many geometrical theories. The text contains an accurate process of division for finding out the square root of numbers.
2. *Suanshu Shu* In Han Dynasty (300-150 B.C.), the earliest Chinese mathematical texts were composed, known as *Suanshu Shu* (A book on Arithmetic) which consisted of around 200 bamboo strips. It is by an anonymous author and contains collection of arithmetic problems, including operations with fractions, the determination of proportional payments, the calculation of areas of fields etc. (Joseph: 2011. p.193).
3. *Jiu Zhang Suan Shu (The Nine Chapters on the Mathematical Arts*, 300 BC-200 AD): It is a longest surviving and most influential Chinese math book composed by several generations of scholars. It consists of root extraction; ratios (including the rule of three and the rule of false position), solution of simultaneous equations, areas volumes of various geometrical figures and solids, right-angled triangles.[1]
4. *Liu Hui* (c. 263 A.D.) A significant mathematical advance was made by Liu who wrote his commentary on the *Jiu zhang suan shu* or Nine Chapters on the Mathematical Art. He found approximated value of π^2 using regular polygons with 3×2^n sides inscribed in a circle. His best approximation of π was 355/113($\approx$ 3.14159) which he achieved from a regular polygon of 3072 sides. It is clear that he understood iterative processes and the notion of a limit. Liu also uses Pythagoras's

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[1] *The Crest of The Peacock*, p. 191.

[2] Joseph, Needham, *Science and Civilization in China*: *Mathematics and the Sciences of the Heavens and the Earth*, Cambridge University Press., Cambridge, 1st edn. 1959, reprint, 1970, 1972, 1975, 1979, 1992, 1995, Vol. III, p. 66.

theorem to calculate heights of objects and distances to objects which cannot be measured directly. He Suggested Cavelier's principle to find accurate volume of cylinder.

5. *Xiahou Yang* (about 400 - about 470) was the supposed author of the *Xiahou Yang suanjing* (Xiahou Yang's Mathematical Manual) which contains representations of numbers in the decimal notation using positive and negative powers of ten.
6. *Zhang Qiujian* (430-490 A.D.) wrote his mathematical text *Zhang Qiujian suanjing* (Zhang Qiujian's Mathematical Manual) sometime between 468 and 486. Its 92 problems illustrate the formula for summing an arithmetic progression. Perhaps it is most famous for presenting the 'Hundred fowl's problem' which is an indeterminate problem with three non-trivial solutions.
7. *Zu Chongzhi* (450-500 A.D.) was an astronomer, mathematician, and an engineer. On the basis of his astronomical observations he brought a new calendar and found one year as 365.24281481 days, which is very close to 365.24219878 days as we know today. He determined *pi* to lie between 3.1415926 and 3.1415927, the best approximation till then in the world. His best approximation was between 3.1415926 and 3.1415927, with 355/113 and 22/7 (approximate proportion) being the other notable approximations. He obtained the result by approximating a circle with a 24,576 (= $2^{13} \times 3$) sided polygon. He recommended using $^{355}/_{113}$ as a good approximation and $^{22}/_{7}$ as less accurate one.[1]

History says that there is influence of Indian maths on china due to Buddhist monks travelling to China. Prof Jin Kemu summarized china's indebtedness to India in astronomy, mathematics, medicine and in calendar science in a book[2] published in 1957 in china:

The Chinese were familiar with Indian mathematics, and, in fact, continued to study it long after the period of intellectual intercourse between India and china had ceased.

1.9 Mayan Mathematics

The Mayan civilisation had settled in the region of Central America from about 2000 BC, although the so-called Classic Period stretches from about 250 A.D. to 900 century A.D.

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1 *https://en.wikipedia.org/wiki/Zu_Chongzhi.*

2 Agarwal, M.K., *From Bharat to India,* Chrysee the Golden, iUniverse, Bloomington. Vol. I, 2012, p. 281 *cf.* Sarkar, Benoy, *Hindu Achievements in Exact Science,* Green and Co., New York, London, Longmans, 1918.

Very little evidence is available of this culture to know about it. The chief source of information about Mayans is the writing of Father Diego Delanda (1524-1579) who came to Mexico in 1549. According to available sources Mayan people produced high quality art, architecture and sculpture which needed the knowledge of mathematics, astronomy etc. At its peak, it was one of the most densely populated and culturally dynamic societies in the world.

The Mayans devised a counting system that was able to represent very large numbers by using only 3 symbols, a dot, a bar, and a symbol for zero (shown in the fig 1.6 below), or completion, usually a shell. After the number 19, larger numbers were written in a kind of vertical place value format using powers of 20 like 20, 400, 8000, 160000. It was easy for them to add or subtract using dots and bars. But, there was no fraction. James K. Bidwell (Swetz, 1994) writes that the manuscripts use simple symbols from zero to nineteen in the mixed base system. In some of the manuscripts repeated table of addition are found, which can be called the multiplication table but there was no concept of division. The number would be written from bottom to top unlike today's system from left to right.

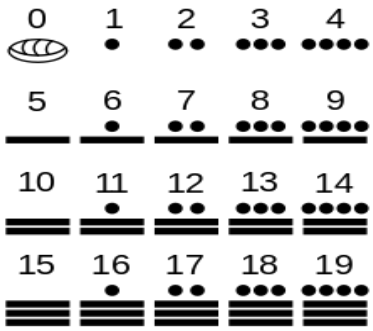

Fig 1.6: Mayan Numerals

A very interesting fact is that the Mayan introduced a variation in the third order. In a perfect vigesimal system, the third term should be 20^2 (=400) but the Maya took 18×20 because 360 was a closer approximation to the length of the solar calendar. In higher orders they continued multiplying by 20. Therefore the first place value is 1; the second, 20, the third 18×20, the fourth is 18×20^2; the fifth 18×20^3, and so forth.

According to C. Gallenkamp (1979) Mayan people used measured sight-lines, or alignments of buildings that served the same purpose to make meticulous records of the movements of the sun, the moon and the planet Venus. They studied solar eclipses in sufficient detail to be able to predict their recurrence.[1]

[1] *The Universal History of Numbers,* p. 297.

The importance of astronomy and calendar calculations in Mayan society required mathematics. The Mayan and other Mesoamerican cultures used a Vigesimal number system based on base 20, probably originally developed from counting on fingers and toes.

Despite not possessing the concept of a fraction, they produced extremely accurate astronomical observations using no instruments other than sticks, and were able to measure the length of the solar year to a far higher degree of accuracy than that used in Europe (their calculations produced 365.242 days, compared to the modern value of 365.242198), as well as the length of the lunar month (their estimate was 29.5308 days, compared to the modern value of 29.53059).[1] Even more fascinating is their use of high numbers for the measurement of time. On a *stela* at Quiriguà in an inscription that mentions the last 5 *alautun*, a period of no less than 300,000,000 years and gives the precise start and end of the period according to the ritual calendar.[2]

The Mayan developed two numeration systems: one for the priests and the other for the common people. These were different in the base value and the symbols to denote the numbers. Numeration used in recording the time by the priest was based on 360 days of the year and hence it was mixed base system using multiples of both 20 and 360 for the place values.[3]

Maya numerals were not invented to deal with the practicalities of everyday reckoning like business or trades but for astronomical observation and the calculation of the time by the priests for their mystical and ritualistic beliefs.

1.10 Indian Mathematics

Indian mathematics emerged in the Indus Valley Civilization from 3000 BC and developed by Jaina, Vedic followed by Buddhist tradition. The earliest evidence of mathematics is found among the ruins of Indus Valley civilization, which goes back to 3000 BC.[4] The classical period of Indian mathematics (400 CE to 1600 CE), in which important contributions were made by scholars like Āryabhata I (476 AD), Varāhmihira (500 AD), Bhāskara I (600-700 AD), Brahmagupta (598 AD), Mahāvīra (850 AD),

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1 *The Universal History of Numbers,* p. 297.

2 *Ibid,* p. 298.

3 *From Five fingers to Infinity,* p. 72.

4 *The Crest of The Peacock,* p. 312.

Bhāskara II (1114 AD), Mādhava of Sangamagrāma (13140-1425 AD) and Nilakaṇṭha Somayā (1445-1545 AD). (In this work however we are more concerned in the period up to 500 AD.). The decimal number system in use today[1] was first recorded in Indian mathematics with zero having place value before 300 AD.[2] In addition, trigonometry was further advanced in India, and, in particular, the modern definitions of sine and cosine were developed there.[3] These mathematical concepts were transmitted to the Middle East, China, and Europe and which became foundation for many branches and further developments of mathematics.

1.11 Pre-Vedic Period of Mathematics (3000-1500 BC)

Excavations at Harappa, Mohenjo-daro and other sites of the Indus Valley Civilisation have uncovered evidence of the use of "practical mathematics". The people of Indus Valley Culture manufactured rectangular bricks whose dimensions were in the proportion 1: 2: 4 and 1: 3: 9[4] considered favourable for the stability of a brick structure. A number of plumb bobs of uniform size and weight are found in Harappa culture. If we consider the weight of plumb bob approximately 27.584 grams as a standard and supposing it 1, we find that standardized system of weights based on the ratios was used: 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, 200, and 500.[5] It strongly indicates existence of the numerate system with well-established centralized system of weights and measures.

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1 The measure of the genius of Indian civilisation, to which we owe our modern (number) system, is all the greater in that it was the only one in all history to have achieved this triumph. Some cultures succeeded, earlier than the South Asian cultures, in discovering one or at best two of the characteristics of this intellectual feat. But none of them managed to bring together into a complete and coherent system the necessary and sufficient conditions for a number-system with the same potential as our own.
The Universal History of Numbers, p. 346.

2 ...our decimal system, which (by the agency of the Arabs) is derived from Hindu mathematics, where its use is attested already from the first centuries of our era. It must be noted moreover that the conception of zero as a number and not as a simple symbol of separation) and its introduction into calculations, also count amongst the original contribution of the Hindus.
Bourbaki, Nicolas, *Elements of the History of Mathematics*, Springer-Verlag, Berlin, Heidelberg, and New York, 1998, p. 46.

3 *Ibid*, p. 126.

4 *Basic Mathematics*, Vol. I, p. 17.

5 *The Crest of The Peacock*, p. 318.

1.12 Mathematics in Vedic Period

Mathematical records are available in various sections of Vedic literature. Vedic literature can be divided in four streams[1]: the *Samhitā* (1000 BC), the *Brāhmaṇa* (800 BC), the *Āraṇyaka* (700 BC) and the *Upaniṣada* "(600-500 BC) [Joseph, 323]. *Taittiriya Saṁhitā, Satapatha Brāhmana* (suggests any date around 4000 BC, 3300 BC, 3000 BC)[2] and *Yajur-veda and Atharva-Veda* (estimated date of both 1200-1000 BC). *Ṛg-Veda* (c. 1500–1200 BC)[3] and *Saṁhitā* include some mathematical work for the rituals. Of these the *Taittiriya Saṁhitā* and *Ṛg-Veda* are considered the oldest and contain rules for the construction of great fire altars. [Joseph, 2000, p. 325] Written in Vedic Sanskrit the Vedic works, Vedas and *Vedāṅga*-s (and later *Sulbasūtras*) are primarily religious in content, but embody a large amount of astronomical knowledge and hence a significant knowledge of mathematics.

The solution to fraction was known to the *Ṛgvedic* People as states as:

tri-pāda-ūrdhva udaita-pūruṣah pādo-asyehā-bhavatpunah
tato viṣvaṅga vyakrāmat-sāśanānaśane abhi (Ṛgveda, 10.90.4)[4]

i.e. The three feet of the *puruṣa* is raised high above (in transcendental realm), and His one foot becomes the creation again and again.

It means *puruṣa sūkta* with three-fourths *puruṣa* went up: one-fourth of him again was here.

Yajur-Veda gives special names for the power of ten from one 10^0 (*eka*) to trillion 10^{12} (*parārdha*). Rāmāyaṇa, one of the most respected Hindu epic, mentions the very large numbers for the army of Rāvaṇa. He has in total $10^{12} + 10^5 + 36(10^4)$ =1,000,000,460,000. While the Rāma,[5] the hero of the epic in opposition has the larger army which is counted as $10^{10} + 10^{14} + 10^{20} + 10^{24} + 10^{30} + 10^{34} + 10^{40} + 10^{44} + 10^{52} + 10^{57} + 10^{62} + 5$ [Joseph, 2000, p. 340].

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1 Bag, A.K, *Mathematics in Ancient and Medieval India,* Chaurkhamba Orientalaya, Varanasi, 1st edn., 1979, pp. 3-4.

2 *The Crest of The Peacock*, p. 368.

3 *Ibid*, p. 4.

4 *Ṛgveda Mantra Saṁhitā*, ed. by R. L. Kashyap and S. Sadagopan, Sri Aurobindo Kapali Shastry Institute, Banaglore, 1998.

5 *The Crest of The Peacock*, p. 340.

In the Vedic period, records of mathematical activity are mostly to be found in Vedic texts associated with ritual activities. To some extent early mathematical developments in India mirrored the developments in Egypt, Babylon and China as it was impelled by social considerations. The requirement for mathematics was (at least at first) twofold, as R Gupta discusses:

...The need to determine the correct times for Vedic ceremonies and the accurate construction of altars led to the development of astronomy and geometry.

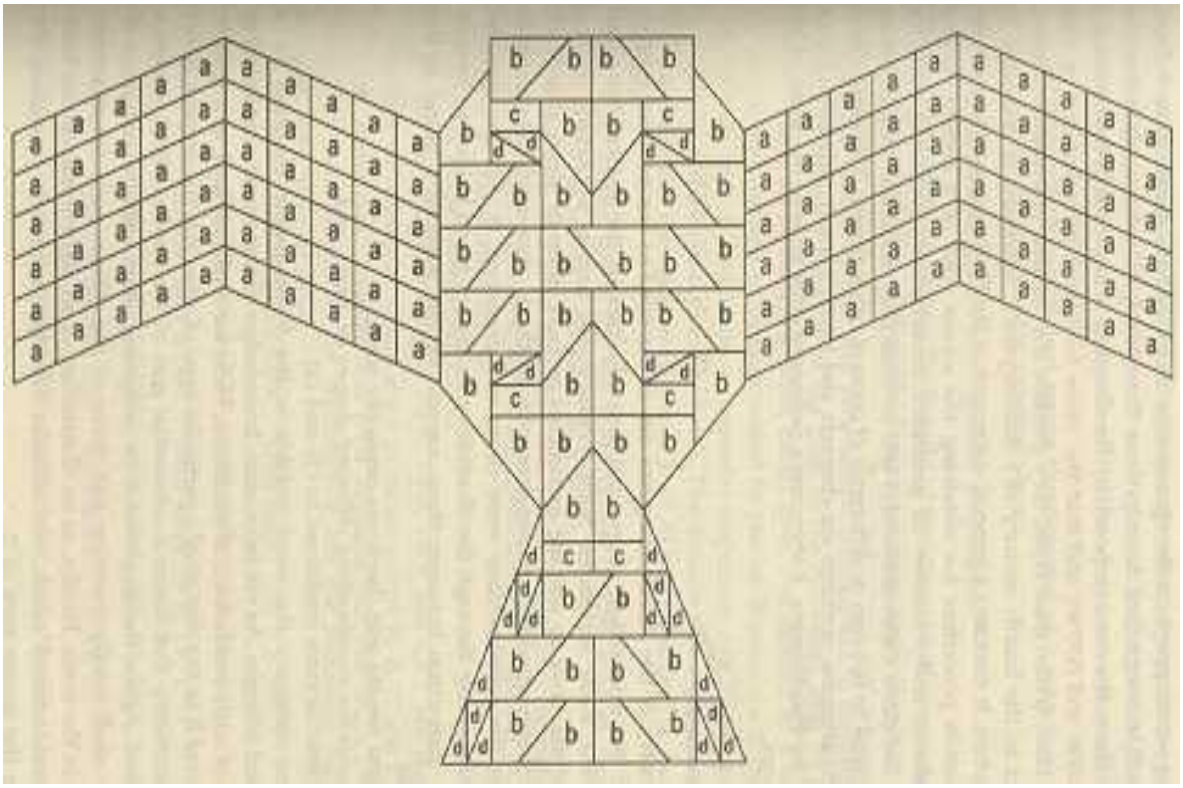

Figure 1.6: First layer of a Vedic sacrificial altar (in the shape of a falcon).

Geometric use (*rekhā-gaṇita*) is found in *Sulba-Sūtra*-s of Baudhāyana (800 BC) and Apasthmaba (600 BC) in which the techniques of the construction of altars used during the Vedic era are described.

Baudhāyana (c. 8th century BC) composed the *Baudhāyana Śulba Sūtra*. It list rules for the construction of sacrificial fire altars.[1] *Śulba Sūtra* contains examples of simple Pythagorean triples, such as:(3, 4, 5), (12, 5, 13), (15, 8, 17), (7, 24, 25), and (12, 35, 37)[2] as well as a

[1] Staal, Frits, *Greek and Vedic Geometry*, Journal of Indian Philosophy, 1999, Vol. 27 (1–2), pp. 105–127.

[2] *Mathematics in Ancient and Medieval India*, p. 123.

statement of the Pythagorean theorem for the sides of a square: "The rope which is stretched across the diagonal of a square produces an area double the size of the original square." Baudhāyana gives a formula for the square root of two[1] :

$$\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34} \approx 1.4142156$$

The formula is accurate up to five decimal places, the true value being 1.41421356…

Kātyāyana (c. 3rd century BCE) is the last of the Vedic mathematicians. He wrote the *Kātyāyana Śulba Sūtra*, which presented much geometry, including the general Pythagorean theorem and a computation of the square root of 2 correct to five decimal places.

One of the prominent scholars of the post-Vedic period, who contributed to mathematics, is **Pingālā** (fl. 300–200 BCE); a musical theorist who authored the *Chaṅdas śāstra* , a Sanskrit treatise on prosody. Pingālā stumbled upon both the Pascal triangle (called *mātrāmeru*) and Binomial coefficients, although he did not have knowledge of the Binomial theorem itself.[2] *Pingālā's* work also contains the basic ideas of Fibonacci numbers. Although the *Chaṅda sūtra* hasn't survived in its entirety, a 10th-century commentary by Halāyudha is available (1000 AD).

Āryabhaṭṭa I (476–550 A.D., Kusumpura, Bihar), a great mathematician of the Gupta period, wrote the *Āryabhāṭiya*. He described the important fundamental principles of mathematics in 32 *śloka*-s. The treatise contained: alphabet-numeral system of notation, rules for arithmetical operations, methods of solving simple and quadratic equations, indeterminate equations of the first degree, sine function of trigonometry, the value of π (correct to 4 decimal places), and general rule of computing the sum of natural numbers and their square and cubes.[3] He invented a new system (*Kāṭapādiya*) capable of representing very large numbers. He developed solutions of simultaneous quadratic equations etc. in Algebra. He provided a systematic treatment of the position of the planets in space. He was

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1 Cooke, Roger, *The History of Mathematics: A Brief Course*, Wiley, New York, 2005.
cf. Mathematics in Ancient and Medieval India, p. 127.

2 Fowler, David, *Binomial Coefficient Function*, The American Mathematical Monthly, 1996, Vol. **103** (1), pp. 1–17.

3 *The Crest of The Peacock*, p. 374.

the first Indian to state that earth is spherical and rotates about its axis. He found accurate calculations for astronomical constants, such as the solar eclipse and lunar eclipse.[1]

Varāhamihira (505–587 A.D.) is remembered for his trigonometric tables in his text *Pañca Siddhānta* (*The Five Astronomical Canons*). He made important contributions to trigonometry, including sine and cosine tables to 4 decimal places of accuracy. His works on permutations and combinations complemented what had been previously achieved by Jaina mathematicians and provided a method of calculation of ${}^{n}C_{r}$ that closely resembles the much more recent *Pascal's Triangle*.

1.13 Jaina Mathematics

Jaina mathematicians are important historical link between the mathematics of the Vedic period and that of the "Classical period." (500 A.D. - 1600 A.D.). As per Jaina tradition, the 24th *Tīrthaṅkara* Lord Mahāvīra himself had been a mathematician. During the period of the *Brāhmaṇa*-s, the maths was a tool for constructing altars using geometrical knowledge to perform the rituals. According to Joseph the credit for giving maths the form of an abstract discipline goes to the Jaina. Jaina mathematics emerged purely for philosophical and practical purposes. N. Dwary says: ...*According to the religious literature of the Jaina, the knowledge of "saṅkhyāna"* (*i.e., the science of numbers, which included arithmetic and astronomy*) *was considered to be one of the principal accomplishments of Jaina priests.* [ND1, P 39]

Philosophical doctrines also had a profound influence on the development of mathematical concepts and formulations. In Śvetāmbara tradition in addition to *Sūryaprajñapti* (3rd century BC) an important Jaina works on astronomy and mathematics, *Jambūdvīpa Prajñapti* (3rd century BC) explains geographical feature, the *Sthānāṅga Sūtra* (fl. 300 BCE – 200 CE); the; *Bhagavatī Sūtra* (3rd C. BC), *Anuyogadvāra Sūtra* (fl. 200 BCE – 100 CE), and many more are significant from mathematical viewpoint and in Digambara tradition the *Ṣaṭkhanḍāgama* (c. 2nd century BC). *Kaṣāyapāhuḍa, Lokavibhāga, Tiloyapaṇṇatti, Dhavalā* and *Jayadhavalā* deal with mathematics comprehensively. In *Āgama Bhagavatī Sūtra* and *Anuyogadvāra Sūtra* are the masterworks of mathematics

[1] Katz, Victor J., *Ideas of Calculus in Islam and India*, Mathematics Magazine, 1995, Vol. 68(3), pp. 163–174.

which cover plenty of areas of mathematics which will be discussed in the succeeding chapters. Besides JJ, PS, US etc. have considerable mathematical content.

The concept of infinity by Jaina is unique. They were the first to conceive of transfinite numbers, a concept was brought to Europe by Cantor in the late 19th century. The ideas of the mathematical infinite in Jaina mathematics are very interesting indeed and were evolved due to the Jaina's cosmological and metaphysical ideas. In Jaina cosmology cosmos and time both are eternal, infinite and without form. The universe (*loka*) is finite and non-universe (*aloka*) is infinite; it was never created and has always existed. Space pervades everything and is without form. The life span of the living beings especially gods and hellish beings led to a deep interest in very large numbers and definitions of infinite numbers. Infinite numbers were created through recursive formulae, as in the *Bhagavaī* and *Anuyogadvāra Sūtra*. In particular, their fascination with the enumeration of very large numbers and infinities led them to classify numbers into three classes: numerable (*saṅkhyāta*), innumerable (*asaṅkhyāta*) and infinite (*ananta*). Like Cantor they conceived the idea of hierarchy of infinities. In an alternate treatment, they classified five different types of infinity: the infinite in one direction, the infinite in two directions, the infinite in area, the infinite everywhere, and the infinite perpetually.[1]

In addition, Jaina mathematicians devised notations for **simple and fractional powers** (and exponents) of numbers like squares and cubes and Ācārya Mahāvīra (850 A.D.) even gave detailed treatment of indeterminate linear equation and fundamental arithmetical operations.[2] The credit for first use of zero also goes to Jaina Mathematicians.

Ideas about infinitesimals

As Jaina *ācārya*-s were dealing with very large numbers in terms of innumerable and infinities, they were also conversant with concept of very small, i.e. infinitesimals. They came across very small size of the living beings and the space occupied by them which is very small part of the existing measure-scale.

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1 *Basic Mathematics*, Vol. I. p. 12.

2 *Mathematics in Ancient and Medieval India*, p. 23.

The term Infinitesimal here is used to count the size of various beings. For an example, *sūtra* 1506.1-1506.7 and 1507.1-1507.3 state that the minimum height of plant-bodied living (*paryāpta*, *aparyāpta*, and gross) with *audārika* body is infinite-th[1] part of an *aṅgula*.

Which can be expressed as, a_{∞}/I (where I is *ananata*).

Theory of numbers: There is a great fascination in Jaina philosophy with the enumeration of large numbers. Selected examples of 'time periods' mentioned start with $(84000000)^2$ $=7056\times10^{10}$ days, and goes up to $(8400000)^{28}$ called *śīrṣaprahelikā*,[2] and 194 digits number in decimal scale. It seems that the aim of Jaina scholars had been to explain the every detail of *karma* theory through use of mathematics for which they defined many classes of numbers and more interesting they conceived of analogy measures for the time-period human beings are unable to count.

Also found in Jaina works are knowledge of the fundamental laws of indices (both integral and fractional), arithmetical operations, sequences–arithmetic progressions, geometric progressions, geometry, operations with fractions, rule of three, operations with logarithms.[3] Permutations and combinations was the favourite topic for Jaina-s to study and explain the principles of *saptabhaṅgī* (seven-fold standpoints) and *syādvāda* (qualified assertion), so dear to Jaina philosophers. Some concepts very similar to occupancy problem, modular arithmetic and probability theory are found in certain *āgamic* works. Advanced concepts like partition function in context similar to that in modern atomic physics are discussed in *Bhagavatī*. We will explain all these in subsequent chapters.

1.14 Conclusion

Each civilization has unique contribution in the field of mathematics like unit fraction of Egyptian, sexagesimal of Babylonians, mathematical proofs of Greeks etc. Similarly, Jaina developed unique concepts of innumerable, infinites, exponentiation, binomial coefficients, partition theory, probability theory etc. to meet their philosophical and metaphysical requirements.

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1 *Prajnāpanā Sūtra,* ed. by Yuvācārya Miśrīmal ji Madhukar, with Hindi translation and appendices, Agam Prakashan Samiti, Beawar, 1st edn., Vol. I, 2nd edn. 1993, p. 430.

2 *Bhagavaī,* Chief Synod Ācārya Tulsī, ed. by Ācārya Mahāprajña, with text, Sanskrit commentary, and Hindi translation, Jain Vishva Bharati, Ladnun, Vol. I, 2000. p.287.

3 *The Crest of The Peacock*, p. 349.

Chapter-2

Mathematics in Jaina Heritage: *Āgama*

Without mathematics, there's nothing you can do.

Everything around you is mathematics.

Everything around you is numbers.

~ Shakuntala Devi ~

Jainism is mainly a philosophy and science of soul (*ātmā*) which in its pristine form is equated with knowledge (*jñāna*). Hence metaphysics, especially ontology and epistemology became chief concern of Jaina saint scholars. As a result they started meditating deeply upon the cosmology, cosmogony and cosmography and some force '*karma*' that keeps the cycle of existence going on. They found that all the phenomenon can be explained clearly and accurately only based on mathematics. Thus, mathematics became a major plank in disquisition of various tenets. Mathematics became an important aspect of studies for monks. Thus, in Jaina religion, even philosophical concepts are explained mathematically.

Realization that right from miniscule fungus to a most developed human being are endowed with life force – capable of perception and cognition from an infinitesimal degree to omniscience – gave rise to respect for all living beings and this gave birth to the supreme principle of non-violence (*ahiṁsā*). This made them think in probabilistic terms which gave rise to development of permutations and combinations, occupancy problem of probability theory. For existence of species and their perpetration they built an elaborate theory of *karma* which governs the actions of living being by bondage of *karma* particles with the soul. This made them invent processes of growth, decay and annihilation of such bondages which made them invent calculus like processes of *ṣaṭsthānapatita vṛddhi* or *hāni* (6-stage increase or decrease).

Naturally, they started thinking of universe its genesis (cosmogony), evolution (cosmology) and its structure including our (earth's) place in it – cosmography and

geography of *Jambuūdvīpa*. They, thus developed mensuration for liner dimensions, areas, volumes of different entities in it. The enormity of universe (*loka*) emboldened them to think of colossal numbers and even they developed a theory of hierarchy of innumerable and infinites, which resembles with Cantorian thinking on infinite sets.

When they further realized that the universe is teaming with inanimate matter (which is even in infinite times of living beings), they meditated upon the nature of matter (*pudgala*) and stumbled on quite advanced theories like partition theory, fission and fusion of atoms and molecules.

In this chapter and throughout the thesis, we attempt to bring out the mathematical ideas developed in the oldest of Jaina scriptures, *Āgama,* composed during a long period from the late 6th C .BC to 5th C. A.D.

2.1 Mathematics in *Āgama*

Mathematics occupies a special place in Jaina religion. Their canonical literature, as per Śvetāmbara tradition was divided in 4 streams: 1) *caraṇa-karaṇānuyoga* (code of conduct), 2) *dharmakathānuyoga* (ethics), 3) *gaṇitānuyoga* (mathematics), and 4) *dravyānuyoga* (metaphysics). Its cosmology and cosmogony, logic and metaphysics etc. required strong training in mathematics on part of its spiritual leaders (*Ācārya*-s). Its cosmology required expertise in geometry of circles, annular series of type $f\{\ 2^{2^n}\}$, and in dealing with very large numbers. Its *karma* theory was instrumental in development of theory of transfinite and infinitesimals. The earliest evidence of decimal place value system with zero having place value can be traced to Jaina literary sources only. Its logic (*nyāya*), especially the concepts of *anekāntavāda* (theory of multi-dimensional truth) *saptabhaṅgī* (seven-fold statements) and *syādvāda* (theory of conditional dialect) enabled Jaina scholars to develop theory of permutations and combinations. Its ideas about transmigration of souls based on a unique *karma* theory were responsible in enabling them to tackle occupancy problem of probability theory. Lastly, its Atomic theory made them grapple with 20th century concepts of partitions in modern number theory.[1]

[1] Shah, R.S., Lure *(Lore) of Large Numbers,* Arhat Vacana, Vol. 21, No. 4, 2009, pp. 61-78.

2.2 Antiquity of Jainism

India has been the land of spirituality. There have been two prime streams of Indian religions- *Śramaṇa* and *Vedic*. Jainism and Buddhism fall under the first one. Jainism is one of the oldest religious orders under *Śramaṇa* tradition. The roots of Jainism go up to prehistoric age. In this time cycle, Bhagawāna Ṛṣabha was the first *tīrthaṅkara* (preceptor, who establishes the four-fold order) thousands of years ago. After finding the ruins from the excavation of Harappa and Mohenjo-Daro some scholars accepted that Jainism existed in prehistoric age, may be not under the label of Jainism [Doshi,19]. Historians like Heinrich Zimmer,[1] Thomas McEvilley[2] are of the opinion that there exist some link between First Jaina *tīrthaṅkara Ṛṣabha* and Indus Valley Civilization. Zimmer writes that

… there is truth in the Jaina idea that their religion goes back to remote antiquity, the antiquity in question being that of the pre-Aryan, so called Dravidian period, which has recently been, dramatically disillusioned by the discovery of a series of Great Late Stone Age cities in the Indus Valley, dating from the third and even perhaps fourth millennium B.C. [Zimmer, p. 60].

Major J.G.R. Furlong says:

From unknown times there existed in India a highly organized Jaina religion from which later on developed Brahmanism & Buddhism Jainism was preached by 22 tīrthaṅkara before the āryans reached the Ganges. (Short studies in the science of comparative religion)

Almost all Jaina literatures are focused on the spiritual upliftment and attaining liberation. It is so interesting that in spite of being an ancient religion it gave importance to mathematics in all ages.

2.3 Why Mathematics in Spirituality?

It's a genuine question, why did mathematics become necessity for the Jaina saints for the advancement of incorporeal soul. The counter question can be posed how could without

[1] Zimmer, Heinrich, Philosophies of India, ed. by Campbell, Joseph, Princeton University Press, New York, 1969, pp. 60, 208-209.

[2] Thomas, McEvilley, *The Shape of Ancient Thought: Comparative studies in Greek and Indian Philosophies*, All Worth Communication Inc., 2002.

mathematics the philosophy of soul be explained. The concept of *karma* becomes inevitable while talking about soul. How long any *karma* can stay with soul? Does fruition appear immediately or is there any dormant period (*abādhā kāla*) also? The duration (*sthiti*) of *karma* could never be explained without mathematics. For example the maximum duration of pain-pleasure producing (*vedanīya*) *karma* is 30 *koḍākoḍī* (10^{14}) *sāgaropama* and of deluding (*mohaniya*) is 70 *koḍākoḍī* (10^{14}) *sāgaropama* (BS 6.3.34)[1] etc. The dormant period (*abādhā kāla*) for such duration of the *karma* is 3000 and 7000 years respectively. It means if duration of the bonded *karma* is 30×10^{14} *sāgaropama* then it will be passive and not give fruit for 3000 years. Furthermore, it was necessary to explain for the concept of destruction of *karma*-s through penance. They need to develop calculus to measure the exponential growth or decay in *karma* particles and duration which is explained as exponential growth (*ṣaṭsthāna vṛddhi*) and exponential decay (*hāni*).

Knowing the life-durations of living beings was part of the spiritual life of Jaina saints. Therefore, Jainism is always fascinated in describing various aspects of soul (*jīva*) like types of species, size, residence, number of rebirths, life-span etc. To explain the time cycle mathematical calculations were indispensable. The study of Jaina cosmology to know the system of the universe was their subject. How the system of cosmos is regulated? What are the disciplines or norms of this universe? Where do the living beings reside? How many space points are there in cosmos? Where the place of rebirth is located? How many islands and oceans are in universe? Furthermore, meditation is the core part of Jaina *sādhanā* (spiritual practices). Along with Jaina monks lay-followers (*śrāvaka*) too were recommended to meditate on the nature of universe (*lokānuprekṣā*). For such explanations knowledge of very large numbers and transfinite numbers was essential. Thus, cosmology became instrumental in the development of geometry of circle. To measure and figure out the location, dimensions, volume, area of islands, ocean, heavens, hells, mountains, rivers etc. the measure-scales and the treatment of mensuration was also essential.

Permutation and combinations are very important part of Jaina mathematics known as *bhaṅga samutkīrtana* (which can be translated as modern combinatorics) used to explain various possible stages of soul under influence of passions (*kaṣāya*), psychic colour (*leśyā*)

[1] *Bhagavaī,* Synod Ācārya Tulsī, ed. by Ācārya Mahāprajña, with text & Sanskrit commentary, Hindi translation, Jain Vishva Bharati Institute, Ladnun, Vol. II, 2004, pp. 245-246.

etc. To describe the relative number, size of living beings, the concept of maxima and minima has been employed.

Auspicious practices or rites at auspicious time bring success - it is an evergreen belief. To perform the religious activities perfectly knowing about the time was a necessity because Jaina spiritual practices are at some extent dependent upon sunrise, sunset and the motion of the moon (*tithi*). For example, what time a monk should do *swādhyāya* (scriptural study), meditation and when should he go for alms and take his food?[1] What are the sacred days? What can be the auspicious days and hours for the spiritual practices like penances? Such requirements gave birth to Jaina astronomy where the movement of sun, moon, and other heavenly bodies and planets were required to be calculated. In this way, mathematics was a prerequisite to explain the Jaina principles and leading for a spiritual life.

2.4 *Āgama*: The Treasure of Wisdom

Āgama or Jaina canonical literature is a treasure of wisdom. It can be said as an encyclopaedic in nature as it includes various topics like philosophy, history, social and economic principles, cosmology, mathematics, astronomy etc. *Tīrthaṅkara*, delivers the spiritual discourses for his disciples and the followers in the common public language. The one who has attained omniscient knowledge (*kevala jñāna*) and established four fold order of monk (*sādhu*), nuns (*sādhvī*), layman (*śrāvaka*) and lay-woman (*śrāvikā*) is known as *tīrthaṅkara*.[2] The doctrines in the form of preaching by each *tīrthaṅkara*, from Bhagawāna Ṛṣabhadeva to Mahāvīra, are known as *Āgama*. At present the available doctrines and the canonical texts are the pious preaching of Bhagawāna Mahāvīra (527-599 BC) the 24th *tīrthaṅkara*. *Āgama* were preached by *tīrthaṅkara* and vocalized by *gaṇadhara*[3] (the chief disciple of *tīrthaṅkara*). *Gaṇadhara*-s Gautama, Sudharmā etc. would compile them verbally in the form of prose and verse in *ardha-māgadhī* (*prākṛta*) language. Those preaching are called *aṅga* and form part of *Āgama*. These are *aṅga*-s twelve in number.

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[1] *Uttarajhayaṇāṇi,* Synod Ācārya Tulsī, ed. by Ācārya Mahāprajña, with text & Sanskrit commentary, Hindi translation, Jain Vishva Bharati, Ladnun, 5th edn., 2006, p. 412, (*sūtra* 26.12).

[2] *Bhikṣu Āgama Viṣaya Śabda Kośa,* Synod Ācārya Tulsī, ed. by Ācārya Mahāprajña, Jain Vishwa Bharati Institute, Ladnun, 1996, p. 301.

[3] *suttaṁ bhāsati arahā, suttaṁ gaṅthanti gaṇaharā niuṇaṁ* (*gāthā* 86)

Āvaśyaka niryukti of Ācārya Bhadrabāhu, ed. by Samani Kusum Prajñā, JainVishwa Bharati Institute, Ladnun, 2001, p. 34.

In *Samavāyāṅga* (chapter 1, *sūtra* 2) it is said that Bhagawāna Mahāvīra propounded twelve types of canons known as *dvādaśāṅga* (*gaṇipiṭaka*).[1] It means twelve limbs or the treasure for the *gaṇadhara* or *Ācārya*.

These *aṅga*-s are the nucleus of *Āgama* as they contain original preaching of Bhagawāna Mahāvīra as mentioned in *Samavāo* (1.2).[2] These 12 *aṅga*-s altogether are known as *dvādaśāṅga* (twelve *aṅga*-s).The twelve *aṅga*-s are:

1. *Āyāro* (*Ācārāṅga*),
2. *Suyagaḍo*(*Sutrakṛtāṅga*),
3. *Thāṇaṁ* (*Sthānāṅga*),
4. *Samvāo* (*Samvāyāṅga*),
5. *Bhagavaī* (*Bhagavatī Sūtra*),
6. *Nāyādhammakahāo* (*Jñātā-dharma-kathā*),
7. *Uvāsagadasāo* (*Upāsakadaśā*),
8. *Aṅtagaḍadasāo* (*Antakṛta-daśā*),
9. *Anuttarovavāiyadasā* (*Anuttaraupa-pātikadaśā*),
10. *Paṇhāvāgaraṇa* (*Praśna-vyākaraṇa*)
11. *Vivāgasuya* (*Vipāka-Sūtra*)
12. *Diṭṭhivāo* (*Dṛṣṭivāda)*

It is believed and recognized that *Dṛṣṭivāda* (mainly comprised of 14 *pūrva*-s) is extinct now. The *pūrva*-s were too difficult to understand for a common monk. Therefore, in simplified form 11*aṅga*-s were explained and then formulated.[3] The remaining *aṅga*-s being the original teachings of Bhagawāna Mahāvīra were composed around the end of 6th century BC. But scholars believe that the available *aṅga*-s are not the original but compiled, edited and revised by the *Ācārya*-s from time to time. Under the presidentship of

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[1] *Samavāo,* p. 33.

[2] *Ibid*, p. 1.

[3] *jati vi ya bhūtāvāde savvassa vayogatassa otāro,*
ṇijjūhaṇā tadhā vi hu dummedhe pappa itthī ya (*gāthā*, 548).

Viśeṣāvaśyaka bhāṣya of Jinabhadragaṇi Kṣamāśramaṇa, preceptors Ācārya Mahāprajña and Ācārya Mahāśramaṇa, trans. by Sādhvī Muditayaśā in Hindi, Jain Vishva Bharati, Ladnun, Vol. I, 2014, p. 131.

Devardhigaṇī Kṣamāśramaṇa the *aṅga*-s were revised fourth time. Dr Jacobi believes that the *aṅga*-s can be older than end of 4th century and beginning of the 3rd century.[1]

In the course of time, scholar-saints composed some literature with detailed explanation for the understanding of disciples. These were in line with principles propounded by Bhagawāna Mahāvīra, and therefore accepted as *Āgama*. When the number of *Āgama* kept on increasing then to differentiate between the original preaching and supporting texts it became necessary to identify them with different names. The new classification is found in *Anuyogadvāra Sūtra* (4th C. A.D.) and *Nandī Sūtra* (453 A.D.) which are included in *Āgama*.[2]

2.5 *Upāṅga*: Later form of *Āgama*

In *sūtra* 43 of *Nandīsūtra* a new classification of *Āgama* goes as : 1. *aṅgapraviṣṭa* 2. *aṅgabāhya*.[3] *Aṅgapraviṣṭa* is *dvādaśāṅga* itself. *Aṅgabāhya*, in fact, is later creation after Bhagawāna Mahāvīra by subsequent scholarly *Ācārya*-s.

The work created by later erudite scholar *ācārya*-s was also accepted as *Āgama* but with different title as *aṅgabāhya* or *upāṅga*-s viz.

Twelve *upāṅga-s* :

- *Uvvāvaye* (*Aupapātika Sūtra*) ∗
- *Rāyapaseṇīya* (*Rājapraśnīya Sūtra*)∗
- *Jīvājīvābhigamopāṅgaṁ* (*Jīvājīvābhigama Sūtra*)∗
- *Paṇṇavaṇā* (*Prajñāpanā Sūtra*)∗
- *Jaṁbūdīvapaṅṅatti* (*Jambūdvīpaprajñapti Sūtra*)∗
- *Candrapaṇṇatti* (*Candraprajñapti Sūtra*)∗

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[1] Jacobi, Herman, *Sacred Book of the East* (*Jaina Sūtra*-s), Oxford University Press, Oxford, Vol. 22, 1984, p. 31, (preface).

cf. Doctrines of Jainas. pp. 73, 81.

[2] *Anuogadārāiṁ*, chief synod Gaṇādhipati Tulsī, ed., trans. and commentary by Ācārya Mahāprajña, with original text, Sanskrit and Hindi, Jain Vishva Bharati Institute, Ladnun, 1996, p. 11, (preface).

[3] *Nandī Sūtra*, chief synod Gaṇādhipati Tulsī, ed., trans. and commentary by Ācārya Mahāprajña, with text Sanskrit and Hindi, Jain Vishva Bharati Institute, Ladnun, 1997, p.115.

- *Sūrapaṇṇatti* (*Sūryaprajñapti Sūtra*)∗
- *Nirayāvaliyāo* (*Niryāvalikā Sūtra*)∗
- *Kappavadaṁsiā* (*Kalpavasantika Sūtra*) ∗
- *Pupphiyā* (*puṣpikā Sūtra*)∗
- *Pupphacūliyā* (*Puṣpacūlikā Sūtra*) ∗
- *Viṇhidaśā* (*Vṛṣṇidaśā Sūtra*)∗

The authors of all the *upāṅga*-s are not known. Later some more texts, the composition of erudite *Ācārya*-s were also included in *Āgama*. Hence, number of the ancient Jaina canonical literatures went up to 45 which include 11 *aṅga*-s (the twelfth one *Dṛṣṭivāda* is believed as lost), 12 *upāṅga*-s, 4 *Mūla Sūtra*, 6 *Cheda Sūtra*, 2 *Cūlikā Sūtra*, and 10 *Prakīṛṇaka*.

Six *Cheda Sūtra*-s (texts relating to the conduct and behaviour of monks and nuns)

- *Ācāradaśāh (Daśāśrutaskansha)*∗
- *Bṛhatkalpa Sūtra*∗
- *Vyavahāra Sūtra*∗
- *Niśītha Sūtra*∗
- *Mahāniśītha Sūtra*
- *Jītakalpa*

Four *Mūlasūtra*-s (Scriptures which provide a base in the earlier stages of the monkhood)

- *Dasveāliyaṁ* (*Daśavaikālika*)∗
- *Uttarajjhayaṇāṇi* (*Uttarādhyayana Sūtra*) ∗
- *Āvaśyaka*∗
- *Piṇḍaniryukyti*

Ten *Prakīṛṇaka sutra*-s (Texts on Independent or miscellaneous subjects)

- *Catuhśarana*
- *Āturapratyākhyanā*
- *Bhaktaparijñā*
- *Saṁstāraka*

- *Tandulavaicārika*
- *Candravedhyāka*
- *Devendrastava*
- *Gaṇividyā*
- *Mahāpratyākhyanā*
- *Vīrastava*

Two *Cūlikasūtra*-s (The scriptures which further enhance or decorate the meaning of *aṅga*-s) :

- *Nandī Sūtra*∗
- *Anuyogadvāra Sūtra*∗

In Śvetāmbara tradition all the sects are not unanimous about the number of works forming *Āgama*. The idol-worshiper (*mūrtipūjaka*) sect accepts all 45 works whereas *Sthānakavāsī* and *Terāpnathī* believe in 32 works including all 11 *aṅga*-s and other 21 works (in the list aforementioned marked with the sign of asterisk∗) excluding 10 *prakīrṇaka*, last two of *Cheda Sūtra*, and one *Mūla Sūtra*.[1]

Cūlikā Sūtra Anuyogadvāra is supposed to be authored by Āryarakṣita and his period was around beginning of the first century. *Nandī* was believed to be created by Devavācaka (Devardhigaṇī Kṣamāśramaṇa) and its time period goes around 453 A.D.

2.6 Availability of Contemporary *Āgama*

In the age of Bhagawāna Mahāvīra there were no means and tradition of writing. Initially it was oral tradition of transferring the knowledge from one generation to another by the learned *ācārya*-s. Culture of writing and printing was not the part of that era when memory was helping to preserve the knowledge. *Caturdaśa pūrvadhara* (*ācārya*-s having knowledge of 14 *pūrva*-s of twelfth *aṅga*) and 8 *śrutakevalī*-s (quasi-omniscient), the *ācārya*-s of high calibre, after emancipation of Bhagawāna Mahāvīra, were carrying this knowledge of *Āgama*. Thereafter, when the faculty of memorization started deteriorating, the question of preserving of *Āgama* arose. Did *Āgama* survive after that period? Was there

[1] Doshi, Bechardas, *Jaina Sāhitya kā Bṛhad Itihāsa,* Parshvanath Vidyashram Shodh Sansthan, Varanasi, Vol. I, 1966, pp. 27-28.

any way to keep those treasures secured? Here come two different views of two traditions of Jaina school Śvetāmbara and Digambara. The former says *aṅga*-s except the twelfth *Dṛṣṭivāda* are surviving till today whereas in the views of latter, after 683 years[1] of the emancipation of Bhagawāna Mahāvīra all 12 *aṅga*-s were lost but some part of *pūrva*-s related to *karma* theory is retrieved.

Digambaras claim that all *aṅga*-s are lost except a small portion of *pūrva*-s was retrieved by Ācārya Puṣpadanta and Bhūtabali who hailed from 2nd C. A.D. and who put it down in written form (called *Ṣaṭkhaṇḍāgama*). Another small portion of *pūrva* was composed in the form of *Kaṣāya Pāhuḍa* by *Ācārya* Guṇadhara who hailed from around the same period.

On the other hand, Śvetāmbara tradition believes the availability of all *aṅga*-s except Dṛṣṭivāda. However, some portion of *pūrva*-s relating to *karama* is believed to be retrieved in the form of *Karama Prakṛti* by Ācarya Śivaśarma Sūrī (c. 3rd-4th C. A.D.).

2.7 Process of Preserving the *Āgama* Through Councils (*vācanā*)

Śvetāmbara-s claim that 11 *aṅga*-s with some variations are preserved by their saint-scholars through summoning councils from time to time. The first council was held almost about 160 years after emancipation of Bhagawāna Mahāvīra at Pāṭaliputra (present Patna) in Bihar under the presence of many scholar *Ācārya*-s after a severe famine. The revised eleven *aṅga*-s were transmitted from generation to generation.

Second council was held simultaneously at two places Mathurā and Vallabhī (Gujarat) around 300 A.D. chaired by Ācārya Skandila and Ācārya Nāgārjuna respectively. The two versions of councils (*vācanā*-s) came to be known as Māthurī *vācanā* and Vallabhī *vācanā*. But it was maintained through oral tradition only. So, in due course of time the need was felt of preserving the *Āgama*. The third council was held at Vallabhī in 450 (or 463) A.D., presided over by Ācārya Devardhigaṇī Kṣamāśramaṇa.[2] The very first time in the history Jaina scriptures put into written form by Ācārya Devardhigaṇī Kṣamāśramaṇa. It appears that his *vācanā* relies more on earlier Māthurī *vācanā*.

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[1] *Ācārāṅgabhāṣyam,* chief synod Gṇādhipati Tulsī, ed. and commentary by Ācārya Mahāprajña, trans. in English by Nathmal Tatia, Muni Dulahraj, Muni Mahendra Kumar, Jain Vishva Bharati, Ladnun, 2001, p. X (Foreword).

[2] Ācārya Hastīmal, *Jaina Dharma kā Maulika Itihāsa,* Jain Itihasa Samiti, Jaipur, Vol. II, 1974, p. 718.

It is generally accepted by modern scholars of Jainaology that *āgamic* literatures available today represent the state of science, mathematics etc. till 450 (or 463) A.D. when it was edited and compiled and put in writing last. But it may be fair to assume that the numeration and other mathematical ideas must have been established a couple of centuries earlier.

2.8 *Āgama* with Mathematical Contents

Śvetāmbara texts like *Thāṅaṁ* (TS), *Bhagawaī* (BS), *Samavāo* (SAM), *Jambūdīvapaṅṅatti* (JDP), *Sūrapaṅṅatti* and *Candrapaṅṅatti* (SCP), *Anuogadārāiṁ* (ADS), *Uttarajjhayaṅāṅi* (US), *Paṅṅavaṅā* (PS), *Jīvājīvābhigama* (JJ), and in Digambara *Ṣaṭkhaṇḍāgama* (SK), *Kaṣāya Pāhuḍa* (KP), etc. contain many mathematical ideas.

In Śvetāmbara tradition works like *Karma Prakṛti*, *Jyoitṣa Karaṇḍaka* (JK), *Jīva Samāsa* (JS), *Viśeṣāvaśyaka Bhāṣya* (VB), *Bṛhatkṣetra samāsa* (BKS), *Tattvārthādhigama Sūtra Bhāṣya* (TSB) form non-*āgamic* literature but almost contemporary or little later to *Āgamic* works compiled by Devardhigaṇī Kṣamāśramaṇa in 5th C. A.D. In *Digamabara* tradition works like *Loka-vibhāga* (LV) and *Tiloyapaṇṇatti* (TP) fall under non-*āgamic* category. This literature contains very valuable and advanced ideas in mathematics of that period.

Considerable mathematical treatment is found in the *Kṣetrasamāsa* (collection of the places) by Jinabhadragaṇī (500 A.D.) who wrote two works of the same class: a bigger one called *Bṛhatakṣetra Samāsa* and a smaller one called *Laghukṣetra Samāsa*.

Since this thesis is limited to the works up to around 500 A.D., therefore later works are not included.

2.9 A Bird's Eye-view of Mathematics in *Āgama*

How significant mathematics is in Jaina philosophy can be understood by the illustration found in the ninth chapter of *Ṭhāṇaṁ* (9.28), where it describes nine such branches of knowledge that a person if gets mastery in even any one of the subjects will be recognized as an erudite scholar. Out of them the very first one is mathematics (*saṁkhāṇe*)-*ṇava ṇeuṇiyā vatthu paṇṇattā, taṁ jahā- saṅkhāṇe,...*[1] i.e. there are nine types of experts viz. mathematician etc. A number of Jaina texts of mathematical importance have yet to be

[1] *Ṭhāṇaṁ*, p. 855.

studied. *Sūrya prajñapti*, *Jambūdvīpa Prajñapti*, *Sthānaṅga Sūtra (Thāṇaṁ)*, *Uttarādhyayana Sūtra*, *Bhagavatī Sūtra* and *Anuyogadvāra Sūtra* etc. are the oldest canonical literature containing a treasure of mathematical ideas. A deep survey of mathematical ideas available in the Śvetāmbara *Āgama* is given below.

2.9.1 *Bhagavaī* (*Bhagavatī Sūtra,* BS)

Jaina canonical literature, known as *Āgama*, form the treasure of the preaching of Bhagawāna Mahāvīra. Out of 12 *aṅga*-s, *Bhagavatī Sūtra* known as *Vyākhyāprajñapti* holds the fifth place. Being the largest available *Āgama*, which elucidates various aspects of Jainism, it is an ocean of knowledge to understand Jaina philosophy. It describes several topics like culture, ontology, metaphysics, ethics, and cosmology, along with nature of matter, atoms, etc., mathematical concepts, biological ideas and many more making it encyclopaedic in character. This text is in the form of dialogues between Bhagawāna Mahāvīra and his chief disciple Gaṇadhara Gautama. Available BS consists of 15752 *sūtra*-s and is in the form of questions and answers. Scientific ideas hidden within philosophical concepts enhance the value of this *Āgama*. In short it is an encyclopaedic work. It was compiled and edited from time to time between 6^{th} century B.C. and 5^{th} century A.D.

Concept of Zero and Large Numbers: BS describes mathematical measures to explain the philosophical ideas. We find profuse use of decimal place value system in this *Āgama*. In BS we do not find clear use of zero for counting. In *śataka* 1 (BS, 1.2.104-111) explaining the *saṅsthāna kāla*[1] (the life-duration of a living being in that realm) the word *śūnya* is used.

The largest countable number in BS is stated as *śirṣaprahelikā* which is equivalent to $(8400000)^{28} \approx 7.58\times10^{193}$, and is consisting of 194 digits having 54 digits followed by 140 zeros. It then states that arithmetic ends here and the calculations beyond this can be made in the form of analogy measure of *palyopama* and *sāgaropama*.

[1] *neraiyasaṁsārasañciṭṭhaṇakāle ṇaṁ bhante! kativihe paṇṇatte? Goyamā! tivihe paṇṇatte, taṁ jahā – sunnakāle, asunnakāle, missakāle*.

–*Bhagavaī* , pp. 64-65.

Classification of Numbers: The positive numbers are classified in 4 categories viz. 4n+1, 4n+2, 4n+3 and 4n+4, for n=0, 1, 2, 3…Later the same are classified in 16 categories: 16n+1, 16n+2, 16n+3… 16n+16 for n =0, 1, 2, 3 …

Decimal place value system: We find the use of decimal place value system at various places in BS for defining the age, and height of the living beings and while describing the various entities in the universe. The decimal scale used is *eka* (1), *dasa* (10), *saya* (100), *sahassa* (1000), *dasasahassa* (10,000), *sayasahassa* (100,000), *dasasayasahassa* (10, 00,000), *koḍi* (10^7), *dasakoḍi* (10^8) etc. Therefore, we can say that this system was prevalent in the era of Bhagawāna Mahāvīra also. In BS we find this scale was used to measure the time, height and distance BS (6.7.132/134).

This *Āgama* also describes various time units and the smallest time unit is '*samaya'* which is the order of infinitesimal.

***pṛthaktva*:** A unique concept found frequently in Jaina *āgamic* canonical texts and other works also is the concept of decimal scale separation or *pṛthaktva*. *Puhatta* (*pṛthaktva*)[1] means separation. It was defined by commentators as any number between 2 to 9 [BS (24.1.96)] and not as 1 to 9, as 1 was not considered as a number. Similarly, *dasa-puhatta* (ten-separator) as a number between 10 and 90 etc. *sayapuhatta* (hundred-separator) [25.6.447] as a number between 100 and 900, *sahassapuhatta* (thousand-separator) [25.6.446] between 1000 and 9000, *koḍipuhatta* [25.6.450]. Numerous use of *pṛthaktva* shows that they were well-versed in counting.

In this thesis I have given more plausible explanation of the concept of *pṛthaktva.*

saṅkhyeya, asaṅkhyeya and ananta: BS divides the numbers in three categories: *saṅkhyeya* (numerable), *asaṅkhyeya* (innumerable) and *ananta* (infinite), *Āgama*. This is a unique contribution of Jaina mathematical philosophy. While defining medium of motion (*dharmāstikāya*), and medium of rest (*adharmāstikāya*) it is said (BS 2.10.134-135)[2] that

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[1] *jīvassa ṇaṁ bhante! āṇaya-devatte, noāṇayadevatte, puṇaravi āṇayadevatte pucchā. Goyamā! savvabandhaṁtaraṁ jahaṇṇeṇaṁ aṭṭhārasa sāgarovamāiṁ* ***vāsapuhatta****-mabbhahiyāiṁ, ukkoseṇaṁ aṇantaṁ kālaṁ- vaṇassaikālo. evaṁ jāva accue, navaraṁ jassa jā ṭhitī sā savvabandhaṁtaraṁ jahaṇṇeṇaṁ* ***vāsapuhatta****-mabbhahiyā kāyavvā, sesaṁ taṁ ceva* (8.9.401).

–Bhagavaī, Vol. III, 1st edn. 2005, p. 155.

[2] *Bhagavaī*, Vol. I, p. 291.

asaṅkhejjā dhamatthikāyapadesā i.e. *dharmāstikāya* consists of innumerable space points. space (*ākāśāstikāya*),sentient beings (*jīvāstikāya*) and matter (*pudgalāstikāya*) consist of infinite space-points. The hell is explained as extended up to numerable as well as innumerable *yojana* (BS 13.1.3).[1]

Infinitesimals: As Jaina scholars were trying to tackle the innumerables and infinities, they were groping with concepts of very small, i.e. infinitesimals. The smallest unit of space (length) is called *pradeśa*. BS (III, 6.7.134, p. 288) asserts that "*anantānanta*" (infinite) *paramāṇu*-s make one small unit of length called *utslakṣaṇslakṣṇa* and 8^9 = 134217728 of these make one *aṅgula*. For practical purposes *aṅgula* is regarded as smallest unit of length (literally meaning finger). Now '*pradeśa'* is also equivalent to space occupied by a *paramāṇu*. Thus we can see that *pradeśa'* is infinitesimally infinitesimal.

Fraction: *śataka* 8 mentions the effect of heat of the sun. It says that the heat of the sun goes up to 47263 plus 21 out of 60 parts of *yojana*.[2] It can be written as- $47263\frac{21}{60}$ *yojana*. In this way we find the use of fraction at many places.

Pi Value and geometry: In all of *āgamic* literature value of π is assumed as $\sqrt{10}$ (=3.1622776----)[3]. As the diameter of *Jambūdvīpa* is 10^5 *yojana*, its circumference is calculated by assuming the correct formula as $\pi\times$diameter = $\sqrt{10}\times10^5=\sqrt{10^{11}}$. This value is given in BS (20.9.81) (in many other *āgamic* works) as 316227 *yojana*, 3 *kośa*, 128 *dhanuṣa* and little above $13^1/_2$ *aṇgula*-s (equivalent to $\sqrt{10}$ = 3.162277660168... correct to 10 decimal places against the correct value 3.162277660177).

Combinatorics: In this *Āgama* permutations (*bhaṅga*) and combinations (*vikalpa*) are widely used to explain the spiritual, ethical and metaphysical concepts. For illustration, *sūtra* 1.5.217[4] talks about possible combinations of the hellish beings having anger,

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[1] *Bhagavaī*, Vol. IV, 2007, p. 109.

[2] *Goyamā! egaṁ joyaṇasayaṁ uḍḍhaṁ tavaṅti, aṭṭhārasa joyaṇsayāiṁ ahe tavaṅti, sīyālīsaṁ joyaṇsahassāiṁ doṇṇi ya tevaṭṭhe joyaṇsae ekkavīsaṁ ca saṭṭhibhāe joyaṇassa tiriyaṁ tavaṅti.*

Bhagavaī, Vol. III, pp. 132,133, *sūtra* 8.8.339.

[3] *Gaṇitānuyoga*, ed. by Muni Knahaiyālāla Kamal, Agam Anuyog Trust, Ahmedabad, 2nd edn., 1986, p. 13 (Preface).

[4] *Bhagavaī*, Vol. I, pp. 108-109.

conceit, deceit and greed. In *śataka* 8 (5.237), 49 possible combinations of *pratikramaṇa*[1] (self-introspection) are mentioned.

Occupancy Problem in Probability Theory: In BS the concept equivalent to occupancy problem in probability theory is explained by Bhagawāna Mahāvīra. *śataka* 9 [Vol. III. (32.88-100). pp. 228-257] of BS describes about the possible numbers of entry of 1,2,3,4,…..numerable…..innumerable…..,infinite living beings into one to seven hells. In later *sūtra*-s they have described about the entry of the souls in other realms also.

Partition theory: Modern partition theory developed by Euler, Hardy and Rāmānujana etc. is found in its most elementary form in BS. In number theory and combinatorics, a partition of a positive integer *n*, also called an integer partition, is a way of writing *n* as a sum of positive integers. In *śataka* 12 (4.69-80) the possible combinations of fission of aggregates consisting up to 10 *paramāṇu*-s are evaluated.

Geometry: Few geometrical figures and shapes are also introduced like *trayasra* (triangle), *caturasra* (square), *āyata* (rectangle), *valayavṛtta* (annular) etc. *śataka* 5 and *uddeśaka* 2 explain the shape of *Lavaṇasamudra*[2] (LS) etc. as annular. While explaining *tamaskāya* (BS 6.5.70-78)[3] and *kṛṣṇarāji* (BS 6.5.90)[4] their complex geometrical shapes are also mentioned.

Figurate numbers: BS 24.3 states about number of points required for representation of any geometrical figure or shape, which is very similar to treatment of figurate numbers. It even attempts to represent shapes like semi-circle, circle, sphere, annular, torus etc. by certain number of points for which there is no figurate number representation.

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[1] *Goyamā !tivihaṁ vā tiviheṇaṁ paḍikkamati, tivihaṁ vā duviheṇaṁ paḍikkamati, evaṁ ceva jāva egaviheṇaṁ vā egaviheṇaṁ paḍikkamati.1.tivihaṁ tiviheṇaṁ paḍikkamamāṇe na karei, na kāravei, kareṁtaṁ ṇāṇujāṇai maṇasā vayasā kāyasā….49. ahavā kareṁtaṁ ṇnāṇujāṇi kāyasā.*

–*Ibid,* Vol. III, pp. 85-86, *sūtra* 8.5.237.

[2] *Bhagavaī*, Vol. II, p. 142, *sūtra* 5.2.155.

[3] *Ibid*, Vol. II, pp. 269-270.

[4] *Ibid*, pp. 272-273.

2.9.2 *Samavāo* (SAM)

Samavāo or *Samavāyāṅga* is the fourth *aṅga* which deals with multitude of subjects. The subject matter is described on numerical basis, not on alphabetical basis. It starts from one and goes upto *koṭā-koṭī sāgaropama.* This text introduces the subject matter of all twelve *aṅga*-s, *karma* , food, breath-rate of deities of various heavens, area, height and width of the mountains, historical figures like *cakravartī* (conqueror of 6 continents), *baladeva* (conqueror of 3 continents), *vāsudeva* (younger brother of *baladeva*), geographical features, time-cycles, twenty-four *tīrthaṅkara*-s, metaphysical concepts etc. Good amount of mathematical ideas and calculations are employed to describe these concepts. It is an encyclopedia like *Thāṇaṁ* of Jaina philosophy consisting of 1667 *sūtra*-s at present in 100 chapters and one miscellaneous.

Sūtra 18.5 mentions the words like number (*aṅka)* and arithmetic (*gaṇiya*). Out of eighteen types of writing scripts two are named as *aṅkalivi* (language of numbers) and *gaṇiyalivi*[1] (language of arithmetic). *gaṇiyalivi* means the special numerals used for mathematical purpose specifically.

Decimal place value system seems prevalent in that era to describe the life spans of the beings, the duration of *karma* and the number of verses in *Āgama*. The 94 *sūtra* of *prakīrṇaka* enumerates that *Nāyādhammakahāo* consists *addhaṭṭhāo akkhāiyakoḍīo*[2] i.e. 3.5×10^7 (crore) stories. Like BS, it talks of *saṅkhyāta, asaṅkhyāta* and *ananta* and specifies the entities falling in these categories.

Geometrical calculations are found in the text to explain the natural entities. For illustration, *sūtra* 53.2[3] mentions that the length of chord (*jīvā*) of mountains *Rukma* and *Mahāhimavāna* is *…tevannaṁ joyaṇasahassāiṁ nava ya egatīse joyaṇasaechacca ekkūṇavīsaibhāe joyaṇassa* ($= 53931\frac{6}{19}\ y$) each and *sūtra* 57.5[4] their southward arc is ... *sattāvaṇṇaṁ joyaṇasahassāiṁ doṇṇi ya teṇaue joyaṇasae dasa ya egūṇavīsaibhāe*

[1] *Samavāo,* p. 105.

[2] *Ibid*, p. 323.

[3] *Ibid*, p. 219.

[4] *Ibid*, p. 224.

*joyaṇassa…*i.e. $57293\frac{10}{19}$. Therefore, it shows that they were acquainted with fractions also which are used in many places.

2.9.3 *Thāṇaṁ* (TS)

Thāṇaṁ (600 BC) is the third canonical text which characterizes mathematics as subtle and deep subject. It is very rich in content because of various subjects, around 1200, which again goes up to thousands of topics by sub-classifications. Like *Samavāo* it deals with great number of topics and in the form of numerical lexicon discusses things falling in 1, 2, 3…. etc. categories. As a result, all mathematical ideas falling in particular class are mentioned which were elaborated in other scriptures. It has 10 chapters containing 3600 verses.

Geometry was necessary to explain the geographical features and measures in Jaina philosophy. *sūtra* 248[1] of Chapter 1, mentions the shapes while explaining the mountains and the oceans.

Jaina saints have always been interested in long counting which goes upto transfinite numbers. Same as in other *Āgama* here also mention of *saṅkhyeya, asaṅkhyeya, ananta, palyopama* and *sāgaropama* is made frequently.

Four kinds of measurement in chapter 2 *sūtra* 125 are found as *davvappamāṇe* (measurement of the substance), *khettappamāṇe* (measurement of the area), *kālappamāṇe* (measurement of time) and *bhāvappamāṇe* (measurement of quantities).

In Chapter 4, *sūtra* 364 we find references to theory of numbers. As it mentions *cattāri jummā paṇṇattā, taṁ jahā -kaḍajumme, teyoe, dāvarajumme, kalioe* i.e. there are four types of *rāśi* (set of numbers) *kṛta yugma*, *tryoja*, *dvāpara yugma* and *kalyoja*.[2] There are two types of numbers *yugma* and *oja*. Ācārya Mahāprajña in his commentary of this text mentions that according to *Sthānāṅga vṛtti* (*patra* 226)[3] by Abhayadeva sūrī (10^{th} C. A.D.) *yugma* is the set of even numbers while *oja* is the set of odd numbers. Therefore there are two types of *yugma*: *kṛta* and *dvāpara* and two types of *oja*: *tryoja* and *kalyoja*. This classification of numbers with more detail is found in BS.

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[1] *Ṭhāṇaṁ*, pp. 16,17.

[2] *Ibid*, p. 390.

[3] *Ibid*, p. 515.

The 10th chapter *sutra* 24 of the *Sthānāṅga Sūtra* mentions about 10 types of *suhume* (fine things) and out of them two is *–gaṇiyasuhume* (mathematics), *bhaṅgasuhume* (combination) i.e. *gaṇita* (mathematics) and *bhaṅga*[1] (permutation). In the same chapter the (TS 10.100, p. 926) verse is very significant from mathematical standpoint which introduces 10 types of *saṅkhyāna* (mathematical operations) as following:

1. *Parikarma* (Arithmetic)
2. *Vyavahāra* (Series)
3. *Rajju* (Geometry)
4. *Rāśi* (Set Theory)
5. *Kālasavanna* (Fraction)
6. *Yāvat-tāvat* (Simple algebraic equation)
7. *Varga* (square)
8. *Ghana* (Cube)
9. *Varga-varga* (square of square)
10. *Kalpa* (Permutation & Combination)

2.9.4 *Jambūddīvapaṇṇatti* (JDP)

JDP is the sixth *upāṅga* which holds a significant place in Jaina canonical literature. The central subject of this text is reflected by name of the text itself. It is a composition of ancient knowledge with reference to cosmology, cosmography and astronomy. Geography of *Jambūdvīpa* (JD) the central island or earth we live on is elucidated in detail. Along with *Jambūdvīpa*, however, is given the biography of Bhagawāna Ṛṣabha, the last Progenitor (*kulakara*), first Emperor (*cakravarṭi*) Bharat, and first Preceptor (*tīrthaṅkara*) Ṛṣabha of current descending eon (*utsarpiṇī*). Historians of astronomy were searching for a link between Vedic astronomy (*jyotiṣa*) [up to 6/5th C. B.C.] and *siddhāntic* (five-fold) astronomy (from 5th C. A.D. onwards) a period of almost 1000 years called dark-age of Hindu astronomy. Now, it is recognized by all scholars (occidental and oriental) that this gap is just and exactly filled in by Jaina astronomy.[2] It has 7 chapters with 5240 *sūtra*-s.

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[1] *Ṭhāṇaṁ*, p .907.

[2] *Jambūddīvapaṇṇtti, (Uvaṅgasuttāṇi)*, synod Ācārya Tulsī, ed. by Yuvācārya Mahāprajña, Jain Vishva Bharati, Ladnun, Part-2, 1989, trans. by Shah, R.S., Jain Vishva Bharati Institute, Ladnun (Under Publication), p. 1, (Introduction).

Decimal Place value system, Large Numbers: The text shows total familiarity with decimal place value system. The diameter and circumference of JD, chords, bows and arms of continents and continental mountains, lengths of orbits of Sun, Moon, planets, asterisms and stars etc. are expressed in decimal place value notation. The biggest number encountered in JDP is the number of stars in JD which is 13 395 000 000 000 000 000 (*sūtra* 7.159).[1] *Sūtra* 7.83 mentions that when king Bharat after conquering the entire continent entered his capital number of associates with him was very large. The entourage consisted of, in that order, 8 auspicious emblems, 2 flags *Vijaya* and *Vejayanta*, divine scepter, parasols, throne, 7 one-sensed jewels, 9 treasured jewels, 16000 gods, 32000 subordinate kings, 5 five-sensed jewels [the General, Chief Quarter Master, Chief Engineer, Chief Priest, and Chief Queen (*Subhadrā*), 32000 maidens, 32000 ladies,32000 actors, 360 Chief Cooks, (the army consisting of) 8, 400, 000 horses, 8, 400, 000 elephants, 8, 400, 000 chariots, 960,000, 000 foot soldiers, princes, sheriffs, provincial governors, heads of important families, tradesmen, mayors, commanders, heads of caravans, uniformed servants, entertainers, brigade of 800 small elephants rode by fighters, 800 chariots with fighters, infantry-men carrying weapons etc. In total it was 8+2+1+1+1+7+9+16000+ 32000 + 5 + 32000 + 32000 + 32000 + 360 + 8,400,000 + 8,400, 000 + 8,400,000 + 960,000,000+800+800 =985345634=9.85×10^8. Such large was his retinue.

Here also the time scale goes up to *sīsapaheliyā* as mentioned in BS. We also find the evidence depicting necessity of simile-measure in chapter 2, *sūtra* 24, *etāva tāva gaṇie, etāva tāva gaṇiassa visae, teṇaṁ pare ovamie*[2] – counting is possible till this and beyond that things can be measured using simile (analogy).

Zero with place value and pi: There is no mention of zero or its representation as having place value anywhere in JDP. However, there are large scale examples of extraction of square roots of quite big numbers in JDP. E.g. the circumference C of a circle is calculated as follows: C = $\sqrt{10}d$ = $\sqrt{10}\times100000$ = $\sqrt{100000000000}$ = $\sqrt{10^{11}}$. It seems that $\sqrt{10}$ value of π was used for the calculation. It is described in *sūtra* 4.91 that in the very centre

[1] *Jambūdvīpaprajñapti Sūtra*, ed. by Yuvācārya Miśrīmal ji Madhukar. with Hindi translation and appendices, Agam Prakashan Samiti, Beawar, 1st edn., 1994, p. 319.

[2] *Ibid*, p. 27.

of pond *Gaṅgāppavāyakuṇḍa* there is an island named *Gaṅgādīva*[1] with diameter 8 *y* and circumference slightly over 25 *y*. The actual value of circumference is $8\times\sqrt{10} \approx 25.298$ *y*.

Fractions: JDP contains numerous instances of total familiarity with operations involving fractions. In chapter 1 and 4, the north-south spans of continents and continental mountains involving fractions with denominator 19 are properly determined. Chapter 7 deals with complicated fractions. For example, *sūtra* 7.16[2] says, the diameter of the orbit, third from the innermost one, is *…ṇavaṇauiṁ joyaṇasahassāiṁ chacca ekkāvaṇae joyaṇsae ṇava ya egasaṭṭhibhāe joyaṇassa āyāma vikkhaṁbheṇaṁ…i.e* 99651 $\frac{9}{61}$ *y*. The Sun crosses one circular orbit after another, increasing diameter of the orbit by $5\frac{35}{61}$ *y* (*pañca pañca joyaṇāiṁ paṇatīsaṁ ca egasaṭṭhibhāe…*) and circumference by 18 *y* per circular lane, and ultimately reaches the outermost orbit. Furthermore, *sutra* 7.21[3].... mentions that when the sun is moving across the orbital lane next to the innermost one, it covers a distance of *pañca pañca joyaṇsahassāiṁ doṇṇi ya egāvaṇṇe siyālisaṁca saṭṭhibhāge joyaṇassa* i.e. $5251\frac{47}{60}$ *y* per *muhutta*. At that time a human being there (in the continent of *Bharaha*) observes the Sun from a distance of $47179\frac{21}{60} + \frac{19}{60\times61}$ *y* (*sīyālīsāe joyaṇasahassehiṁ eguṇāsīe joyaṇasae sattāvaṇṇāe ya saṭṭhibhāehiṁ joyaṇassa saṭṭhibhāgaṁ ca egasaṭṭhihā chettā egūṇavīsāe cuṇṇiyābhāgehiṁ…*). Here *y* stands for *yojana.*

Extraction of square-roots: chords c of various continents and continental mountains are calculated by using the formula $\left(\frac{c}{2}\right)^2 = h\ (d - h)$ or $c^2 = 4h\ (d - h)$, which involves extraction of square-roots.

Geometry: Besides the circular earth and annular islands and oceans many of the mountains, ramparts, mansions, and temples are also described as circular in shape. *Jambūdvīpa* is 100000 *j* in diameter and with a circumference of 316227 *j* 3 *k* 128 *d* and slightly over 13 $\frac{1}{2}$ ạ (chapter 1, *sutra* 3)[4] which when converted to *yojana* comes as

[1] *Jambūdvīpaprajñapti Sūtra*, p. 186.

[2] *Jambūddīvapaṇṇtti* (*Uvaṅgasuttāṅi*), p. 554.

[3] *Ibid*, p. 555.

[4] *Jambūdvīpaprajñapti Sūtra*, p. 5.

316227.766017578125 *yojana*. This gives the Jaina value of π as $\sqrt{10}$ = 3.16227766017578125… correct to 10 decimal places.

The geometry of circle was developed to quite an advanced level in the Jaina canon. As all the continents and continental mountains are depicted to be running east-west length-wise, it became necessary to calculate their southern and northern chords. Of course, northern chord of a continent becomes southern chord of next Continental Mountain.

North-south width of Bharata continent (1.11) is …*doṇṇi aṭṭhatīse joaṇasae tiṇṇi a egūṇavīsaibhāge joyaṇassa vikkhaṁbheṇaṁ*… i.e. $238\frac{3}{19}y$.[1] We find more such examples in the text.

The formula is not stated anywhere in JDP, but the values given exactly fit with results obtained by using the formulas =$\sqrt{c^2 + 6h^2}$, which is stated by later *Ācārya*-s and commentators.

JDP describes shape of *kāgaṇī* jewel and gives its vertices, edges and faces reminding us of Eulers' famous formula viz. *v+f-2=e* (JDP, *Uvaṅgasuttāṇi,* 3.94) . Like BS, it gives value of π as $\sqrt{10}$.

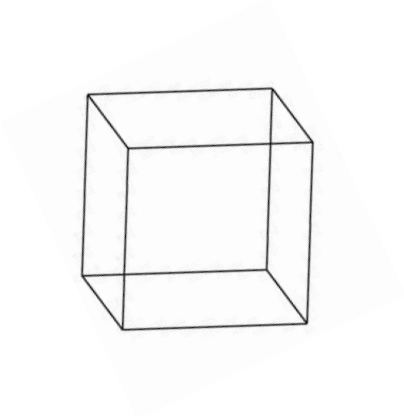

Fig-1 : Kāgaṇi Jewel

Infinitesimal: *sūtra* 103 of 4th chapter mentions about infinitesimals. The width of Gandhamādana Mountain near Mountain Meru tapers to innumerable part of *aṅgula*.[2]

[1] *Jambūdvīpaprajñapti Sūtra*, p. 9.

[2] *Ibid*, p. 209.

2.9.5 *Anuogadārāiṁ* (ADS)

Anuyogadvāra Sūtra is a valuable canonical work composed sometimes between 300 B.C. and 300 century A.D. It consists of 2162 *sūtra*-s in 13 chapters (*prakaraṇa*). It contains mathematical concepts quite advanced for those times and which are far advanced compared to other *āgamic* works. These are sufficient to establish that decimal place value system was in place any time between 300 B.C. and 5th century A.D. A study of ADS reveals that ideas about rules of indices (including fractional indices) permutations and combinations, geometry, mensuration etc. were clearly crystallized then. While discussing measures of space and time there is a valiant attempt to formulate ideas somewhat parallel to the treatment of ordinals, cardinals, and transfinite numbers as enunciated by Cantor (19th C. A.D.). We get theoretical treatment of many mathematical concepts in ADS.

Decimal Place Value System and Zero: There are numerous references in ADS which clearly establish that decimal place value system was well established by then. In *sūtra* 231,[1] while dealing with measures of counting (*gaṇitamāna*), mention is made of the scale: one (*ego*), ten (*dasaṁ*), hundred (*sayaṁ*) and goes upto ten billion (*dasakoḍisayaṁ* $=10^{10}$). In *sūtra* 219 (p.142), this goes up to *śīrṣaprahelikā* and continued with analogy measure.

Again, *sūtra* 574[2] deals with counting measures (*gaṇanā*) where ideas on innumerable and infinites are dealt in detail (This will be discussed in the next chapter). The "analogy-measures" are also described in detail to deal with very big uncountable numbers.

In *sūtra* 490, there is mention of number of physical bodies which are bound by *karma* and it specifies that this number is equal to 2^{96} and it has 29 digits. If we write down this number in full we get a zero in it (it is explained in chapter 3 in detail).

Infinitesimal: on one hand ADS deals with very large numbers, simile-measures and transfinite numbers and on other hand it describes very small numbers (infinitesimals) like space-points, atoms, molecules etc.

Rules Of Indices: ADS shows quite familiarity with rules of exponentiation, even in respect of fractional indices. While calculating a certain human population, *sūtra* 490 states that it is equal to product of 6th successive square of 2 with 5th successive square of 2, i.e.

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[1] *Anuogadārāiṁ*, p. 144.

[2] *Ibid*, p. 309.

$2^{64} \times 2^{32} = 2^{96}$, showing the use of rules of indices. Furthermore, *sūtra* 463 gives the spatial measure of number of transformable bodies of bonded hellish beings as product of first square root of *aṅgula* and second square root of *aṅgula* and says it is equal to cube of second square root of *aṅgula*. i.e.

$$a^{\frac{1}{2}} \times a^{\frac{1}{4}} = a^{\frac{3}{4}} = (a^{\frac{1}{4}})^3$$

Spatial and Temporal Innumerables: Besides classification of numbers in three main classes as numerable (*saṅkhyāta*), innumerables (*asaṅkhyāta*), and infinites (*ananta*) for measurement of space-points, ADS has another set of innumerables and infinities for measuring time (*sūtra* 418-440). Among innumerables it gives *palyopama* (pit-measure) and *sāgaropama* (sea-measure). *Palyopama* and *sāgaropama* are treated thereby further dividing in 3 sub-classes viz. *uddhāra/addhā/khetta* and their outcrops namely *ossappiṇi*, *kappa*, *ussappiṇi*. ADS also specifies that since universe is external- without beginning or without end, the past time (*atītaddhā*) and future time (*anāgataddhā*) are infinite and hence all time (*savaddhā*) too. A very detailed treatment of both type of classification is given in ADS.

Permutations and Combinations: Profuse use of permutations and combinations is made to explain many things like viewpoints (*naya*), valid cognition (*paramāṇa*), substance (*dravya*) etc. ADS, *sutra* 150,[1] says about obtaining the number of all possible arrangements of six substances (*dravya*-s) excluding the first (serial) and the last (reverse) which is the process of permutation. In many more *sutra*-s we find the use of this concept. It appears that at the time of composition of ADS certain rules about permutations and combinations like $P_n = n!$ and ${}^nC_r = \frac{n!}{(n-r)!\, r!}$ were well-known.

Geometry: While explaining about *saṅkhyāta* ADS talks about cylindrical pit. ADS, *sūtra* 513, mentions 5 shapes: *parimaṇḍala* or *valaya* (ring or torus), *vṛtta* (circle or sphere), *tryasra* (triangle or tetrahedron), *caturasra* (square or cube), and *āyata* (rectangle or parallelepiped).

2.9.6 *Paṇṇavaṇā* (PS)

Prajṇāpanā is the 5th *upāṅga* and holds same status in *upāṅga* as BS in *aṅga*-s. While editing *Āgama* in 450 C. A.D. Devardhigaṇī quotes 2.10.6 *Paṇṇavaṇā* many times in BS. *Ārya* (*Ācārya*) Śyāma, the 23rd successor of Mahāvīra, has authored this text. Thus time of

[1] *Anuogadārāiṁ*, p. 94.

composition can be reckoned as beginning of 2nd C. A.D. It basically explains the concept of living beings (*jīva*), inanimate matter (*ajīva*), *karma* etc. form various perspectives, very minutely and in depth. Along with ontology we also find many mathematical ideas like numerable, innumerable, infinite, fractions, time scale, analogy-measures, permutations, combinations, geometrical shapes etc. This text contains 7887 verses in the form of question and answer.

Decimal Place value system: There is a clear evidence of use of decimal place value system in PS. Though we do not find the entire measure scale of counting as it is mentioned in ADS and BS but counting scale found to measure different entities show that decimal system was established by that period. The higher counting goes as *sahassāṅaṁ* (thousand), *saya-sahassāṅaṁ* (*lākha*, 10^5) and *koḍākoḍī* (10^{14}) etc. These are used several times at various places while dealing with the life-span of the living beings and explaining the location of the deities' etc. *sutra* 196 of chapter 2 mentions the number of air-ships of celestial (*vaimānika*) deities is …*caurāsīi vimāṇāvāsasatasahassa sattāṇauiṁ ca sahassā tevīsaṁ…* (i.e. 84,97,023). Further the circumference of *īṣatprāgbhārā pṛthvī* is explained using decimal place value system as little more than 1,42,30,249 *yojana*.

Large Numbers, Concept of Zero, Dyadic scale: PS *sūtra* 12.921.1[1] tells about human population like ADS showing familiarity with decimal scale and dyadic scale. Like ADS it states that minimum number of human corporeal bodies is *saṅkhyātakoḍākoḍī* which is a number with 29 digits and the product of 5th and 6th successive squares of 2. This is an indirect evidence of knowledge of zero with place value, since one of 29 digits is zero.

Numerable, innumerable, infinite: To explain the concept of *jīva* and *ajīva* several times these mathematical terms are used in this *Āgama*. For example, it states, the number of *pratyeka vanaspati* (plants) living beings is *asaṅkhyāta;* there are *asaṅkhyāta jīva* in the root, stem, skin, branch and leaves of multi-seeds trees (*sūtra* 1.41, p.49). The diameter of *saudharma* celestial heaven is …*asaṅkhejjāo joyaṇakoḍākoḍīo āyāma-vikkhaṁbheaṁ…* i.e. innumerable *koḍākoḍī* (10^{14}) *yojana* (2.197.1).Chapter 1, *sūtra* 54.10[2] tells about the existence of *ananta* (infinite) souls of *nigoda* in one body.

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[1] *Prajnāpanā Sūtra*, Vol. II, 1st edn., 1984, p. 113 .

[2] *Ibid*, Vol. I, p. 63.

Chapter 15 deals with the six types of living beings in detail. *sūtra* 986.5 of this chapter mentions that earth-bodied living beings (*pṛthvīkāya*), which are innumerable (*asaṅkhyāta*) in number, get accommodated in the numerable (*saṅkhyāta*) space-points of space. It specifies further that this innumerable is equal to space points in innumerable cosmos (*loka*).

Fractions: In *āgamic* literature fractions were tackled very proficiently. In chapter 2, *sūtra* 29 while explaining the height of the liberated soul is stated to be 2/3rd of the height of the existing human body at that moment. The maximum height of human being can be 500 *dhanuṣa*. Therefore maximum height of liberated soul can be 333 $\frac{1}{3}$ *dhanuṣa*. The medium height can be $4\frac{2}{3}$ *hātha* and the minimum can be 1 *hātha* and 8 *aṅgula* (i.e. $1\frac{1}{3}$ *hātha*). There are numerous such examples found in this text.

Maxima and Minima: This concept is frequently used to explain the relative number or magnitudes of living beings from various perspectives. For illustration, the comparison of number of living beings in different directions is given in *sūtra* 3.213.[1] The west (W) direction consists of least number of souls. East (E) contains slightly more (*viśeṣādhika*) number of souls than the previous and south (S) and north (N) hold slightly more from the preceding one. i.e.

$$E = (1+v)\times W \text{ (where } 0<v<1)$$

$$S = (1+v)\times E;\ N=(1+v)\times S$$

Another example quoted here (3.232, p. 217) is: Mobile (M) living beings are the minimum. Fiery-bodied (F) are innumerable times of the mobile beings. Earth-bodied (E) are slightly more than fiery-bodied. Water-bodied (W) are slightly more than the earth-bodied. Air-bodied (A_r) are slightly more than the water-bodied. Bodiless (B) are infinite times of air-bodied. Plant-bodied (P) are infinite times of the previous one. All living beings (L) with body are slightly more than the previous one. i.e.

$$F=A\times M;\ E=(1+v)\times F;\ W=(1+v)\times E;\ A_r=(1+v)\times W;\ B=I\times A_r;\ P=I\times B;\ L=(1+v)\times P$$

where, I is infinite, and $0<v<1$

Geometrical figures: When asked about the shapes or configurations Mahāvīra answered Gautama that there are five types of configuration: *parimaṇḍala* (ring), *vṛtta*, (circle), *tryasra* (triangle), *caturasra* (square) and *āyata* (rectangle) (10.791, vol. II, p.29).

[1] *Prajnāpanā Sūtra*, Vol. I, p. 201.

Infinitesimal: The infinitesimal is described by the size of various beings which are miniscule. For an example, *sūtra* 1506 and 1507 (vol. II, p.429, 430) state that the minimum linear dimension of plant-bodied living [developed (*paryāpta*), undeveloped (*aparyāpta*), and gross with physical (*audārika*) body] is infinite[1] part of an *aṅgula*.

Which can be expressed as, a_{∞}/I (where I is infinite)

Similarly, the length of *sammūrchima āsāliyā* (agglutinative reptile) is *aṅgulassa asaṅkhejjibhāga* i.e. innumerable-th part of an *aṅgula*= a_{∞}/A (where A is innumerable) (1.82).[2]

Further PS states in *sūtra* 18.1337 that maximum duration of a soul having *nīla leśyā* (blue psychic colour) is *ukkoseṇaṁ dasa sāgarovamāiṁ paliovamāsaṅkhejjaibhāgabbhiyāiṁ* i.e. 10 *sāgaropama* and innumerable-th part of *palyopama* i.e. P/A (where A is innumerable).

***Pṛthaktva*:** How long a living being can stay or take birth in the same form? To measure the time-span use of *pṛthaktva* is found. A *sajñī* (living being endowed with mind) can take birth in the *sajñī* form a maximum *śata* (100) *sāgaropamapṛthaktva*[3] (i.e. between 100 to 900 *sāgaropama*). Later in chapter 21 it is used to measure height too. Maximum height of flying birds (*khecara*) is *dhanuṣa pṛthaktva* (i.e. between 1 to 9 *dhanuṣa*).

Though this text is not a mathematical work, yet to explain the qualities of the sentient beings and non-sentient things mathematical ideas are used effectively.

2.9.7 *Jīvājīvābhigama* (JJ)

Jīvājīvābhigama is the third *upāṅga* or *aṅga-bāhya*. The name of the author is unknown but it must be written by some scholarly *ācārya*-s. It seems that some part of this text is lost. Earlier it had 20 *uddeśaka* but now 18 are available with 4750 *sūtra*-s[4] form. It's very rich work which explores many aspects of living beings (*jīva*) and non-animate (*ajīva*). At

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[1] *Prajnāpanā Sūtra*, Vol. II, p. 430.

[2] *Ibid*, Vol. I , p. 79.

[3] *Ibid*, vol. II, p. 368, *sutra*, 18.1389.

[4] *Jīvājīvābhigama Sūtra*, chief ed. Yuvācārya Miśrīmal ji Madhukar, with Hindi translation and appendices, Agam Prakashan Samiti, Beawar, Vol. I, 1st edn., 1989, p.16 (preface).

present the text available explains in detail about all the classes of *jīva* divided in 9 *pratipatti* (chapters). In this explanation at many places mathematics is used abundantly.

The author appears to be very clear about fractions, extractions of root, some types of geometrical figures, decimal place value system, zero, large numbers, numerable, innumerable, infinites etc. While dealing with the life-spans of the living being dimensions of various hells and heavens as well as of JD, other islands and seas have been discussed.

Decimal Place value system: Numerous example of use of decimal place value system is the clear evidence of knowledge of this concept in that period. In, *sūtra* 3.2.81 (Vol. I, p. 222) while explaining the dimension of all the seven hells place value system is used with large numbers. The width of first hell *ratnaprabhā* is *...asiuttara joyansayasahassabāhallāe ...*1, 80, 000 *yojana.* This text enumerates the number of stars in various islands and oceans. For illustration, in *Kālodadhi Ocean* (3.3.175)[1] there are 2812950×10^{14} stars and in *puṣkarvaradvīpa* (3.3.176a, vol. II, p.40) the number of stars mentioned is, *chhaṇṇaui sayasahassā cattālīsaṁ bhave sahassāiṁ cattārisayā… pukkharavara tārāgaṇakoḍikoḍīṇaṁ* i.e. 9644400×10^{14}. The geometry of entire hell is well explained and large numbers are widely used to explain the dimension of each hell etc.

Numerable, innumerable, infinite: This concept is found very widely used in this text. *Sūtra* 3.2.86(b) (vol. I, p.232) mentions that in the first hell maximum numerable or innumerable living beings can take birth in one *samaya*. The population of hellish beings is so large that if in each *samaya, asaṅkhyāta* beings are taken out then even in *asaṅkhya utsarpiṇī* and *avasarpiṇī* it cannot be exhausted.

Fractions: While explaining the geographical measures and height of the hellish beings fractions are used profusely and perfectly. The height of the hellish beings of the first hell is a minimum of *asaṅkhyāta* part of *aṅgula* (*ą*/A) and a maximum of 7 *dhanuṣa*, 3 *hātha* and 9 *ą* [3.86(3), p. 233].

Geometrical shapes: The shapes of hells are also discussed here. *uddeśaka* 1 and *sutra* 5 mentions that the shape of hells are circle (*vaṭṭā*), triangle (*taṁsā*), square (*cauraṁsā*).[2] The residence of hellish beings in the first hell is round from inside and outside it is square.

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[1] *Jīvājīvābhigama Sūtra*, Vol. II, 1991, p, 37.

[2] *Ibid*, Vol. I, p. 225.

Analogy-measure: The life span of beings of first hell is minimum 10,000 years and maximum 1 *sāgaropama,* which keeps on increasing and that of 7th hell is minimum 22 *sāgaropama* and maximum 33 *sāgaropama.*

Maxima and Minima: Many a times this concept of maxima and minima is used to compare the numbers of living beings. *sūtra* 4.209[1] describes comparative number of various living beings from five-sensed to one sensed.

Pañca$_s$<*Catur*$_s$<*Tri*$_s$<*Dvī*$_s$< *Eka*$_s$ i.e. five sensed are the minimum in number. Four sensed are slightly more than five sensed; three sensed are slightly more (*viśeṣādhika*) than the previous one, two sensed are slightly more than the previous and one sensed are infinite times the number of two sensed beings.

sūtra 221.a(1) of chapter 5 compares the number of gross living beings[2] in the universe. There it says, the minimum are gross *trasakāya* i.e. T_s

$T_{j=tejaskāya} = A\times T_s$; $V_{p=prtayeka\ vanaspati} = A\times T_j$; $N_{g=nigoda}=A\times V_p$; $P_{r=pṛthvīkāya} = A\times N_g$; $A_{p=apakāya}=A\times P_r$;$V_{y=vāyukāya} = A\times A_p = V_{b=bādar\ vanaspatikāya} = I\times V_y$. [Where, A= innumerable times, I=infinite times]

or, $T_s<T_j<V_p<N_g<P_r<A_p<V_y<V_b$

In chapter four, five and six we find numerous illustrations explaining maxima and minima of living beings from various perspectives.

***Pṛthaktva*:** To describe the heights, concept of rebirth and life, *pṛthaktva* is profusely used. For example, *sūtra* 36 of *pratipatti* 1 (vol.I, p. 85), mentions that the maximum height of animal is *gāuyapuhuttaṁ* [*gavyūti* (*kośa) pṛthaktva*, 2-9 *kośa*]. And the maximum height of agglutinative (*sammūrchima)* reptiles and birds is *dhanuṣa pṛthaktva* (2-9 *dhanuṣa*). *sūtra* 2.48(b) states that a female animal can take birth as any female animal for maximum duration of *pūrvakoṭipṛthaktva*[3] (2-9 *pūrvakoṭi*, where *pūrva* = 7056×10^{10} and *koṭi*=10^7).

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[1] *Jīvājīvābhigama Sūtra,* Vol. II, p. 121.

[2] *Ibid*, Vol. II, p. 133.

[3] *Ibid*, Vol. I, p. 133.

2.9.8 *Uttrajjhayaṇāṇi* (US)

Uttrādhyayana sūtra is one of the *āgamic* works considered under the category of *mūla sūtra*, which is based on preaching of Bhagawāna Mahāvīra and the later *Ācārya*-s. It consists of 36 chapters and 2100 verses.[1] Though this *āgamic* work focuses more on ethical practices of a monk including some inspirational spiritual life stories; we find use of mathematical concepts also while explaining the living things and other entities.

Decimal Place value system: We find the scanty use of decimal place value system. *sūtra* 58 of chapter 36 mentions that the diameter of *siddhaśīlā* is 45,000,00 *yojana*.

Numerable, innumerable, infinite: Words *saṅkhyeya*, *asaṅkhyeya* and *ananta* are used good number of times in this text. It says in chapter 10 that a living being can remain in the form of earth-bodied (*pṛthvīkāya, sūtra* 5), water-bodied (*apkāya* 10.6), and fiery-bodied (*taijaskāya* 10.7) for *asaṅkhyeya* years; in the form of plant-bodied (*vanaspatikāya* (10.9)) for *ananta* years; as two-sensed, three-sensed, and four-sensed living beings for *saṅkhyeya* (10.10-12) years. We find the use of *ananta* in *sūtra* 36.82 where it says that *antara kāla* (interregnum of a living being between the two births) of *pṛthvīkāya* and that of *apkāya* (36.90) is for a maximum of *ananta-kāla* etc.

Fractions: Use of fractions at various places is also found here. For example, in chapter 36 while explaining about the location of the liberated soul, *sūtra* 62[2] says that in the $1/6^{th}$ part of the uppermost last 1 *yojana*, the end of the *loka*, *siddha*-s are located. *Sūtra* 64 in the same chapter says that the height of the liberated soul[3] is $1/3^{rd}$ less than the height of the person in his last life during liberation. Furthermore, it says, the life span of flying living beings (*khecara*)[4] is innumerable-th part (*asaṅkhyāta bhāga*) of a *palyopama*.

Analogy-measure: As in other canonical texts analogy-measures are also used in this text. For example, maximum duration of *jñānavarṇīya* (knowledge obscuring) *karma* is 30 *koṭā-koṭī sāgara*[5] ($=30\times10^{14}$ *sāgaropama*).

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[1] Jain, Anupam, *Ardhamāgadhī Sāhitya me Gaṇita,* Jain Vishva Bharati Institute, Ladnun, 2008, p. 39.

[2] *Uttarajjhayaṇāṇi*, p. 606.

[3] *Ibid*, p. 607.

[4] *Ibid*, p. 621, *sūtra,* 36.191.

[5] *Ibid*, p. 574, *sūtra,* 33.19.

Geometrical shapes: Five shapes of *pudgala* (matter)[1] like annular (*parimaṅḍala*), circle (*vaṭṭa*), triangle (*taṅsā*), square (*cauraṅsaṁ*), rectangle (*āyayā*) are mentioned.

2.9.9 *Sūrapaṇṇatti* and *Candrapaṇṇatti* (SCP)

Sūryaprajñapti and Candraprajñapti both fall in the category of *upāṅga*. This is the oldest available *upāṅga* but the author is unknown.

This is a great work dealing with mathematics and astronomy. It explores about sun, moon, stars and their movements. This combined text consists of one chapter divided into 20 sections known as *prābhṛta* with 2200 verses. It's accepted with respect by all sects of Jaina religion. This text is completely dedicated to astronomy. It explains about calculation of day, night, fortnight, year according the movement of sun. How does sun rise and move, its illuminated area, location, number of suns, moons, planets, asterism (*nakṣatra*) and stars, size of their air-ships etc. are well explored.

Decimal System and Large Numbers: This text contains large numbers with decimal system in good amount to count the number of family members of astral bodies in *Jambūdvīpa, Lavaṇasamudra, Dhātakīkhaṇḍa, Kālodadhīsamudra* etc. For example, the number of stars in half *Puṣkara*[2] island is *chaṇṇautiṁ sayasahassāiṁ coyālīsaṁ sahassāiṁ cattāriya sayāīṁ tārāgaṇakoḍikoḍīṇāṁ* i.e. 9644400×10^{14} ($\approx 9 \times 10^{20}$) stars.

Fraction: Use of fraction was their need to calculate the days, motion etc. accurately. For example, the number of *muhūrta* in one *nakṣatra* month is, *tā aṭṭha egūṇavīse muhuttasae sattāvīsaṁ ca saṭhibhāge muhuttassa āhie tti vadejjā*[3] (1.1.8) i.e. $819\frac{27}{67}$ *muhūrta*,

There are 1830 days and 67 *nakṣatra* month in a *yuga* (5 years). And one day and night has 30 *muhūrta*.

67 *nakṣatra* month = 1830 days

therefore, 1 *nakṣatra* month = $\frac{1830}{67} = 27\frac{11}{67}$ days

30 *muhūrta* =1 day

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[1] *Uttarajjhayaṇāṇi*, p. 602, *sūtra,* 36.21.

[2] *Sūrapaṇṇatti Caṅdapaṇṇatti (Uvaṅgasuttāṇi),* synod by Ācārya Tulsī, ed. by Yuvācārya Mahāprajña, Jain Vishva Bharati, Ladnun, Vol.II, 1989, p. 686, *sūtra* 19.15.

[3] *Sūryaprajñapti Candraprajñapti*, ed. by Yuvācārya Miśrīmal ji Madhukar, with Hindi translation and appendices, Agam Prakashan Samiti, Beawar, 1st edn., 1995, p. 5.

Therefore the number of *muhūrta* in one *nakṣatra* month = $\frac{1830}{67} \times 30 = 819\frac{27}{67}$*muhūrta*

Similarly the number of *muhūrta* in sun-month (*sūrya māsa*),[1] moon-month (*candra māsa*)[2] and in seasonal-month (*ṛtu-māsa*)[3] are also calculated as 915, $885\frac{30}{62}$, and 900 respectively. Fractions generally appear in calculation of the length of the day and night, heat-area of the sun length of shadows etc.

Geometry: Without geometry explanation of circular motion, angular degree, calendar etc. is not possible. What is the shape of the air-ships of moon and sun? They have listed sixteen[4] opinions about the shape in the chapter 4 like square, equilateral, parallelogram, rhombus irregular-rhombus, quadrilateral, ring, irregular-ring, semi-circular ring, umbrella-like, mansion-like, shop-like, palace-like, temple-like, theatre-like, pinnacle-like, bed-like and play ground-like.

2.9.10 *Ṣaṭkhaṇḍāgama* (SK)

The *Ṣaṭkhaṇḍāgama* of *Ācārya* Puṣpadanta and Bhūtabali holds an important place among Jaina literature. As per the belief of Digambara tradition this is the only text surviving today and written on the basis of *pūrva* (of 12th *aṅga*) and hence it is as respectful as *Āgama*. It is an extract of second *pūrva agrāyaṇī* and some portions of the fifth *aṅga Viāhapaṇṇatti* (*Bhagavaī*) taught by Ācārya Dharasena (2nd CE) to Puṣpadanta and Bhūtabali. Puṣpadanta composed the first 177 *sūtra*-s and Bhūtabali wrote the rest, the total being 36000 *sūtra*-s [In fact, first five sections (*khaṇḍa*-s) contain 6000 *sūtra*-s (on which Vīrasena wrote encyclopedic commentary Dhavalā) and 6th *khaṇḍa* called Mahābandha contains 30000 *sūtra*-s and for long time called Mahādhavala). It is an ocean of knowledge. It talks about fourteen various stages of spiritual development (*guṇasthāna*) of the souls, their numbers in different realms, area, life-span, *kaṣāya*, types of *karma* , various combination of karmic bondage, knowledge, right faith (*samyaktva*), aggregation of atom (*paramāṇu*), ontology etc. The beauty of this text lies in explaining the same concept of

[1] *Sūryaprajñapti Candraprajñapti*, p.145, *sūtra* 10.72.3.

[2] *Ibid*, p.144, *sūtra* 10.72.2.

[3] *Ibid*, p.145, *sūtra* 10.72.4.

[4] *Ibid*, pp. 41-42.

soul through four different perspectives –substance (*dravya*), space (*kṣetra*), time (*kāla*) and state of the soul (*bhāva*). Six commentaries have been written on it and very famous is written by ācārya vīrsena in 816 A.D. is *Dhavalā*.[1]

The subject matter of *Ṣaṭkhanḍāgama* resembles to a great extent with *Paṇṇavaṇā* which is earlier to it.

Large Numbers and Decimal system: The number of developed (*paryāpta*) deluded souls mentioned is very large number. It is stated as *koḍākoḍākoḍīe uvari koḍākoḍākoḍākoḍīe heṭṭhado chaṇhaṁ vaggāṇamuvari sattaṇhaṁ vaggāṇaṁ heṭṭhado*[2] (1.2.45), which means it lies between $10^7 \times 10^7 \times 10^7$ and $10^7 \times 10^7 \times 10^7 \times 10^7$.

They have also talked of the number of bound physical human-bodies as in ADS and PS. In SK it is given as: Let's say the number of humans is H. Then,

$$2^{2^6} < H < 2^{2^7}$$

or, $2^{64} < H < 2^{128}$

It shows that *Ācārya*-s in that period were well-versed with such huge calculations and the relation between decimal and dyadic scale.

saṅkhyāta, asaṅkhyāta* and *ananta: The section *davvapamāṇa* is completely dedicated to number as its name depicts *davva* means number i.e. *saṅkhyāta, asaṅkhyāta* and *ananta*. How many souls are endowed with perverted faith (*mithyā dṛṣṭi*)? The author says, *ananta* (*sūtra* 1.2.2).[3] The numbers of such souls in heaven is *asaṅkhejjā*. The number of deities in *sarvārthasiddha vimāna* is *saṅkhejjā.* The number of souls between 6th *guṇasthāna* to 13th is *saṅkhejjā* (*sūtra* 1.2.105, p.73). The number of souls endowed with clairvoyance (*avadhi jñāna*) and mind-reading knowledge (*manah-paryava jñāna*) in each of 6th to 12th *guṇasthāna* is *saṅkhyāta* (1.2.145-146, p.78).

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[1] *Dhavalā* commentary on *Ṣaṭkhanḍāgama* of Puṣpadanta and Bhootabali by Vīrasenācārya, trans. in Hindi by Hiralal Jain, Jain Sanskrit Sanraksha Sangha, Solapur, Vol. I, 2000, p. ii (preface).

[2] *Ṣaṭkhanḍāgama* of Ācārya Puṣpadanta and Bhūtabali, trans. by Sumatibai, Balchand Devachand Shah, Satara, 1965, p. 65.

[3] *Ibid*, p. 54.

Analogy-measures were also in use to explain the duration. For example, the maximum duration of *jñānāvaraṇīya, darśanāvaraṇīya, asātā vedanīya* and *antarāya karma* is 30 *koḍākoḍī sāgaropama* (1.9-6.4).[1]

Pṛthaktva: There are plenty of usages of this concept in this text. For example, the number of souls in 6th *guṇasthāna* is *koḍipuhuttaṁ* which means between 1×10^7 to 9×10^7 *pṛthaktva*. *Sūtra* 1.2.14[2] states the number of souls in 13th *guṇasthāna* is *sadasahassapudhhattaṁ* (10^5 *pṛthaktva*).

Rules of Indices: It is very clear that rules of indices were known. The author writes that the number of perverse souls in hell if explained in terms of area is equal to space-points in *asaṅkhyāta jagaśreṇī* (innumerable world-lines) which form innumerable-th part of world-plane (*jagapratara*) and the width of these world-lines is equal to product of 1st square root of *sūci-aṅgula* (index-finger) and 2nd square root of index-finger (1.2.17),[3] i.e. if we denote *sūci-aṅgula* byα, then

$$\text{width} = \sqrt{\alpha} \times \sqrt{\sqrt{\alpha}} = \alpha^{\frac{1}{2}} \times \alpha^{\frac{1}{4}} = \alpha^{\frac{3}{4}}$$

Infinitesimal: The text is full of references to infinitesimal quantities. Gross air-bodied beings are equal to the space-points existing in numerable-th part of the *loka* (1.2.94, SK, p.72). The living beings in 2nd to 5th *guṇasthāna* are accommodated in innumerable-th part of the cosmos. Similarly, the accommodation of other beings is also articulated in the text. The 10th *bhāgābhāgāṇugamo* part of second section is dedicated to explain the number of souls in comparison to other souls.

Maxima and Minima: *alpabahutvānugama*[4] of SK has many references of comparative magnitude. For example, the relation between possible numbers of the souls in various *guṇasthāna* is explained in this manner; the number of souls in 6th *guṇasthāna* (G_6) is *saṅkhyāta* times of those in 7th (G_7) one. Souls in 5th (G_5) are *asaṅkhyāta* times of those in 6th (G_6), those in 2nd (G_2) are *asaṅkhyāta* times of those in 5th (G_5), those in 3rd (G_3) are *saṅkhyāta* times of those in 2nd (G_2), those in 4th (G_4) are *asaṅkhyāta* times of those in 3rd

[1] Śāstrī, Bālacandra, *Ṣaṭkhaṇḍāgamapariśīlana,* Bharatiya Gyanpitha Prakashan, 1st edn., 1987, p. 301.

[2] *Ṣaṭkhaṇḍāgama,* p. 58.

[3] *Ibid*, p. 60.

[4] *Ibid*, p. 55.

(G_3) and those in 1st (G_1) are *ananta* times of those in 4th *guṇasthāna* (G_4). In notation it can be written as :

$$G_6 = S \times G_7,\ G_5 = A \times G_6,\ G_2 = A \times G_5,\ G_3 = S \times G_2,\ G_4 = A \times G_3,\ G_1 = I \times G_4,$$

(Where S= *saṅkhyāta,* A= *asaṅkhyāta*, I= *ananta* and G_s denote the number of souls in *guṇasthāna*[1] indicated by subscript of G) (1.8.9-14, p. 229)

Many other concepts like *kaṣāya, leśyā,* knowledge, deities, and hellish beings are explained with reference to their relative size.

2.10 Mathematics in Non-*āgamic* Text

2.10.1 *Tiloyapaṇṇatti* (TP)

Tiloyapaṇṇatti is the creation of Ācārya Yativṛṣabha who seems to be a great mathematician also. The date of this text is estimated to be 5th/6th century A.D. This text gives a systematic explanation of basic mathematics to explain the cosmography. It gives a glance of Jaina history. The work contains a total of 5677 verses divided into 9 chapters. Along with the decimal place value notation we find his contribution in this work on the subjects like measurement system, number system, geometry, solid geometry, series, and logarithms.

Analogy-Measure: *sūtra* 93 of this text there are 8 types[2] of simile measure- *palyopama, sāgaropama, sūcyaṅgula, pratarāṅgula, ghanāṅgula* (cube of an *aṅgula*), *jagaśreṇī, lokapratara and loka. palya* is further divided in three- *vyavahāra, uddhāra* and *addhā* where the first one is used to measure the number, second to get the number of oceans and islands and third one to calculate the life-span. The author attempts to calculate *uddhāra palyopama* as $\approx 10^{44}$.

Yativṛṣabha has prepared the scale of time (*kālamāna*) and *samaya* is the smallest unit of time which is defined in a unique way. Starting from *samaya*, the smallest unit of time, he goes up to gigantic number *acalātma*. 1 *acalātma* = $(84)^{31} \times (10)^{90}$ years :

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[1] *Ṣaṭkhaṇḍāgama*, p. 229.

[2] *Tiloyapaṇṇatti* of Yativṛṣabha, ed. by Chetanprakash patani, Viśuddhamati Mātājī, Chandraprabh Jain Digambara Jain Atishaya Kshetra, Alwara, Vol. I, 3rd edn., 2008, p. 21.

$= 4494256631493854619752955668188751627516065267245169602723 84\times 10^{90}$

$= 4.494\times10^{59}\times10^{90} = 4.49\times10^{149}$ years

Under the category of uncountable number, *palyopama*, *sāgaropama* etc. are defined; some of them are given here :

1 *vyavahārapalya* = $19/24 \times 43 \times (2000)^3 \times (4)^3 \times (24) \times (500)^3 \times (8)^{21}$

$= 413,452,630,308,203,177,749,512,192 \times 10^{18}$

$\approx 4.13\times10^{44}$

1 *vyavahārapalyopama* =100 *vyavahārapalya*,

1 *uddhārapalya* = *vyavahārapalya*×uncountable crore years.

Similarly, *addhāpalya*, *vyavahāra playopama*, *uddhāra sāgaropama* etc. are also defined.[1]

To describe the area of the cosmos (*loka*) the scale defined is:

For example,

infinite *paramāṇu* = 1 *uvsannāsanna skandha*

8 *uvsannāsanna skandha* = 1 *sannāsanna*

8 *sannāsanna* = 1 *truṭireṇu skandha*

In this way by keep multiplying the previous *skaṅdha* by 8 (*sūtra* 102-106, Vol. I, p. 23) the resulting *skandha* are consecutively *trasreṇu, rathreṇu, uttam bhogabhūmikā bālāgra, Madhya bhogabhūmikā bālāgra, jaghanya bhogabhūmikā bālāgra, karma bhogabhūmikā bālāgra, līkha, jūṁ, jau,* and *aṅgula (utsedha aṅgula)*. Like ADS *aṅgula* is further divided in three –*utsedha*, *pramāṇa* and *ātmāṅgula* which are used to measure the different things. Furthermore, the scale goes up to *yojana* from *aṅgula*. These are used to calculate the height of human beings, animal, hellish beings and the deities.

Mensuration: TP proceeds to calculate the volume of all three parts of *loka* –upper, medium and lower with every small detail. *sūtra* 118 (vol. I, p. 26) describe the area of all three kinds of *palya* (pit) with the diameter of one *yojana paramāṇu yojana* y^3

[1] Jain, Anupam and N. Shiva Kumar, *Ācārya Yativṛṣabha and His Mathematical Contributions*, Arhat Vacan, April-Sep, 2005, pp. 31-42.

Circumference of the *palya* (pit) = $\sqrt{D^2 \times 10} = \sqrt{10^{10} \times 10} = \sqrt{10^{11}}$ where it takes $\sqrt{10} \approx \frac{19}{6}$.

∴ Volume of the *palya* = $\pi r^2 h = \frac{19}{6} \times \frac{1}{2} \times \frac{1}{2} = \frac{19}{24}$ cubic *pramāṇa yojana* (y^3)

The cubic volume of all 7 hells (*sūtra* 174-191, pp. 47-56) and that of the heavens are also calculated. For calculation of area of vertical cross-section of universe *Ācārya* Yativṛṣabha employs number of different methods.

Large Numbers and Decimal Place Value System: TP counts the number of *bālāgra* (hair-tip) *vyavahāra palya* which is $10^{44.616}$ (against Śvetāmbara value $\approx 10^{44.3}$). The numbers are used to record the history of all the *tīrthaṅkara* from various perspectives. It calculates the duration of the order (*Tīrtha-kāla*) of all the 24 *tīrthaṅkara*-s of current *ossappiṇi*. E.g. *Tīrtha-kāla* (duration of religious order) of Bhagawāna Ṛṣabha survived for one *pūrvāṅga* (8400000 year) plus five *koṭā koṭī* (5×10^{14}) *sāgaropama,* that of Bhagawāna Ajitanātha for three *pūrvāṅga* and 3×10^{14} *sāgaropama* and so on. It provides rich statistics in respect of number of monks and nuns, number of other schools, number of *pūrvadhara*, teachers, dynasties, important personalities and clairvoyants etc. and thus have great historical value.

It is very interesting to note that TP contains 2 relations [1, 131-132], viz.

$$(palya)^{\frac{log_2(addhāpalya)}{asaṅkhyāta}} = sūcyaṅgula.$$

$$(ghanāṇgula)^{\frac{log_2(addhāpalya)}{asaṅkhyāta}} = jagaśreṇī.$$

which shows advances achieved in mathematics in TP

Zero: Zero as a place value was in use by then. As mentioned in *gāthā* 123-124 (p. 28):

aṭṭhārasa ṭhāṇesuṁ suṇṇāṇiṁ do ṇavekka do ekko
paṇa-ṇava-caukka-satta, saga-sattā ekka-tiya-suṇṇā.

do aṭṭha suṇṇa-tia-ṇaha-tiya-chakkā doṇṇi-paṇa-caukkāṇi
tiya ekka caukkāṇiṁ, aṅka kameṇa pallaromassa.

i.e. 18 zeros at the end, two, nine, one, two, one, five, nine, four, seven, seven, seven, one, three, zero, two, eight, zero, three, zero, three, six, two, five, four, three, one and four are the digits of *palya* in order.

2.10.2 *Lokavibhāga* (LV)

LV is work, which explains about cosmos (*loka*) in detail. It consists of 10 sections, 1737 *śloka*-s and additionally 117 proses included from other literatures like TP, *trilokasāra*, and JDP etc.in Sanskrit. It is authored by *Ācārya* Siṁhatilakasūrī (12th C. A.D.). Siṁhatilakasūrī clearly says that his work in Sanskrit is literal translation of work in Prakrit of Muni Sarvanandi who composed it in *śaka* era 350 (458 AD). This work is completely mathematical. It verifies that the author was well acquainted with the knowledge of universe (*loka*) and mathematics. The subjects it deals with are all three sections of *loka* comprising mountains, continents, rivers, forests, human world (*ḍhāī dvīpa*), oceans, islands, time-cycle, astronomy, hells, heaven etc. with their dimensions. It's an ancient work which gives the knowledge of cosmos.

saṅkhyāta, asaṅkhyāta* and *ananta: Here we find the time-scale for *saṅkhyāta* but it differs from the BS and ADS. Here the number goes up to *hasta-prahelita* and *acalātmaka* (= $(84)^{31} \times (10)^{90} \approx 4.49\times10^{149}$, as against *Śīrṣaprahelikā* of Śvetāmbara tradition). Beyond that starts *asaṅkhyāta.* The word *saṅkhyāta* and *asaṅkhyāta* are used many times. E.g. after *asaṅkhyāta koṭi* years the second *kulkara* took birth (5.39, p. 88); *saṅkhyāta āvalikā* equals one *ucchvāsa.* In chapter ten while describing the heaven and their celestial planes *saṅkhyāta,* and *asaṅkhyāta* are mentioned frequently. The very first *śloka* of chapter 9 says *anantadarśanajñānān* i.e. infinite perception and infinite knowledge.

Decimal System and Large Number: The order of the number written in the text shows the well-familiarity with decimal system. For example, *lakṣasthānād kramād grāhyah sapta dve dve ṣaḍekakaṁ, trīṇi cāsya parikṣepo yojanānāṁ pramāṇatah* (1.8)[1] - here *lakṣasthānād kramād grāhyah* means start the number from the place of lac that 6th place of the digit and the whole number becomes 3,16,227 *yojana.* It is the direct proof of the use of decimal system in that early age. At many places we can see this evidence. Large numbers are also used many times. In *sūtra* 1.256 (p. 31) the number of goddesses of each *lokapāla* deity is 35×10^6. In *sūtra* 4.34 the inner circumference of *Nandīśwara dvīpa*[2] mentioned as *trīṇi pañca ca saptaiva dve śūnyaṁ dve ca rūpakaṁ ṣaṭ trīṇi gaganaṁ caikamantah paridhirucyate* i.e. three, five, seven two zero, two, one six, three, zero and one (written as

[1] *Lokavibhāga* of *Ācārya Siṁhasūrī*, trans. in Hindi by Balcanda Siddhānta Śāstrī, Gulabchand Hirachand Doshi, Solapur, 1962, p. 2.

[2] *Lokavibhāga*, p. 76.

right to left and to be read from left to right, 10361702753) *y*. At many places the dimension is calculated as such large number without help of any gadget.

Lakṣa (*lākha*, 10^5) word is also found at many places as mentioned in *sūtra* 4.32, *caturśīti ca lakśāṇi triṣaṣṭiśatakoṭayah*[1] i.e. 84 *lākha* and one hundred sixty three crores (=1638400000). It shows that *lakṣa* was in use by 458 C. A.D.

Zero: LV is credited with earliest evidence of use of zero having place value. There are numerous examples of this in this work. For example, in *sūtra* 3.56 (p. 68) *saptaṁ dvikaṁ catuṣkaṁ ca śūnyaṁ śūnyaṁ ca saptakam ekamekaṁ*- the circumference of *puṣkara* island, is a number (read from right to left as) seven, two, four, zero, zero, seven, one and one (i.e. 11700427 *yojana*) where it clearly uses Sanskrit word *śūnya* for zero. In *sūtra* 6.122 it is mentioned as *śūnyatrikāṣṭkaikena*[2] i.e. zero, three, eight and one (=1830) which is written from left to right. (see previous paragraph for another example.)

Mathematical Operations: Many mathematical operations are clearly mentioned here, like, *guṇasaṅkalena rūpeṇa*, which means by adding the given numbers. In *sūtra* 3.10 (p. 61) *dvikṛti* is mentioned for square of two and for multiplication *śataṁ pūrṇaṁ sahasra guṇitaṁ- guṇitaṁ* is used. It shows that the author was highly conversant with all mathematical operations.

Fraction: Fractions are found employed throughout LV. For example, the length of southern Bharat is *dveśate triṅśadaṣṭau ca kalāstisraśca,* which can be expressed as $238\frac{3}{19}$ *yojana* (1.47, p. 5). The chord of half *Vijaya* is $10720\frac{11}{19}$ *yojana*. *Sūtra* 1.114 (p. 13) mentions that the dimension of Mountain *Vijayārdha* etc. is *sahsra vistṛtā mule madhye tattyurhīnakāh*. i.e. at the base it is 1000 *yojana* and in the middle 1/4th less than its base, which will be 1000-1000/4=750 *yojana*. Complicated fractions are also tackled here. For E.g. in *sūtra* 6.124 we find, *ṣaṭ ca hāni trisaptati śataṁ bhāgāh saptādaśaśataṁ punah, caturviṅśati yutaṁ*[3]*...*i.e.$173\frac{1724}{1830}$. We find many such examples of fraction throughout the text.

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[1] *Lokavibhāga*, p. 76.

[2] *Ibid*, p. 119.

[3] *Ibid*, p. 119.

Infinitesimal: Furthermore, it (10.293)[1] the celestial gods of *anuttara* air-ships can know the infinite-th part of the *karma*.

2.10.3 *Jyotiṣakaraṇḍaka* (JK)

It is an astronomical work, authored by unknown *Ācārya*. The available text is with the commentary by *Ācārya* Malayagiri. JK explains about the motion of sun, and moon, waxing and waning of moon, their paths, number of stars, asterism, and length of day and night. It is a *prakirṇaka* having 21 chapters (*adhikāra*) and 266 proses. The word *gaṇita* (math) appears many times in the text as *cande muhuttagaṇitaṁ*[2] (2.45).

Finite and trans-finite numbers: JK gives the time scale starting from *samaya* to *śīrṣaprahelikā*. However, the value of *śīrṣaprahelikā* found in this text is far higher here viz. $(8400000)^{36}$ i.e. a 250 digits number. The author classifies the numbers in three categories mentioning from *sutra* 7 it says: *saṅkhejjamasaṅkhejjo aṇanta kālo u nidiṭṭho*[3] a) *saṅkhyāta*: from *samaya* to *śīrṣaprahelikā* are *saṅkhyāta*, b) *asaṅkhyāta*: simile measure like *palyopama* is and c) *ananta*: time as a whole is *ananta* where *ananta utsarpiṇī*-s and *avasarpiṅī*-s are elapsed. Height and distance measure scales are also discussed.

Decimal Place Value System and Large numbers: Any counting represented here is in the form of decimal notation. The family of moon in JD, *Lavaṇasamudra* (LS) and other islands are introduced here. For example, the total number of asterism, stars etc. under one moon in JD mentioned in *sutra* 124 is *cāyatthiyam ya sahassa nava ya saya pañca sattara honti. egasasi parivāro tārāgaṇa koḍīkoḍīṇaṁ*.[4] i.e. $669750000000 \times 10^{14}$ and in LS it doubles. The order of the numeric words written in *prākṛta* is in the form of decimal place value system supposed to be read from right to left.

Fraction: Five types of years are described in Jaina astronomy namely *āditya, karma, candra, nakṣatra,* and *abhivṛddhi*. These years with number of days in each are explained in detail with precise values. For example, *sūtra* 2.45 (p. 18) tells *bisaṭṭhibhāgā ya*

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[1] *Lokavibhāga*, p. 209.

[2] *Jyotiṣkaraṇḍaka,* with commentary by Malayagiri in Sanskrit, Shri Rishabhdeva ji Kesharimal ji Shvetambara Sanstha, Ratalam, 1928, p. 18.

[3] *Ibid*, p. 4.

[4] *Ibid*, p. 71.

pannāsaṁ i.e. 50/62. *Sūtra* 2.60 (p. 37) mentions that life-span of animals, humans, deities and hellish beings, the duration of *karma* , time cycle etc. are to be calculated using *sūrya saṁvatsara*.

2.10.4 *Jīvasamāsa* (JS)

Jīvasamāsa is one of the oldest texts of Jaina philosophy. The author is not known. It covers various subjects like Jaina astronomy, geography, cosmology, time-measurement etc. along with the current prevailing system of measurement and various types of species of the living beings. This text has 8 sections known as *dvāra* literally meaning door but connotes a section containing 286 *sūtra*-s. Out of them five sections–*parimāṇa dvāra* (measurement section), *kṣetra dvāra* (space section), *sparśana dvāra* (contact section), *kāla dvāra* (time section), *ananta dvāra* (infinite section) and the last *alpabahutva dvāra* (maxima-minima section) are dedicated to various mathematical measurements.

Decimal System and Large Numbers: *sūtra*-s 40-44 of the first chapter state the number of lineages and species of all the living beings. For example,

> *bārasa satta ya tinni ya satta kulakoḍi sayasahassāiṁ*
> *neyā puḍhavidagāgaṇivāūṇaṁ ceva parisaṅkhā* (JS, *sūtra* 40)[1]

The number of lineages of *pṛthvīkāya* is 12×10^{12} (*bārasa kulakoḍi sayasahassa*), that of *apkāya* is 7×10^{12} (*satta kulakoḍi sayasahassa*) of *tejaskāya* is 3×10^{12}, of human beings is 12×10^{12} and so on. So the total number of lineages is 197.5×10^{12}. It gives the number of lineages in various realms (*gati*) of living beings and arrives at the figure of 8400000 as the total number of species. Thus it appears as a common belief in Vedic, Jaina and Buddhist scriptures of total of 8400000 species.

The entire time scale from *samaya* up to *śīrṣaprahelikā* (*sutra* 114,115)[2] is introduced with its value same as that in ADS.

***saṅkhyāta, asaṅkhyāta,* and *ananta*:** *sūtra* 136 elucidates three types of number field (*gaṇita saṅkhyā*)[3]- *saṅkhyāta, asaṅkhyāta,* and *ananta*. In next *sūtra asaṅkhyāta* is further

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[1] *Jīvasamāsa*, trans. by Sādhvī Vidutprabhā in Hindi, Parshvanath Vidyapith, Varanasi, 1st edn., 1998, p. 34.

[2] *Ibid*, pp. 88-89.

[3] *Ibid*, p. 100.

classified into nine categories. Treatment of these three classes is mentioned in *sūtra* 139. While detailing procedure for arriving at *asaṅkhyāta* it stipulates 2 measure-pits viz. *śalākākuṇḍa* and *pratiśalākākuṇḍa,* while in ADS there is only one measure-pit viz. *śalākākuṇḍa*. *sūtra* 71 tells that human beings and animal (*tiryañca*) having the age of *asaṅkhyeya*[1] years can have four psychic colours (*leśyā*-s)–black (*kṛṣṇa*), blue (*nīla*), grey (*kāpota*) and red (*tejo*). There are two types of *paramāṇu*, subtle (*sūkṣma*) and gross/practical (*vyavahārika*) and the number of fine atoms (*paramāṇu*-s) in a gross atom (*vyavahārika paramāṇu*) is *ananta (sūtra* 95, p.82). These units are used to count the time. For example, *asaṅkhyāta samaya* makes one *āvalikā* and *saṅkhyāta āvalikā* makes one breath (*śvāsocchavāsa*).

Measurement-scale: In ancient age goods were measured or weighed by specific objects. Five types of measure-scales are mentioned (*sūtra* 88, p. 78):

1. *māna* – scaled receptacles were used to measure the grains or liquid.
2. *unmāna* – some weighing tools to weigh the solid goods.
3. *avamāna* – some linear scale to measure the land etc.
4. *gaṇima* – numerals to count the numbers of objects.
5. *pratimāna* – highly precise weighing tools to weigh the precious things like gold etc.

Analogy-measure: Like *Tiloyapaṇṇatti* the scale of simile measure is mentioned in *sūtra* 98 *paramāṇu, trasareṇu, rathareṇu, uttama bālāgra, līkha, jūṁ, jau,* and *aṅgula (utsedha aṅgula)* which goes up to *yojana* with the classification and definition of *aṇgula*. *utsedha aṅgula* was used to measure the height of the body, *ātmāṅgula* for house, bed etc. (*kṣetra pramāṇa*), *aṅgula yojana* to measure islands and oceans (*sūtra* 104, p.86). The concept of *palyomama* and *sāgaropama* are also explained here.

Rules of Indices: Number of living beings in various realms is calculated by the formula involving square and higher roots. E.g. the following *sūtra* quotes the number of mansion dwelling gods (*bhavanapati*), hellish beings in first hell (*dhammā*) and number of celestial gods of first heaven (*saudharma*) :

seḍhīsūipamāṇaṁ bhavaṇe dhamme taheva sohamme
aṅgulapaḍhamaṁ viyatiyasa-maṇantaravaggamūlaguṇaṁ.

(*sūtra* 157, p. 132)

[1] *Jīvasamāsa*, p. 60.

i.e. the number of mansion dwelling gods (*bhavanapati*) is the product of the first and first square root of the number of space-points in the *śreṇī* (= *asaṅkhyāta yojana pramāṇa aṅgula*). i.e. $a \times \sqrt{a} = a^{\frac{3}{2}}$

The number of hellish beings in first hell (*dhammā*) is the product of the first and second square root of the number of space-points i.e. $\sqrt{a} \times \sqrt{\sqrt{a}} = a^{3/4}$

The number of celestial gods of first heaven (*saudharma*) is the product of the first and third square root of the number of space-points i.e. $\sqrt{\sqrt{a}} \times \sqrt{\sqrt{\sqrt{a}}} = a^{3/8}$.

Similarly, the number of hellish beings of other hells and the celestial gods of other heavens are also determined through the same process.

Maxima, Minima: The eighth section of this text explains about maxima and minima. *sūtra* 271 mentions that among all the living beings the number of humans (H) is minimum, hellish beings (H_L) are *asaṅkhyāta* times the humans, deities (D) are *asaṅkhyāta* times the hellish beings, liberated souls (L) are infinite times the deities and animals (A_n) are infinite times the liberated souls.[1] i.e.

$H_L = A \times H$; $D = A \times H_L$; $L = I \times D$; $A_n = I \times L$ (where A *asaṅkhyāta* is and I is *ananta*)

$H < H_L < D < L < A_n$.

2.10.5 *Sabhāṣyatattvārthādhigama Sūtra* (TSB)

Sabhāṣyatattvārthādhigama Sūtra of Ācārya Umāsvāti is a well-respected and accepted text by both of the Jaina traditions Śvetāmbara and Digambara. The available sources claim that he was the first Sanskrit author in the Jaina history. He also composed a commentary on it and appended it to main work. He has discussed ontology, metaphysics, epistemology, *karma*, geography, astronomy and ethics in his work. The text contains 10 chapters and 375 *sūtra*-s. Though it's not a mathematical work, yet there is no doubt that Ācārya Umāsvāti was well-versed with mathematical formulas and calculations. In the first chapter eight ways to know the real entities (*tattva*) are mentioned and out of them six like *saṅkhyā, kṣetra, kāla*[2] etc. are related with mathematics. Some of them are presented here.

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[1] *Jīvasamāsa,* p. 223.

[2] *Sabhāṣyatattvārthādhigama Sūtra* of Ācārya Umāswāti, with his commentary, trans. by Khubachanda Siddhanta Shastri in Hindi, Shrimad Rajchandra Ashram, Agas, 1992, p. 31.

Decimal system and Large Numbers: Umāsvāti in his commentary of *sūtra* 4.15 (p.210) gives decimal scale which differs from that in ADS. After *pūrvāṅga* it goes up to *hūhū* only (omitting quantities with suffix *aṅga*) and ends at $(8400000)^{11}$. For geographical measurements decimal system was a necessity. For example, the dimensions of hells go up to *asaṅkhyāta koḍākoḍī* (10^{14}) *yojana.* Height of the first hell *ratnaprabhā* is 1,80,000 lac *yojana* (*sūtra* 3.1, p. 140). While dealing the life span of all the living beings simile measure is mentioned. Life span of the living beings in heaven from 1 to 7 is 1, 3, 7, 10, 17, 22, and 33 *sāgaropama* respectively (*sūtra* 3.6, p.155). Similarly, number of residences in each hell, life span of hellish beings, dimension of Mt. Meru, number of stars under a moon and many more things represent the large numbers with decimal system.

***saṅkhyāta, asaṅkhyāta,* and *ananta*:** Many times these words are used in the text. Explaining the right faith, it is asked how many right faiths are there? The answer is *asaṅkhyāta* right faiths (*samyak darśana*) and *ananta* right vision (*samyak dṛṣṭi*) are there. There are six types of clairvoyance knowledge and one is *hīyamāna* clairvoyance, which diminishes gradually. The person having this knowledge in the beginning can know *asaṅkhyāta*[1] oceans and islands (*sūtra* 2.52).

Sūtra 5.7 (p. 253) tells about the *asaṅkhyāta* number of *pradeśa* of *dharmāstikāya* (medium of motion) and *adharmāstikāya* (medium of rest).

Infinitesimal: As mentioned earlier decreasing (*hīyamāna*) clairvoyance keeps diminishing. It can diminish up to *aṅgulasaṅkhyeya bhāgāt pratipatyeva* i.e. innumerable-th part of an *aṅgula* (*a*/A). The holder of such knowledge can know the material things existing in the area up to *a*/A in the universe.[2]

Fractions: The use of fractions is found at many places especially to explain the geography. In middle cosmos, *Jambūdvīpa* is located in the center. The dimension of seven major areas in JD is mentioned precisely with the fractional value. For example, *sūtra* 3.11 (pp. 166-168) mentions the north-south span (*iṣu*) of *Bharata* is *pañca yojana śatāni ṣaḍaviṅśāni ṣaḍcaikonaviṅśatibhāgā* i.e. $526\frac{6}{19}$ *yojana.* Its east-west chord (*jyā*) is *caturdaśa sahasrāṇi catvāri śatānyekasaptatīni ṣaṭ ca bhāgā viśeṣā* i.e. little more than

[1] *Sabhāṣyatattvārthādhigama Sūtra*, p. 132.

[2] *Ibid*, p. 46.

$14471\frac{6}{19}$ *y* and southward arc (*dhanupṛṣṭha*) is *caturdaśa sahasrāṇi śatāni pañcāṣṭaviṅśānyekādaśa ca bhāgāh sādhikā* i.e. little more than $14528\frac{11}{19}$ *y*.

***pṛthaktva*:** *sūtra* 4.22 mentions that the deities with the life of one *palyopama* desire to have food after *pṛthaktva* days. The maximum height of the body of the liberated soul is 500 and *pṛthaktva dhanuṣa*.

Geometry: In commentary of *sutra* 3.11, Umāsvāti gives 3 formulas:

a) $C = \sqrt{10d^2}$

b) $c = \sqrt{4h\,(d-h)}$

c) $s = \sqrt{d^2 + 6h^2}$

Where C is circumference of circle, c = chord of circle and *h* is height of segment bounded by c. For c he writes take four times product of arrow *h* and excess of diameter over arrow i.e. $(d-h)$. This is equal to square of the chord c of the circle i.e. $c^2 = 4h(d-h)$.

Maxima-minima: TSB gives a simple example. *Sūtra* 2.39,40 (pp. 112, 113) states that space-points (*pradeśa*-s) of physical (*audārika,* A_u) body are least in number. Innumerable times of these are of transformational (*vaikriya,* V) body, innumerable times of these are of assimilative (*āhāraka,* A_h), while infinite times of these are of fiery (*taijasa,* T) body and infinite times of these are of karmic body (K). i.e.

$V = A \times A_u$; $A_h = A \times V$; $T = I \times A_h$; $K = I \times T$

$A_u < V < A_h < T < K$.

2.10.6 *Viśeṣāvaśyaka Bhāṣya* (VB)

VB is a commentary on *āvaśyaka*, which is *aṅgabāhya Āgama*, written by Jinbhadragaṇī Kshamāśramaṇa (489-504 A.D.). He has authored many texts and few of them like VB, *Bṛhatsaṅgrahaṇī, Bṛhatkṣetra samāsa* are mathematical in nature. These works clearly show that Jinbhadragaṇī was a scholar of great mathematical prowess. VB is in fact treasure of many Jaina philosophical ideas on epistemology, ethics, ontology, *karma*, *anekāntavāda*, history and cultural values etc. To present the concepts precisely and perfectly mathematical treatment becomes effective. It's so encyclopaedic that thing like cure of disease is also explained by giving the examples from mathematics. For example, some number if divided by any number then it gets reduced and if not divided remains

same. Similarly, any disease is cured if medicines are taken properly and precautions are considered, otherwise it remains as it is.

Mathematics in Jainism started far before the era of Mahāvīra in the age of Ṛṣabha, which is mentioned in *gāthā* 1618 (vol.I, p.348) that *gaṇitaṁ saṅkhāṇaṁ sundarīye vāmeṇa uvadiṭṭhaṁ* i.e. He (Bhagawāna Ṛṣabha) taught mathematics (*gaṇita*) and number (*saṅkhyā*) to Sundarī by left hand.

In *gāthā* 404 (vol.I, p. 96) nine ways to explain the perceptual knowledge (*ābhinibodhika jñāna*) are mentioned in the text and out of them 7 are related with mathematics viz. *dravya* (substance), *pramāṇa* (measurement), *kṣetra* (area), *sparśanā* (contact), *kāla* (time), *antara* (interregnum), *bhāva* (state of soul) and *alpabahutva* (minima-maxima). Some of the mathematical concepts explained in the text are mentioned below.

saṅkhyāta, asaṅkhyāta, anant: At many places these terms are used to express the numbers. For example, the alphabets can be combined in infinite forms (*gāthā* 443,106). Furthermore, *gāthā* 315 mentions, *vatthumatthavayaṇādi pajjayāṇaṅtasattisaṁpaṇṇaṁ* i.e. every object is possessed of infinite attributes (p.74). The words of low voice of a speaker get destroyed after spreading *saṅkhyeya yojana* in the cosmos (*gāthā* 379, p.90).

Exponential growth and decay: Jainism believes that each soul is endowed with knowledge but the degree varies. The knowledge of all *caturdaśa pūrvadhara*-s[1] is not same. It means because of the variance in destruction-cum-subsidence of *karma* (*kṣayopaśama*) in various souls the degree of available knowledge differs exponentially in them and it is mentioned in *gāthā* 141 *jaṁ coddasapuvvdharā chaṭṭhāṇagatā paropparaṁ honti*.[2] According to Jaina philosophy the degree of clairvoyance knowledge is not necessarily stable. It can grow or decay in the same holder. This growth and decay from the perspective of time and area is exponential at the rate of *ạ*/A, and can be in six ways (*ṣaṭsthānapatita*) as mentioned in *gāthā* 724-726 (vol.I, pp. 168-169).

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[1] *coddasupuvvīṇaṁ ajiṇāṇaṁ jiṇasaṅkāsāṇaṁ savvakkharasannivādīṇaṁ jiṇoviva avitahaaṁ vāgaramāṇāṇaṁ* i.e. they are not *tīrthaṅkara* but almost equal to *tīrthaṅkara;*equally truth-speaker and endowed with the power of understanding whole word through a letter only (*sarvākṣara sannipāta labdhi*)

–Śrī Bhikṣu Āgama Viṣaya Śabda koṣa, p. 427.

[2] *Viśeṣāvaśyaka bhāṣya*, Vol. I, p. 34.

Infinitesimal: *gāthā*[1] 348 says that the minimum size visible by eyes is numerable-th part of *aṅgula* (*g*/S). It means any object having such a small size is the minimum visibility of eyes. The question is about the extent of the clairvoyance knowledge (*avadhi jñāna*) from temporal and spatial considerations. The answer in *gāthā* 665 (vol. I., p.156) is given as: the area it can know is the numerable-th (*saṅkhyāta*) part of the *loka* and from temporal viewpoint it can know up to (past and future) numerable-th (*saṅkhyāta*) part of one *palyopama* (i.e. p/S). Elsewhere it is said that the innumerable-th (*asaṅkhyāta*) part of the spoken sound or words (having power of spreading in the entire cosmos) can travel in innumerable-th (*asaṅkhyāta*) portion of the *loka* (*gāthā* 388, vol. I, p. 92). Furthermore, it is said that at least infinite-th (*ananta*) part of knowledge is possessed by all the worldly living beings (of lowest order).

Combinatorics: Jainism believes in five states of soul (*bhāva*) on the basis of rising or destructing state of *karma*. While describing the time-span of those *bhāva* (*gāthā* 2547-2553)[2] the four possibilities are mentioned on the basis of their beginning, end, beginingless point and endless point. What are the possible arrangements of fire-bodied beings on the space-points of *loka* (cosmos)? Six possible combinations are suggested here in *gāthā* 598.[3]

Decimal system and Geometry: Geometrical ideas are also presented in basic form. In *gāthā* 598 (Vol. I. p.142) three possible shapes in which *jīva*-s can be accommodated on space-points of *loka* are mentioned. Those are *caturaṅsaghaṇaṁ payaraṁ seḍhī* i.e. in cube, plane and linear forms. In *gāthā* 3802/3 the circumference of *iṣatprāgbhārā pṛthvī* is mentioned as, *ekā joyaṇakoḍī bātālīsaṁ bhave satasahassā, tīsaṁ ceva sahassā do ceva sayā auṇapaṇṇā* i.e. 1,42,30,249 *yojana* 1 *kośa*, 1766 *dhanuṣa* and $31\frac{1}{2}$ *aṅgula*. This Jinabhadragaṇi calculates from the expression $\sqrt{10} \times 4500{,}000$. Here we can see the use of **decimal place value** notation involving extraction of square root and fractions.

Space and Time relation: Space and time relation is mentioned in this text. For example, one who knows the innumerable-th (*asaṅkhyāta*) part of an *aṅgula* (ą) knows the innumerable-th (*asaṅkhyāta*, A) part of a *āvalikā* (a_v) (*gāthā* 604, p.144). He also clearly

[1] *Viśeṣāvaśyaka bhāṣya*, Vol. I, p. 83.

[2] *Ibid*, Vol. I, pp. 527-528.

[3] *Ibid*, p. 142.

states the relationship between ultimate units of mass (*paramāṇu*), space and time (*samaya*) discussed later in chapter 5.

2.10.7 *Bṛhatkṣetra Samāsa* (BKS)

From mathematical viewpoint this text holds an important place in Jaina literatures. It is penned by Ācaryā Jinabhadragaṇī Kṣamāśramaṇa, the author of VB. His works *Bṛhatkśetrasamāsa* (BK), *Bṛhatsangrahaṇī* (BS), and *Viśeṣāvaśyakabhāṣya* (VB) (discussed above) are all in poetic form where all numbers are expressed in words and no symbols for numerals appear therein. Yet it is abundantly evident that numerals 0, 1, 2, … 9 were evolved then and operations with them were just like those we perform today. Five chapters and 655 *gāthā*-s of BKS basically focused on Jaina geography in detail. The human-world (*ḍhāī dvīpa*) is explored with the dimension of its various places, mountains, rivers, ponds, caves and time-cycle etc. where the mathematical techniques of various operations are introduced. It reflects his command on mathematics and calculations. Few of ideas are mentioned here.

Decimal System, Large number: In BKS the area of *Jambūdvīpa* (JD) is mentioned as

> *satteva ya koḍisayā, nauyā chappannasayasahassā ya*
> *cauṇauiṁ ca sahassa, sayaṁ divaḍḍhaṁ ca sāhiyaṁ.*
>
> *gāuyamegaṁ pannarasa-dhaṇusayā taha dhaṇuṇi pannarasa,*
> *saṭṭhiṁ ca aṅgulāiṁ, jambūdīvassa gaṇiyapayaṁ.*
>
> [BKS, 9-10, (vol. I, p. 35)]

i.e. the area of JD is 790, 56, 94, 150 *yojana,* 1 *gavyūta*, 1515 *dhanuṣa,* and 60 *aṅgula*. Furthermore, the area of *niṣādha mountain*[1] in *gāthā* 115 is 1,42,54,66,569 *y* less than 18 *kalā* (17 *kalā* and 17 *vikalā*). There are plentiful examples of large numbers. The number of stars under two moons in JD is 13396×10^{15}. Large numbers encountered and obtained while extracting square roots are expressed in word forms, spelling out decimal powers as *egā joyaṇakoḍī, lakkhā bāyāla, tīsa sahassā ya, .. do ceva sayā auṇapannā* i.e. 1,42,30,249 *y*. This is circumference (*parirao*) of the human world. (vol. I, *gāṭhā* 1.5, p. 27). Here is one of the earliest instances of use of '*lakkha*' for *sayasahassa* (i.e. 10^5).

[1] *Bṛhatkṣetra Samāsa* of Jinabhadragaṇī Kṣamāśramaṇa, trans. in Gujarati by Ācārya Nityānandasūrīśwara, Bhadrankar Prakashan, Ahmedabad, Vol. I, 1998, p. 127.

Zero: The use of the word *sunna,* and *asunnaga* are found in this text. The processes described in the text, numbers written digit-wise and calculations involving very large numbers leave not an iota of doubt that some symbol of zero existed then and the system of manual calculations existing then was same as employed at present. In other words, this work presents the earliest proof of operations with zero having place value. For example, to obtain the arc (*bāhā*) of north Bharat the method is mentioned and while getting square root it is said that by ignoring the zeros the remainder is 40715(0) and the divisor is 48392(0).

kalalakkhadugaṁ īyā-lasahassā nava sayā ya saṭṭahiyā
sunnam*avaṇeu aṅsaṁ, cau* ***sunnaga*** *satta ega paṇa.*(83)[1]

cau (4) ***sunnaga*** **(0)** *satta* (7) *ega* (1) *paṇa* (5) – the order of the numbers reflects the familiarity with decimal place value system with zero having place value. There are several such examples for zero with place value.

Fractions: Fractions and operations therein come up in calculations of dimensions of various geographical entities. For example, the distance between the mountains which are close to LS is 72114 $\frac{3}{8}$ *yojana*. Jinbhadragaṇī shows great competence in handling fractions and operations with them. In his works in verse 2.33 (Vol. II, p.133) he would express 3/8 as '*tinaṭṭhabhāgā*' (3 out of 8 parts) or fraction 40/95 as '*paṇanauī bhāgāṇa ya duguṇiya vīṣaṁ*' (2 times 20 out of 95 parts). [Vol. II, g. 37, p.137].

Mathematical operations: Arithmetic operations like multiplication, square, square root, area and volume are found at many places. This work provides many formulas. By multiplying chord of JD by 1, 2, 4, 8, 16, 32, and 64; and by dividing them by 190 the diameter of the places (like Bharat etc.), there existing mountains etc. can be obtained (*gāthā* 28).[2] The cubic area of *Vaitāḍhya* Mountain is 5123076 *yojana,* 6 *kalā,* where *kalā* is defined as 1/19 *y* which is mentioned in *gāthā* 77 (Vol. I, p. 104).

Geometry: This work is full of geometrical treatment as its name depicts that it elucidates the dimensions, areas, volumes of various zones (continents, mountains, island, oceans etc.). Circumference, arc, area, volume and chord etc. have been explained and the mathematical methodology is explained in detail. He attempts boldly to calculate area between two chords of a circle.

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[1] *Bṛhatkṣetra Samāsa,* p. 108.

[2] *Ibid,* Vol. I, p. 91.

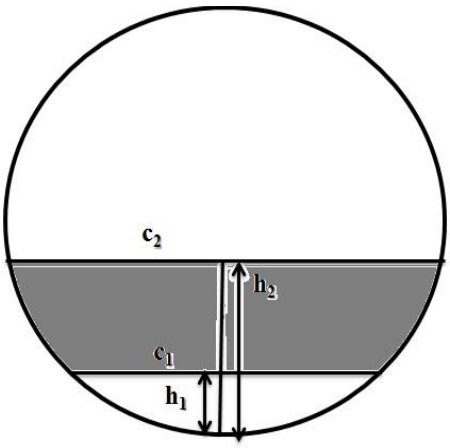

Fig. 2.1 Area between 2 chords

Firstly, he states the area as, $\frac{c_1+c_2}{2} \times (h_2 - h_1)$.

However, he immediately realises that this is the area of trapezium bounded by these 2 chords (c_1, c_2). Hence he calls it approximate formula. He there gives more accurate formula:

$$\text{Area} = \sqrt{\frac{{c_1}^2+{c_2}^2}{2}} \times (h_2 - h_1).$$

Where, instead of mean of c_1 and c_2, he takes their root mean square. And this turns out to be a wonderful approximation.

Therefore, it can be said as the masterwork of Jaina geography with mathematics.

2.11 Conclusion

Being a spiritual philosophy mathematics for the sake of mathematics has never been aim of Jaina saint scholars, yet erudite Jaina scholars have applied mathematical ideas to explain the philosophy in the ancient era. They performed large calculations and arrived at precise results. A glimpse of mathematical ideas from the original sources, the *Āgama* between 500 BC to 500 CE, is presented in this chapter. These mathematical texts from this period have received little attention from historians of mathematics until recent times. We will discuss topics on decimal place value system, large numbers, transfinite numbers, fractions, combinatorics, maxima and minima etc. in detail in the succeeding chapters.

Chapter-3

Number Theory (1): Decimal System

The ingenious method of expressing every possible number
using a set of ten symbols (each symbol having a place value
and an absolute value) emerged in India.

~ P.S. Laplace (1814) ~

Mathematics is a linguistic activity and it aims to precise communication. The language of number is the greatest symbolic creation. But thousands of years ago there were no numbers to represent two or three. Instead symbols like fingers, rocks, sticks or eyes were used to represent numbers. There were neither clocks nor calendars to help keep track of time. By observing the movement of sun and moon, ancient people used to distinguish between times. A numeric system was almost not needed until groups of people formed clans, villages and settlements and began a system of negotiating and trade that in turn created a demand for currency.

3.1 The Need of Counting

As the civilization was developing and communication was growing; there emerged the need for counting. The earliest number systems grew out of the human necessity to count anything. For example, to find out how many members are there in their group or tribe; to make sure that no member is missing. To keep record about the animals if they are increasing or decreasing, necessarily they had some sort of system to express the number or counting. Numbers are abstract ideas used in defining quantities, as in counting or measuring. The origin of numbers is prehistoric.

3.1.1 Counting: From Fingers to Words

Speech is of course co-existential with human life. So, to meet the need of counting our ancestors might have used some words randomly till writing was not developed. When man first began counting and communicating number information, he used his fingers to

establish a one-to-one correspondence with the object. Readily available, the human digits, fingers and toes have always lent themselves to the tasks of counting[1]. The first method of counting was counting on fingers[2]. Soon after language developed, it is safe to assume that humans began counting - and that fingers and thumbs provided nature's abacus.

It's true that since ancient time there was a system to represent the numerals and in the course of time it went through many changes. There came into existence the pictorial numerals in almost all the cultures. When writing became the part of life word numerals were evolved.

It was important to be careful before selecting the word that the object which is referred by the word must exist in constant quantity. It should neither increase nor decrease. For example, to find the word for number '1', the object must exist just one in number like, earth, moon etc. Therefore they used the words like earth, moon etc. to represent '1'. Synonyms were also used. In early forms of speech the number two was expressed by the word 'eyes', five was often expressed by the word for hand, and ten by two hands. Sometimes twenty was expressed as "all the fingers and toes."

In literature may be because of poetic tradition, eventually the representation of numbers took the form of words.

There is no evidence of beginning of this system but according to Rādhācarana Guptā, in India in the early centuries of the Christ era the evidence of using the words to denote the numbers are found[3]. The words used were taken either from the scriptures or the body parts. For example, for thirty two '*daṅta*' (teeth) was used. *Veda* was for four because *veda*-s are four in number. Since, place value system was not developed. There was a rule for the arrangement of the words to express the numbers. It is said that the words should be read from right to left. For example, *khabāṇasūryāh* represents 1250 where, *kha*=0, *bāṇa*=5, *sūryāh*=12 and it is read from right to left. These numerical words were known as *Bhūtasaṅkhyā*.

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[1] *From Five Fingers to Infinity*, p. 52.

[2] *The Universal History of Numbers*, pp. 47–61.

[3] Gupta, R.C., *Prācīna Bhāratīya Gaṇita kī Eitihāsika evam Sānskṛtika Jhalakiyāṅ,* National Council of Educational Research and Training, Delhi, 1997, p. 12.

3.2 What is Number and Numeral?

If mathematics is the language of science, the language of mathematics is numbers, its grammar being represented by the several kinds of numbers and by the ways in which we use them.

The terms number and numeral may often be used inter-changeably, but in some instances finer distinctions or nuances are required. We shall observe the following definitions.

Numbers describe magnitude or position. **Numeral** is a symbol or combination of symbols used to represent a number.

3.2.1 Types of Numeral Systems

There are essentially two types of numeral systems :

1. Unary system
2. Positional system.

1. **Unary system:** The simplest numeral system is the unary numeral system, in which every natural number is represented by a corresponding number of symbols. A natural way of recording number is tally marks.[1] A tally stick is simply a piece of wood, to record and document numbers, quantities, or even messages, in which notches have been made, representing some quantity. If the symbol / is chosen, for example, then the number seven would be represented by **///////**.

   It is the most primitive and basic of all such systems. But for a moderately sized number, we end up with a sprawling collection of tally marks on the page. Therefore extra steps were introduced, like letting a diagonal line through four vertical marks stand for five. But as we write down larger and larger numbers, we find that the notation once again becomes unwieldy. One solution to this problem is to introduce more new symbols. This is precisely how number systems developed historically.

2. **Positional system:** Another type of numeral system is called positional. A **positional system** uses place value to show the value of numerals. Each place of

[1] *From Five Fingers to Infinity*, p. 53.

the number represents some value to the digit. For example, in decimal place value system, the right most number holds the value of unit, next to it in the left holds the value of tens and similarly increases in terms of multiple of tens by which the same digit gets a different value according to its place or position in the sequence. The Hindu-Arabic system is the positional decimal system that we use.

3.3 Ancient Symbols of Numbers

As a basic need of counting each civilization has its own way of representing the numbers. The several original representations differed in breadth, form and concept. In diverse locations, ancient bookkeepers, builders, priests and tax collectors had needs to record information, for their own purposes. Let's have a glimpse of numerals used in various canalizations.

3.3.1 Egyptians Numerals

The **Egyptians** had three systems of writing: hieroglyphic, hieratic, and demotic. Hieroglyphs were the sacred writing used for formal inscriptions, usually on stone. They were written from right to left or left to right, starting from the direction the figures face. Hieratic writing was a simplified version of hieroglyphs used by priests and scribes for common records mostly written on papyrus. Demotic writing was a further simplified method developed from hieratic writing. It was used in everyday life, often to record numbers. Egyptian hieroglyphs scripts developed in the Bronze Age, by 3200 B.C having the symbols for various numbers as:

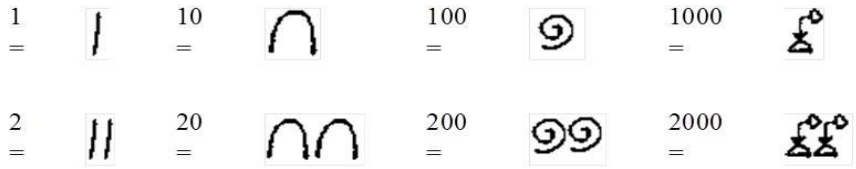
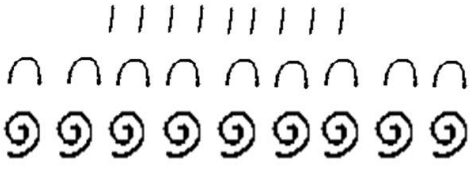


Fig 3.1: Egyptian Numerals

In Egyptian system 999 would be represented as,



Egyptian Numeral: 999

3.3.2 Babylonian Numerals

Early clay tablets show that they did not have systematic positional system for all powers of 60. From around 2000 BC they evolved sexagesimal place-value system using only two symbols[1]. **Babylonian** numbers were written from right to left and used two basic **cuneiform** (wedge-shaped) numerals (symbols) for 1(𒁹) and 10 (𒌋).Other numerals were used by combining these two symbols. The rightmost place was for values from one to fifty-nine, the next place to the left was for multiples of sixty through 3540.

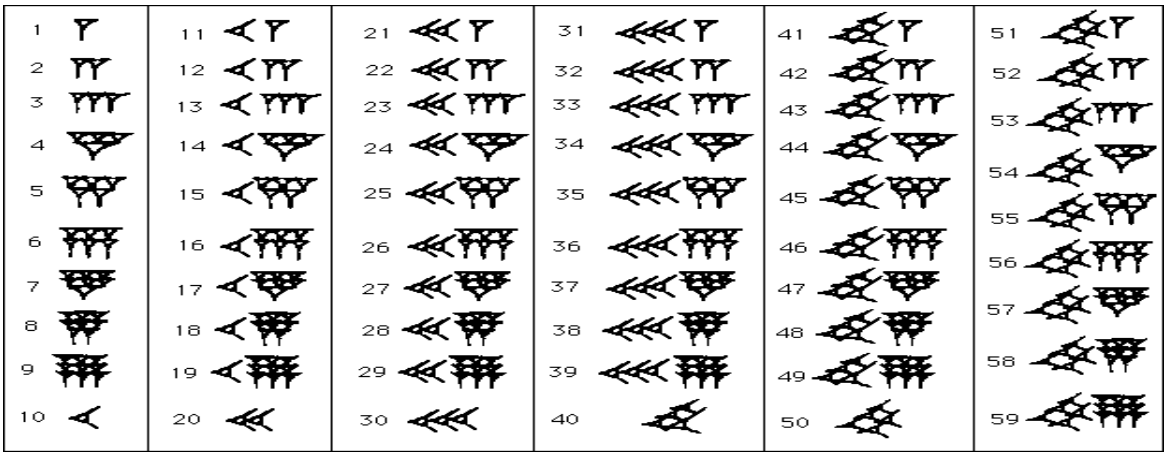
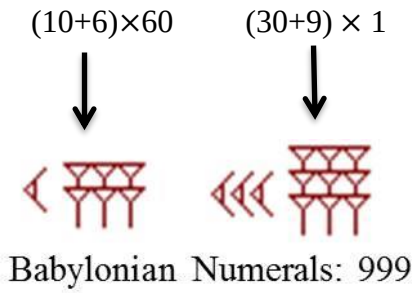


Fig 3.2: Babylonian Numerals

In this way they used base of 60. This base of 60 survives even today in the 60 seconds and minutes of angular measurement, in the 180 degrees of a triangle and in the 360 degrees of a circle.

For example number 999 can be written as



Above number is in base 60 place value. i.e. 16×60 + (30+9) = 999.

[1] *The Crest of The Peacock*, p. 137.

3.3.3 Greek Numerals

The Greek numbering system was uniquely based upon their alphabets. There are two types of numeric system. 1. Acrophonic 2. Alphabetic.

Acrophonic numbers (AKA attic or Herodianic numbers) was the first system of numbers to be used by the ancient Greeks. They were first described in a 2nd century manuscript by Herodian, which is non-positional. 'Acrophonic' means that the symbols for the numerals come from the first letter of the number name. They were derived (after the initial one) from the first letters of the names of the numbers represented. For example, the number 1 is called "Iota" and denoted by the letter "I".

Γ = 500 H = 100 Δ = 10 Γ = 5 I = 1

Number 999 in 'Acrophonic' system will be written as,

ΓHHHHHΔΔΔΔΔΔΔΔΔΓIIII

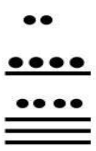
3.3.4 Mayan Numerals

The **Mayan's** used a vigesimal system, which had a base 20[1]. In order to write numbers down, there were only three symbols needed in this system :

1. A dot represented the quantity 1, ●
2. A horizontal bar represented the quantity 5, —
3. A special symbol (thought to be a shell) represented zero. zero

Unlike our system, where the ones place starts on the right and then moves to the left, the Mayan system places the ones on the bottom of a vertical orientation and moves up as the place value increases. For example, 999 will be represented in vigesimal as :

$$2 \times 20^2 + 9 \times 20 + 19 \times 1$$



Mayan Numerals: 999

[1] Thompson, J. Eric S., *The Rise and Fall of Maya Civilization*, University of Oklahoma, Norman, 1966.

The bottom number is 19, middle floor is 180 and the uppermost is 400, as a whole resulting in 999. **Indian Numerals: The Birthplace of Modern Numerals**

In **India** the decimal system was in use since beginning. The Brāhmī script was used from around 50 B.C. Brāhmī numerals have been found in inscriptions and on coins in western India up to the fourth century A.D. As Jawaharlal Nehru has said that the miscalled "Arabic" numerals are found on the Rock Edicts of Aśoka (250 B.C.), a thousand years before their occurrence in Arabic literature. Hindsaa (numerals) in Arabic means from India.[1]

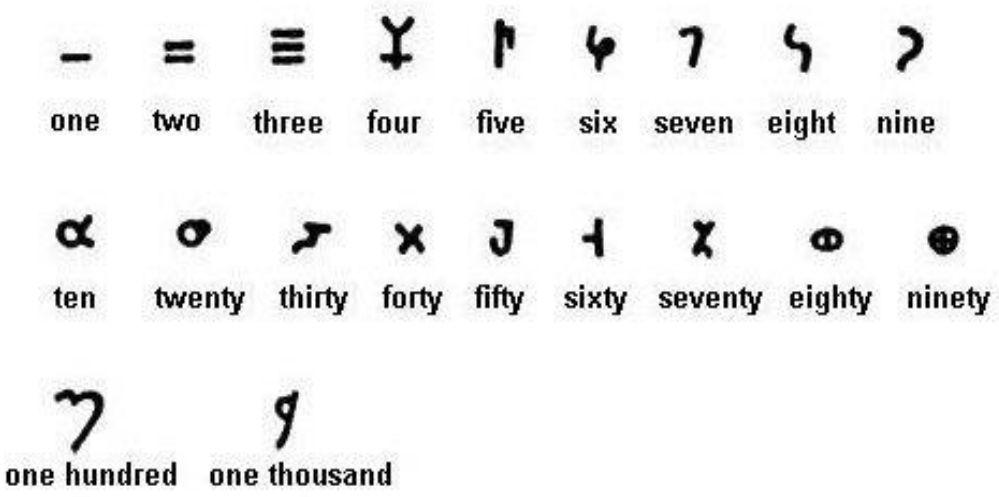
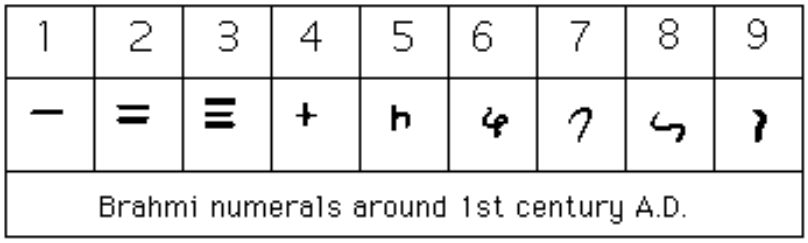


Fig 3.3: Indian Numerals

The Indian Brāhmī number system was decimal, but not a place value system. It used nine numerals for the numbers one through nine, and special symbols for ten, twenty, thirty, etc., and for one hundred and one thousand. The source of the first three numerals seems clear: they are collections of 1, 2, and 3 strokes, in Aśoka's era vertical I, II, III like Roman numerals, but soon becoming horizontal like the modern Chinese numerals.



[1] Nehru, Jawaharlal, *The Discovery of India*, Oxford University Press, Oxford, 1995, p. 216.

1	2	3	4	5	6	7	8	9

Gupta numerals around 4th century A.D.

1	2	3	4	5	6	7	8	9	0

Nagari numerals around 9th century A.D.

In the oldest inscriptions, 4 is a +, reminiscent of the X of neighbouring Kharoṣṭhī, and perhaps a representation of 4 lines or 4 directions. However, the other unit numerals appear to be arbitrary symbols in even the oldest inscriptions.

The gift of modern numerals used today around the world is given by Indian intellects. The modern form of the numerals from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 acquired this form in 15th century in Europe.

3.4 The Birthplace of Numerals

The modern numerals which have become now standard in mathematical and practical world also are the greatest gift by India to the world of mathematics. The question is who is the inventor? The scholars have their own views. Ifrah says that,

The real inventors of this fundamental discovery, which is no less important than such feats as the mastery of fire, the development of agriculture, to the invention of the wheel, writing or the steam engine, were the mathematician and astronomers of Indian civilisation: scholars who unlike Greeks, were more concerned with practical applications and who were motivated by a kind of passion for both numbers and numerical calculations.[1]

There are many grounds to verify India as the real birthplace of modern numerals.

In the Greek year 973 (662 A.D.), Severus Sebokt would know Indian science. In his views, *The Hindus, who are not even Syrians, have made subtle discoveries in the field of astronomy which are even more ingenious than those of the Greeks [sic] and the*

[1] *The Universal History of Numbers*, p. 361.

Babylonians; as for their skilful methods of calculation and their computing which belies description, they use only nine figures.

Ifrah (p.366) writes, *in other words, the Indian system, as described by Severus Sebokt, has an unlimited capacity for representation because it has positional numeration… It might seem curious that Sebokt does not mention the use of zero, but this is probably because he only had an abacus upon which to carry out his mathematical operations.*

It is likely that on his abacus he could only use nine Indian numbers by sprinkling sand on the board. Therefore, zero was not physically represented: the absence of a unit in a given column was communicated by means of an empty space. Furthermore, he describes that Severus Sebokt's evidence proves that the Indian counting system was known and esteemed outside India by the middle of the seventh century[1].

P. S. Laplace writes that,

The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India[2].

Abu Ja 'far Muhammad ibn Musa al Khwarizmi wrote in *kitab al jam' wa'l tafriq bi hisab al hind* (*"Indian technique of addition and subtraction"*), of which there are Latin translations dating from the twelfth century. He[3] writes as:

"... we have decided to explain ***Indian*** *calculating techniques using the nine characters and to show how, because of their simplicity and conciseness, these characters are capable of expressing any number."*

There is a large body of evidence in inscriptions found at many places which verify the decimal place value system with use of zero.

The very first evidence of numerals with symbol for zero having place value is from Malaysian[4] inscription of *śaka* era 605, 606 and 608 (i.e. 683, 684 and 686 A.D.).

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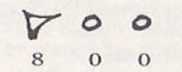
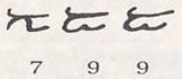
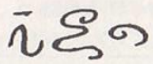
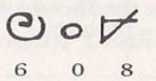
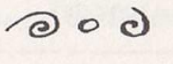
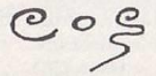
[1] *The Universal History of Numbers*, p. 366.

[2] *Ibid*, p. 361.

[3] *Ibid*, p. 364.

[4] *Ibid*, p. 406.

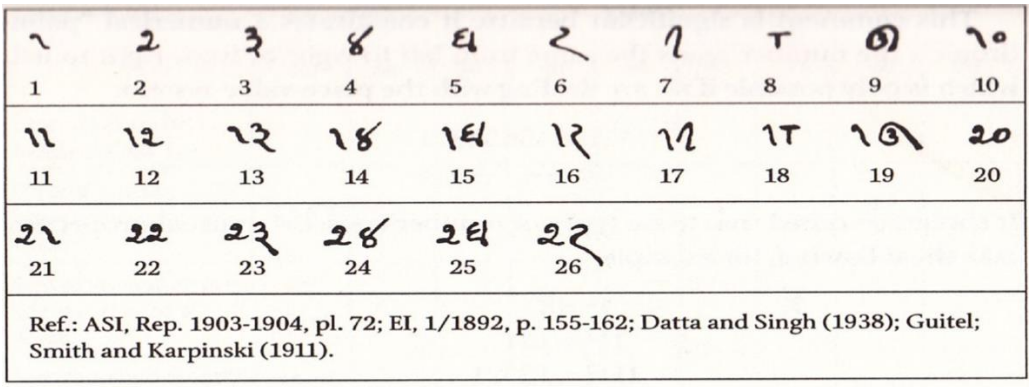
	DOCUMENTS AND SOURCES	HOW THE DATE IS RECORDED IN THE *SHAKA* CALENDAR	
878	Balinese inscription of Mamali. Dated 800 in the *Shaka* calendar (= 800 + 78 = 878 CE).	 8 0 0	Damais, p. 148, f.
877	Balinese inscription of Haliwanghang. Dated 799 in the *Shaka* calendar (= 799 + 78 = 877 CE).	 7 9 9	Damais, p. 148, f.
829	Cham inscription of Bakul. Dated 751 in the *Shaka* calendar (= 751 + 78 = 829).	 7 5 1	IMCC, C 23 ISCC, p. 238 BEFEO, XV, 2, p. 47
813	Cham inscription of Pô Nagar, northwest tower. Dated 735 in the *Shaka* calendar (= 735 + 78 = 813 CE). *This is the first Cham inscription to contain the date written in numerals.*	7 3 5	IMCC, C 37 JA 1891, i, p. 24 BEFEO, XV, 2, p. 47
686	Malaysian inscription of Kota Kapur (isle of Banka). Dated 608 in the *Shaka* calendar (= 608 + 78 = 686 CE).	 6 0 8	BEFEO, XXX, pp. 29ff. Kern VIII, p. 207
684	Malaysian inscription of Talang Tuwo, Palembang (Sumatra). Dated 606 in the *Shaka* calendar (= 606 + 78 = 684 CE).	 6 0 6	BEFEO, XXX, pp. 29ff. ACOR, II, p. 19
683	Malaysian inscription of Kedukan Bukit, Palembang (Sumatra). Dated 605 in the *Shaka* calendar (= 605 + 78 = 683). *This is the oldest inscription in Old Malay to be dated in numerals.*	 6 0 5	BEFEO, XXX, pp. 29ff. ACOR, II, p. 13
683	Khmer inscription of Trapeang Prei, province of Sambór. Dated 605 in the *Shaka* calendar (= 605 + 78 = 683 CE). *This is the first Khmer inscription dated in numerals.*	6 0 5	IMCC, K 127 CIC, XLVII

Note: In Java, the oldest Kawi inscription (written in Old Javanese) to be dated in numerals is of Dinaya, which bears the date 682 in the *Shaka* calendar = 682 + 78 = 760 CE. [Ref: Tijdschrift, LVII, (1976), p. 411; LXIV, (1924), p. 227].

These regions were ruled by *Cola* emperors (Rāyarājā) and Rājendra of south India.

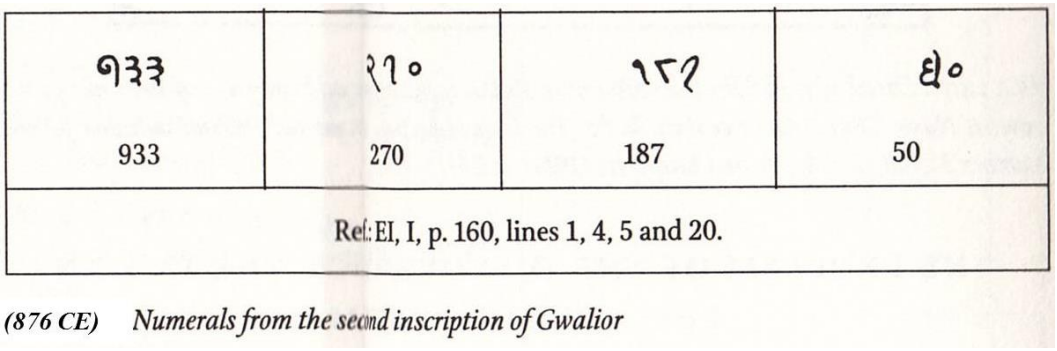
At the end of seventh century (683 A.D.) in Cambodia the vernacular inscription in Old Khmer [Ifrah, 404], in Sumatra in 684 and 683 A.D. bear the witness to its use.

The known Indian documents to verify the use of zero and decimal place value system date back to the second half of the ninth century A.D. These are two stone inscriptions discovered near the town of Gwalior. The first one is dated *vikrama* 932 (=875 AD) It is in Sanskrit consisting of 26 stanzas in *Nāgari* numerals shown below in the figure. [Ifrah, 400]



Ref.: ASI, Rep. 1903-1904, pl. 72; EI, 1/1892, p. 155-162; Datta and Singh (1938); Guitel; Smith and Karpinski (1911).

(875 CE) Numerals from the first inscription of Gwalior



Ref: EI, I, p. 160, lines 1, 4, 5 and 20.

(876 CE) Numerals from the second inscription of Gwalior

The second inscription shown above is dated *vikrama* 933(=876 A.D.). It is written in Sanskrit prose which gives an account of the offerings to *Viṣnu* in numbers.

The picture below shows the systematic development of Indian numerals and their spread to various cultures.

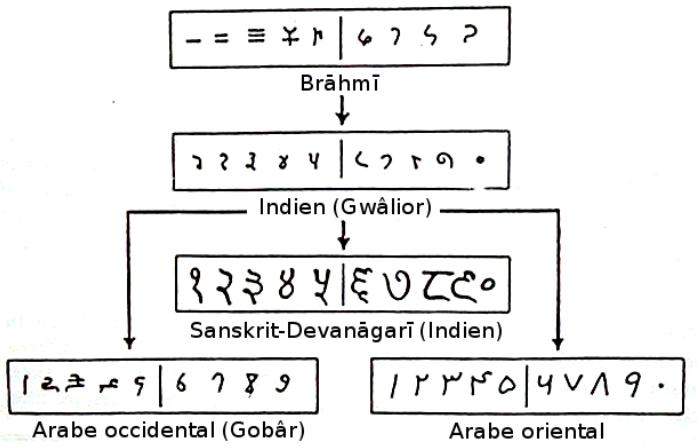


Fig 3.5 Development of Indian Numerals

3.5 Word Numerals in Jaina *Āgama*

Many civilizations like Egypt, Greek, and Indus use decimal numeral system. Use of decimal system is also found in Jaina *Āgama* to record the large numbers. Such large numbers would require numerals to make the job easy. Surprisingly, we do not find in *Āgamic* literature any symbols for numerals. We get an impression by studying all *Āgama* that no symbolic or pictorial numerals as Babylonian, Egyptian, Greek etc. are employed anywhere. The numbers are written in words like, is *ego* (1), *dasa* (10), *saya* (100), *sahassa* (1000), *dasa sahassa* (10,000), *saya sahassa* (100,000), *dasa saya sahassa* (10,00,000), *koḍi* (10^7), *dasa koḍi* (10^8) etc. Therefore, we can say that decimal number system was prevalent in the era of Bhagawāna Mahāvīra also. And it was possible then to count and measure the things of high magnitude without any symbol.

3.6 Classification of Numbers in Mathematics

A number is a mathematical object used to count, measure, and label. In modern mathematics we find many types of numbers, basically classified as shown in the table

Natural numbers ($\mathbb{N}$): The counting numbers {1, 2, 3, …}, are called natural numbers. Some authors include 0, so that the natural numbers are {0, 1, 2, 3, …}.

Whole numbers: The numbers {0, 1, 2, 3, …}.

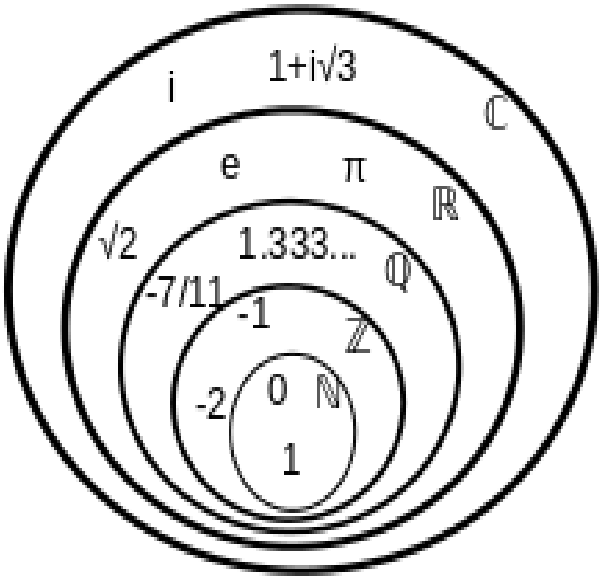
Integers ($\mathbb{Z}$): Positive and negative counting numbers, as well as zero:{…, -2, -1, 0, 1, 2,…}.

Rational numbers ($\mathbb{Q}$): Numbers that can be expressed as a ratio of an integer to a non-zero integer. All integers are rational, but the converse is not true.

Real numbers ($\mathbb{R}$): Numbers that have decimal representations that have a finite or infinite sequence of digits to the right of the decimal point. All rational numbers are real, but the converse is not true.

Irrational numbers : Real numbers that are not rational.

Complex numbers ($\mathbb{C}$): Includes real numbers, imaginary numbers, and sums and differences of real and imaginary numbers.



Beside the above classification there are more types of the numbers:

Even and odd numbers: An integer is even if it is a multiple of two, and is odd otherwise.

Prime number: An integer with exactly two positive divisors: itself and 1. The primes form an infinite sequence 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ….(where 2 is the only even prime number)

Composite number: A number that can be factored into a product of smaller integers. Every integer greater than one is either prime or composite.

Polygonal numbers: These are numbers that can be represented as dots that are arranged in the shape of a regular polygon, including Triangular numbers, Square numbers, Pentagonal numbers, Hexagonal numbers, Heptagonal numbers, Octagonal numbers, Nonagonal numbers, Decagonal numbers, and Dodecagonal numbers etc.

Infinitesimals: Nilpotent numbers. These are smaller than any positive real number, but are nonetheless greater than zero. These were used in the initial development of calculus, and are used in synthetic differential geometry.

Transfinite numbers: Numbers those are greater than any natural number.

Ordinal numbers: Finite and infinite numbers used to describe the order types of well-ordered sets.

Cardinal numbers: Finite and infinite numbers used to describe the cardinalities of sets.

3.7 Where Does Number Start From in *Āgama*?

In modern mathematics numbers are both positive and negative and natural numbers start with zero, while in Jainism 0 and 1 are not regarded as numbers, because by multiplying they do not enhance the value of resulting quantity. *sūtra* 11.574[1] (ADS) deals with counting measures (*gaṇanā*). It says, '*se kiṁ taṁ gaṇaṇāsaṅkhā?gaṇaṇāsaṅkhā–ekko gaṇaṇaṁ ṇa uvei, duppabhie saṅkhā. taṁ jahā-saṅkhejjae, asaṅkhejjae aṇaṇante'* i.e. 1 is not to be included in counting. For, mathematical purpose it cannot change the value of other number if multiplied or divided. For example, multiplication of any number with one will not result in the new number. If we divide any number with one the resultant number will remain same. Therefore, for mathematical measures number should be counted from 2. Numbers start from 2 and can be classified in three categories: (1) Numerable (*saṅkhyāta*), (2) Innumerable (*asaṇkhyāta*) and 3) Infinite (*anaṅta*). It will be discussed in the next chapter.

3.8 Classification of Numbers in *Āgama*

In *Thāṇaṁ*, the classification goes as: *cattāri jummā paṇṇattā, taṁ jahā -kaḍumme, teyoe, dāwarajumme, kalioe* i.e. there are four types of *rāśi* (set of numbers) *kṛta yugma, tryoja, dvāpara yugma* and *kalyoja*[2]. There are two types of numbers *yugma* and *oja*. Ācārya Mahāprajña in his commentary of this text mentions that according to *Sthānāṅga vṛtti* (11th C. A.D.) (*patra* 226)[3] *yugma* is the set of even numbers while *oja* is the set of odd numbers. Further, there are two types of *yugma*: *kṛta* and *dvāpara* and two types of *oja*: *tryoja* and *kalyoja*. This classification of numbers is found in BS, with greater details.

In his commentary of TS Ācārya Mahāprajña[4] explains *kṛta, tryoja* etc. in mathematical notation, but modular notation is not stated there.

The 18th *śataka* and *sūtra* 89-90 of *uddeśaka* 4 (BS vol. V, p. 78) talk about the types of numbers. It introduces the classification of numbers like *kṛta-yugma, tryoja, dvāpara* and *kalyoja*.

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[1] *Anuogadārāiṃ*, p. 309.

[2] *Thāṇaṁ*, p. 390.

[3] *Ibid*, p. 515.

[4] *Ibid*, p. 515.

1. *kṛta* is a set of such numbers where remainder is 4 or 0 after dividing a positive integer by the multiple of 4. Let's say K_0 (*kṛta*) is such set, then

   It can be expressed in mathematical form as

   kṛta-yugma $K_0 \equiv \{4n+4, n = 1, 2, 3\ldots\ldots..\}$, i.e. { 4, 8, 12, 16, 20,…..}

   or, in modern modular notation $K_0 \equiv 0 \bmod 4$

2. *tryoja*: It is the set of numbers where remainder is 3 after dividing a positive integer by the multiple of 4.. Let's say K_3 (*tryoja*) is such set, then

   $K_3 \equiv \{4n+3, n = 1, 2, 3\ldots\ldots.\}$i.e. {3, 7, 11, 15, 19,…….. }

   or, $K_3 \equiv 3 \bmod 4$.

3. *dvāpara* : It is the set of numbers where remainder is 2 after dividing a positive integer by the multiple of 4. Let's say K_2 (*dvāpara*) is set of such members, then

   $K_2 \equiv \{4n+2, n = 1, 2, 3\ldots\ldots..\}$ i.e. {2, 6, 10, 14,….}

   or, $K_2 \equiv 2 \bmod 4$.

4. *kalyoja*: : It is the set of numbers where remainder is 1 after dividing a positive integer by the multiple of 4. Let's say K_1 (*kalyoja*) is set of such members, then

   $K_1 \equiv \{4n+1, \text{where, } n = 1, 2, 3\ldots\ldots..\}$i.e. {1, 5, 9, 13}

   or, $K_1 \equiv 1 \bmod 4$

   yugma (*jumma*) is defined as the number produced of or resulting from the even number is *jumma* which means literally pair[1].

Jaina *ācārya*-s have always been interested in finding the magnitudes or quantities of various entities. Therefore this classification is made to describe the number of beings of various realms (e.g. gods, human beings, animals and hellish beings).

When Gautama *gaṇadhara* asked that in which set the hellish beings would fit? Then Bhagawāna Mahāvīra replied that the minimum number of hellish beings could be in *kṛta-yugma*, maximum in *tryoja* and intermediate numbers can be in any of the four aforementioned categories. (BS, 18.4.91)[2].

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[1] *Ardhamāgadhi Kośa*, ed. by Muni Ratnacandra, Sardāramal Bhandārī, Indore, 1927, p. 680.

[2] *Bhagavaī,* Vol. V, 2015, p. 80.

The minimum number of living beings from 2 senses to 4 can be *kṛta-yugma,* the maximum can be *dvāpara* and intermediate can be any of the above categories (BS, 18.4.93)[1].

In what category does the number of *pradeśa*-s (space-points) in a line (*seḍhi*) fall? The number of *pradeśa*-s in line vertical or horizontal falls in *kaḍajumma* (BS, 25.3.87)[2].

This classification gives an idea of number of living beings in different realms and about the space-points of the inanimate matter. BS explains about all five substances in this context and their set is mentioned. For example, Number of *pradeśa*-s in *dharmāstikāya* and *dharmāstikāya* etc. are *kaḍajumma* (BS, 25.4.113)[3].

There are few more classifications found in BS as *khuḍḍa jumma* and *mahā jumma*, where *khuḍḍa* means small and *mahā* means large. Thus, this classification is on the basis of small and large numbers.

The minor classification (*khuḍḍa jumma*) in four categories is given below:

Sets of *khuḍḍa jumma*[4](BS, 31.1.2):

1. *kṣullaka kṛtayugma* = {4, 8, 12, 16….S, A}
2. *kṣullaka tryoja* = {3,7,11,15,…S, A}
3. *kṣullaka dvāpara yugma* = {2,6,10,14,….S, A}
4. *kṣullaka kalyoja* = {1,5,9,13,…..S, A}

This classification is done to explain the more specifically about the number of beings entering to hell from various realms and going from hell to the next birth-place. For example, how many hellish beings *kṣullaka kṛtayugma* class die at a time (*samaya*) (BS 31.1.4)? Four, eight, twelve, sixteen,…*saṅkhyāta* or *asaṅkfhyāṭa* hellish beings can die together in one *samaya.*

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[1] *Bhagavaī*, p. 80.

[2] *Bhagavaī (Aṅgasuttāṇi)*, synod by Ācārya Tulsī, ed. by Muni Nathmal, with original text, critically edited, Jain Vishva Bharati, Ladnun, Vol.II, 1st edn., 1974, p. 914.

[3] *Ibid*, p. 916.

[4] *Ibid*, p. 998.

There is another type of major classification (*mahā jumma*[1]) in 16 categories (BS 35.1.1) having the sets as below :

1. *kṛtayugma- kṛtayugma* ≡ 0 mod (16); {16, 32, 48…….16n; n≥1}
2. *kṛtayugma- tryoja* ≡ 3 mod (16); {19, 35, 51…….16n+3; n≥1}
3. *kṛtayugma- dvāpara yugma*≡ 2 mod (16); {18, 34, 50…….16n+2; n≥1}
4. *kṛtayugma- kalyoja* ≡ 1 mod (16); {17, 33, 49…….16n+1; n≥1}
5. *tryoja- kṛtayugma*≡ 12 mod (16); {12, 28, 44…….16n+12; n≥1}
6. *tryoja- tryoja* ≡ 15 mod (16); {15, 31, 47…….16n+15; n≥1}
7. *tryoja- dvāpara yugma* ≡ 14 mod (16); {14, 30, 46…….16n+14; n≥1}
8. *tryoja- kalyoja* ≡ 13 mod (16); {13, 29, 45…….16n+13; n≥1}
9. *dvāpara yugma- kṛtayugma* ≡ 8 mod (16); {8, 24, 40…….16n+8; n≥1}
10. *dvāpara yugma- tryoja* ≡ 11 mod (16); {11, 27, 43…….16n+11; n≥1}
11. *dvāpara yugma-dvāpara yugma* ≡ 10 mod (16); {10, 26, 42…….16n+10; n≥1}
12. *dvāpara yugma- kalyoja* ≡ 9 mod (16); {9, 25, 41…….16n+9; n≥1}
13. *kalyoja- kṛtayugma*≡ 4 mod (16); {4, 20, 36…….16n+4; n≥1}
14. *kalyoja- tryoja* ≡ 7 mod (16); {7, 23, 39…….16n+7; n≥1}
15. *kalyoja- dvāpara yugma* ≡ 6 mod (16); {6, 22, 38…….16n+6; n≥1}
16. *kalyoja- kalyoja* ≡ 5 mod (16); {5, 21, 37…….16n+5; n≥1}

There is detailed description of quantities of the living beings (one-sensed to five-sensed) that belong to these categories. For example, from which realm *kṛtayugma- kṛtayugma ekendriya jīva* come and take birth in this from (BS 35.1.3). They can come from any of the realms of human, animal or heaven.

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[1] *Bhagavaī (Aṅgasuttāṇi)*, p. 1024.
c.f. Dravyānuyoga, ed. by Muni Kanhaiyālāl, with text and Hindi translation, Agam Anuyoga Trust, Ahmedabad, part III, 1995, pp. 1575-1576.

3.9 Decimal Place Value System and Zero

3.9.1 Why Place Value System?

Where place value system came from? What is it, though? Place value is the notion that where a digit sits in a number says something about its value. For example say 628 means $6\times100 + 2\times10 + 8\times1$ in decimal system.

Counting was the need and the symbol-system was meeting the needs. Then why did the place value system originate? In fact there were some shortcomings or defects in symbolic notation. How to symbolize zero when it comes in any of the middle places? As the trade grew, amounts of taxation and revenues became very large and counting became very cumbersome. The existing system was not sufficient as it would need entire page for a large number like million to be written and it was time-consuming also. Symbols had to be repeated to express the large numbers. For instance, using the original hieroglyphic script of Egyptian numerals where | = 1 the number 4 would be expressed: ||||. For expression large numbers, we can imagine the length of the expression. Therefore, place value system became need of the era. There again was ambiguity for zero. Blank space was being left for it and it could only be understood through the context and sometimes even not.

It turns out that, mathematicians first thought of zero in the context of writing numbers down - zero was first a placeholder. Before mathematicians understood the notion of zero, there was much ambiguity about written numbers. For instance, if the symbol for 5 was written down, there was no way to distinguish among 5, 50, and 5,000,000. **Zero** was introduced as a placeholder to avoid these ambiguities.

The Babylonian numeration which had only three figures, one for 1, and one for 10; so that a number, say, 999, would require 27 symbols, namely, nine of each of the symbols (see sections 3.4.2).

The positional system is not possible in the Roman numeral system which had no expression or symbol for zero. A number, say, 101,000, would have to be written only by 101 consecutive M’s. The Egyptians had no zero and never reached the idea of expressing all numbers with ten digits.

To calculate and to keep the record of large numbers the place value system was originated.

In India the use of large number was very common not only in the trade but in religious field also. Jainas were always keen to calculate the number and position of the stars, to find

the dimension of cosmos (*loka*), number of deities, life-spans, dimensions of mountains and oceans etc. Therefore, in Jaina *Āgama* extremely large numbers are found. Without decimal place value system it would have been difficult to calculate and express very large quantities. Though they did not use the symbols but positional or place value system was always the part of their mathematical usages.

3.9.2 A Brief History of Zero

The concept of zero is surprisingly deep, and it took human thinkers quite a long time to come up with the notion of zero. Zero is one of the two greatest inventions in the history of mankind. Wheel was the first greatest invention and it is a co-incidence that both the inventions are 'circular'. Both the inventions changed the life of mankind. The wheel made a revolution in transportation and machinery, while Zero brought a revolution in mathematics and calculation.

Further attempts have been made to attribute the first use of a place value system to the ancient Babylonian civilisation of Mesopotamia. It is true that the Babylonians used a place value system, which was sexigesimal (base 60). They would keep some blank space to indicate zero, but there was no symbol for zero. Moreover, it was to be understood by the context only. For example number 408 was expressed as, which is equivalent to $6\times 60 + (30 + 10) + 8$. For them, however, zero was like a concept in ancient Mesopotamia, but was not a separate number. Instead of a separate number, they used a blank space for Zero.

Ifrah (p.310-311) gives the table of astronomical days calculated by Mayans, which is table of synodic revolution of Venus. Through it he concludes that Mayans had really invented a) place value system and b) zero.

However, due to their mixed base system of numeration, lots of ambiguities were created in employing zero with place value Thus, place value system was there but zero with place value was absent.

The number zero is the subtle gift of India of antiquity to mankind. It was the Indian genius who first used Zero as a number, and used probably a circle or dot for it. Later, the Arabs adopted Indian Zero and used it in their mathematics. From Arabs, Zero went to Europeans and then it spread worldwide.

3.9.3 What is Zero?

Now the question is who evolved zero as a place value. Historians have their own views. In India, zero as a concept probably predated zero as a number by hundreds of years and the word '*śūnya'* is found in *Veda* and *Āgama* also, which means 'nothing' or 'empty'.

Zero is the transliteration of the Sanskrit word *śūnya* meaning 'void', 'absence' or 'empty'. The word is probably derived from *sunna* which is the past participle of *svi*, "to grow". In *Ṛgveda*, there is another meaning: the sense of "lack" or "deficiency"[1]. Even Buddhist doctrine also describes the word *śūnyatva* (emptiness) used for spiritual practice of emptying the mind of all impressions.

In Jaina doctrines it is found as *śūnyasthāna,* and *śūnyakāla* etc. Joseph writes (p.345), '*Just as emptiness of space is necessary condition for the appearance of any object, the number of zero being no number at all is the condition for the existence of all numbers.*" Furthermore he writes three roles of zero or *śūnya* as :

1. The concept of the *śūnya* within a place-value system
2. The symbols used for *śūnya* and
3. Mathematical operation with the *śūnya.*

It was soon recognized that the *śūnya* denoted notational place as well as the 'void', or absence of numerical value in a particular notational place. And perhaps this beauty and uniqueness lies only in the number *śūnya*.

The word *śūnya* used in philosophical literature became the boon for mathematics and entire human world.

3.9.4 Zero with Place-value

It will be our attempt to show that first evidence of use of concept of zero with or without positional value is found in Jaina scriptures.

In *śataka* 1 (BS, 1.2.104-111) explaining the *saṅsthāna kāla*[2] in *sūtra* 1.2.104, (the life-duration of a living being in a particular realm) the word *sunna (śūnya*) is used. *saṅsthāna*

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[1] *The Crest of The Peacock*, p. 344.

[2] *neraiyasaṁsārasañciṭṭhaṇakāle ṇaṁ bhante ! kativihe paṇṇatte? Goyamā ! tivihe paṇṇatte, taṁ jahā – sunnakāle, asunnakāle, missakāle*.

–*Bhagavaī*, Vol. I, pp. 64-65.

kāla of hellish beings, deities, human beings and animal kingdom is explained. It is classified as *śūnya kāla, aśūnya kāla* and *miśrakāla. śūnya kāla* is the period when there is no being in the realm in a particular time-span. All the living beings leaving that realm take birth in another realm. None comes from outside. In other words, if the particular realm is devoid of living being which means empty, then that period will be known as *śūnya kāla* for that realm. Here we can say that it is the period when there exist zero or *śūnya* beings in the realm. So it is clear that conceptually *śūnya* had evolved then.

Ācārya Guṇadhara, belonging to early part of 2nd C. A.D. in his *āgamic* work (as per Digambara tradition) *Kaṣāya Pāhuḍa* (KP), in verse 49 states:

"*chavvīsa sattāvisā tewīsā pañcavīsa bāvīsā,*

ede suṇṇatthāna avagavavedassa jīvassa[1]"

i.e. for an *apagatavedi* (one who is free of 3 types of sex-consciousness) there is absence (*śūnya*) of 26th, 27th, 23rd, 25th, and 22nd sub-categories (*uttara prakṛti*) of deluding karmas.

Similarly, there are 14 *suṇṇaṭṭhāṇa* in respect of *jīva* with asexual consciousness; i.e. 19th, 18th, 14th, and from 11th downwards up to 1st mentioned in the following verse:

uguvīsaaṭṭhārasayaṁ coddasa ekkārasādiyā sesā,

ede suṇṇaṭṭhāṇā ṇavuṁsae coddasā honti.

(KP, *gāthā* 50, p.278).

Next few *gāthā*-s mention the word *suṇṇa* which is used for absence.

From aforementioned reference it is clear that, the concept of *śūnya* was evolving then. Though there is direct evidence of use of zero with place value in fifth century, but there is no direct evidence that any symbol for zero was used or not by that period. Since Jaina-s had an interest in the enumeration of very large numbers, there was need of their philosophical exposition. In ADS we have an indirect evidence of the same. Like SK, ADS tackles the problem of minimum number of bound corporeal bodies of humans. ADS *sūtra* 490 makes significant statements about large number with 29 places (*eguṇatīsaṁ ṭhāṇāiṁ*) which is already mentioned above in the section of large numbers. In modern notation it will be 2^{96} = 79228162514264337593543950336. Although this figure is not given and there is no mention of word for zero anywhere in ADS; it is clear that they were aware of

[1] *Kaṣāya Pāhuḍa* of Ācārya Guṇadhara, with commentary by Yativṛṣabha, trans. and ed. by Hiralal Jain, Veer Shasana Sangha, Calcutta, 1955, p. 277.

one certain place out of 29 digits where there is no digit (from 1-9). If we write it, we can notice that there is one 'zero' in it. In that period in place of zero they might have used any symbol or blank space. Even if we assume that this passage is added at the time of editing in the third redaction at Vallabhī it becomes obvious that decimal place-value system with zero was prevalent in India in its full grown form prior to 453 A.D.

SK by Puṣpadanta and Bhūtabali is assigned to 2nd century A.D. it shows complete familiarity with decimal and dyadic systems of numeration and their mutual relationship. But there is no direct or indirect reference to zero as a number anywhere in it. This shows that fully developed decimal place-value system must have started somewhere between 200 and 300 A.D.

Probably the first clear evidence of zero with place value can be found in *Lokavibhāga* by Sarvanandī written in the year 458 A.D exhibiting full familiarity with decimal place-value numeration and as per the research it gets the credit of very first time use of zero in the history. (Ifrah, p. 419). This work was from a period immediately after redaction of *Āgama*-s at Vallabhī in 453 (or 466) A.D. Numerous evidences of zero are found in this text, as :

i) *ārabhya bāhyatah* ***śūnyaṁ*** *prathame ca dvītiyake* (1.135, p. 16) i.e. in the first and second empty sections (*antarāla*) of *vedikā* from outside there is no (*śūnyaṁ*) tree. Here ***śūnyaṁ*** is used to express the absence.

ii) The mathematical use of zero is found in this *gāthā -ṣaṭ catuṣkaṁ ca* ***śūnyaṁ*** *ca saptakaṁ dvau ca pañcakaṁ* (6.98, p. 116) i.e. the circumference of LS at 1/6th of its width is six, four, zero, seven, two, five (52706 as read from right to left) *y*. Here the place of zero is clearly mentioned.

iii) *nava* ***śūnyaṁ*** *catuh pañca navakaṁ pañcamāṅśkaṁ*[1] (6.101) i.e. the circumference of the dark area in 1/6th part of LS in the order of numerals is nine, zero, four, five, zero, and one (105409 if read from right to left) and one fifth part ($\frac{1}{5}$) of *y*.

iv) *gāthā* 10.193 (p. 196) mentions *caturbhya ūrdhve* ***śūnyebhya****strīni dve dve ṣaṭ* i.e. four zeros at the end, three, two, two and six (i.e. 62230000, as read from right to left).

[1] *Lokavibhāga*. p. 116.

After about a century, we find in *Bṛhatkṣetra Samāsa* by Jinabhadragaṇi Kṣamāśramaṇa, many instances of use of decimal place-value system including zero, where he exhibits considerable proficiency with arithmetical operations of addition, subtraction, multiplication, division, fractions, extraction of square roots, etc. Thus, we may reasonably conclude that modern numeration and zero (with place-value) must have been invented sometime between 200 and 300 A.D. and operations with it must have been perfected between 400 and 500 A.D. Jinabhadragaṇi calculated square root of 10^{11} which was a big calculation (see chapter 5, section 5.4).

Jinabhadragaṇi Kṣamāsramaṇa, a Jaina scholar-saint (489-594 A.D.), in his work *Bṛhatakṣetra Samāsa* has made abundant use of 'zero' in writing numbers. The very large numbers were mentioned by reading the digits from left to right or right to left.

i) *Gāthā* 69 (p.98) mentions, square of chord of *Bharata* continent which is 75600000000 and spelled as 'seventy five, six, and eight zeros' (… *paṇasayarī chacca aṭṭha* ***suṇṇāiṁ***). In the same *gāthā* square of chord of *Himavanta* mountain 224400000000 is written as 'twenty two, forty-four, and eight zeros'(*du vīsa coyāla* ***sunnaṭṭhaṁ***).

ii) Next *gāthā* 70 (p.99) mentions square of chord of Haimavaṅta continent, i.e. 512400000000 as 'fifty one, twenty four, and eight zeros'(…*igavanna cauvīsaṁ aṭṭha* ***suṇṇa***...), and square of chord of Mahāhimavāna mountain, i.e. 1050000000000 is written as *du visa coyāla sunnaṭṭhaṁ* i.e. 'hundred five with ten zeros'(*pañcahiyaṁ sayamegaṁ…dasa ya suṇṇāiṁ*).

iii) *Gāthā* 71 (p.100) describes chord of Harivarṣa continent, i.e. 19716×10^8 as 'nineteen, seven, sixteen and eight zeros' (*uṇavīsaṁ satta sola suṇṇaṭṭha*); square of chord of *Niṣadha* mountain 32004×10^8 is 'thirty two, two zeros, four, and eight zeros' (*battīsam do suṇṇā, cauro suṇṇaṭṭha*).

There are many more such references in *Bṛhatakṣetra Samāsa* where numbers are described in above manner. Ifrah also writes in *Universal History of Numbers* [2000, p. 399] through an example of *gāthā* 69 given by Jinbhadragaṇi that the Indians knew of zero and the place value system in the sixth century. It is clear that by this period any specific symbol for zero must be in existence.

George Ifrah [2000, p.419] after giving the chronological evidences concludes that (Monday 25 August) 458 C. A.D., the exact date of *Lokavibhāga* , the Jaina cosmological

text: the oldest known Indian text to use zero and the place value system with word symbols. Furthermore, he says *this invention is entirely Indian and born out of a very specific context.*

Siddhasenagaṇī (6th C. A.D.) in his commentary, of *sūtra* 3.11 of *Tattvārthādhigama Sūtra* of Umāswāti, has used zero in his calculation. The following excerpt[1] of the text verifies it,

*… śūnyāṣṭakasyārdhena catvāri śūnyāni śeṣasya mūlameva…*i.e. *the remainder is this 35,34, 400,000,000, the square root of this is extracted; half of the eight zeros are four zeros; the root of the remaining portion is one, eight, and eight; hence the resulting root is 1,880,000.*

Similarly, one more quotation found here is *śūnya catuṣkāpagame bhāga labdhamidaṁ…*i.e. by removing the 4 zeros, the quotient obtained is 100000.

From above quotations it is proved that the zero has been considered as a number in India from the earliest centuries of the Christian era and there existed some symbol to denote this number.

3.10 Decimal -Scale (Temporal)

As we have discussed earlier, Egyptian, Greek, Roman and Indian (Vedic, Jaina and Buddhist) cultures based their counting on decimal system. In chapter 2 it is described that Jaina *Āgama* contains numerous examples of use of decimal system with place value. In entire literature not only Jaina but other Indian also there is no evidence found of use of the system other than decimal. It means the decimal system was common in every part of Indus civilization.

In ADS the number scale or counting is mentioned separately unlike BS. In *sūtra* 231(ADS, p.144), under *gaṇāṇupuvvi* while dealing with measures of counting (*gaṇitamāna*), mention is made for the scale:

- one (*ego*) = 1
- ten (*dasaṁ*)
- hundred (*sayaṁ*)=10^2
- thousand (*sahassaṁ*)=10^3

[1] *Tattvārthādhigamasūtra* of Umāsvāti with his own gloss, elucidated by Siddhasenagaṇi, ed. by Hiralal Kapadia, Jivanachand Sakercahnd Zaveri, Bomaby, Vol. I, 1926, p. 260.

- ten thousand (*dasasahassāiṁ*)=10^4
- hundred thousand (*sayasahassāiṁ*)=10^5
- million (*dasa sayasahassāiṁ*)=10^6
- ten million (*koḍi*)=10^7
- billion (*dasa koḍio*)=10^8
- ten billion (*koḍisayaṁ*)=10^9
- trillion (*dasa koḍisayāiṁ*)=10^{10}

Time scales are mentioned in many *āgamic* works, which are almost similar. The **time units** are explained in BS 6.7.132 starts from *samaya*, the smallest unit of time.

- 1 *samaya*= 1 *param sukṣma kāla* (ultimate unit time)
- *asaṅkhyāta* (innumerable) *samaya* = 1 *āvalikā*
- *saṅkhyeya* (numerable) *āvalikā* = 1 *ucchvāsa* (inhalation)
- =1 *niśvāsa* (exhalation)
- 1 *ucchvāsa* + 1 *niśvāsa* = 1 *prāṇa* (some scholars even take *ucchvāsa* = *niśvāsa* = *prāṇa*)
- 7 *prāṇa* = 1 *stoka*
- 7 *stoka* = 1 *lava*
- 77 *lava* = 1 *muhūrta*
- 30 *muhūrta* = 1 *ahorātra*
- 15 *ahorātra* = 1 *pakṣa*
- 2 *pakṣa* = 1 *māsa*
- 2 *māsa* = 1 *ṛtu*
- 3 *ṛtu* = 1 *ayana*
- 2 *ayana* = 1 *saṁvatsara*
- *saṁvatsara* = 1 *yuga*
- 20 *yuga* = 1 *saya* (=100)
- 10 *saya* = 1 *sahassa* (=1000)
- 10 *sahassa* = 1 *dasa sahassa* (=10,000)
- 10 *dasa sahassa* = 1 *saya sahassa* (=100,000)

Numbers in BS	Powers of 10
84 *saya sahassa* = 1 *pūrvāṅga* =(84×10^{5})	$(=10^{6.92})$
84 *saya sahassa pūrvāṅga* = 1 *pūrva* = $(84\times10^{5})^{2}$ = 7056 $\times10^{10}$	$(=10^{13.84})$
84 *saya sahassa pūrva* = 1 *truṭitāṅga* =$(84\times10^{5})^{3}$ = 592704 $\times10^{15}$	$(=10^{20.76})$
84 *saya sahassa truṭitāṅga* = 1 *truṭita* =$(84\times10^{5})^{4}$ = 49787136 $\times10^{20}$	$(=10^{27.68})$
84 *saya sahassa truṭita* = 1 *aḍaḍāṅga* =$(84\times10^{5})^{5}$ = 4182119424 $\times10^{25}$	$(=10^{34.6})$
84 *saya sahassa aḍaḍāṅga* = 1 *aḍaḍa* =$(84\times10^{5})^{6}$ = 351298031616 $\times10^{30}$	$(=10^{41.52})$
84 *saya sahassa aḍaḍa* = 1 *avavāṅga* =$(84\times10^{5})^{7}$ = 29509034655744$\times10^{35}$	$(=10^{48.44})$
84 *saya sahassa avavāṅga* = 1 *avava* =$(84\times10^{5})^{8}$ = 236072277245952 $\times10^{40}$	$(=10^{55.36})$
84 *saya sahassa avava* = 1 *hūhūkāṅga* =$(84\times10^{5})^{9}$=19830071288659968 $\times10^{45}$	$(=10^{62.28})$
84 *saya sahassa hūhūkāṅga* =1 *hūhūka* =$(84\times10^{5})^{10}$= 1665725988247437312$\times10^{50}$	$(=10^{69.2})$
84 *saya sahassa hūhūka* = 1 *utpalāṅga* = $(84\times10^{5})^{11}$ = 139920983012784734208$\times10^{55}$	$(=10^{76.12})$
84 *saya sahassa utpalāṅga* = 1 *utpala* = $(84\times10^{5})^{12}$ = 1175336257307391767 3472$\times10^{60}$	$(=10^{83.04})$
84 *saya sahassa utpal* = 1 *padmāṅga* = $(84\times10^{5})^{13}$ = 98728245613820908457 1648$\times10^{65}$	$(=10^{89.96})$
84 *saya sahassa padmāṅga* = 1 *padma* = $(84\times10^{5})^{14}$ = 82931726315609563104018432$\times10^{70}$	$(=10^{96.88})$
84 *saya sahassa padma* = 1 *nalināṅga* = $(84\times10^{5})^{15}$ = 69662650105184330073754828 8$\times10^{75}$	$(=10^{103.8})$
84 *saya sahassa nalināṅga* = 1 *nalina* = $(84\times10^{5})^{16}$ = 58516626088354837261954056192 $\times10^{80}$	$(=10^{110.72})$
84 *saya sahassa nalina* = 1 *arthanipurāṅga* = $(84\times10^{5})^{17}$ = 49153965914218063300041407 20128 $\times10^{85}$	$(=10^{117.64})$
84 *saya sahassa arthanipurāṅga* = 1 *arthanipura* = $(84\times10^{5})^{18}$ = 43353797936295338532183652115152896 $\times10^{90}$	$(=10^{124.56})$

84 *saya sahassa arthanipura* = 1 *autāṅga* = $(84\times10^5)^{19}$ = 641719026648808436703426777672843264×10^{95}	(=$10^{131.48}$)
84 *saya sahassa autāṅga* = 1 *ayuta* = $(84\times10^5)^{20}$ = 305904398238499908683087849324518834176×10^{100}	(=$10^{138.4}$)
84 *saya sahassa ayuta* = 1 *prayutāṅga* = $(84\times10^5)^{21}$ = 256959694520339923293793793432595820707 84 ×10^{105}	(=$10^{145.32}$)
84 *saya sahassa prayutāṅga* = 1 *prayuta* = $(84\times10^5)^{22}$ = 215846143397085535566786786483380489394585 6×10^{110}	(=$10^{152.24}$)
84 *saya sahassa prayuta* = 1 *nayutāṅga* = $(84\times10^5)^{23}$ = 181310760453551849876100900646039611091451904×10^{115}	(=$10^{159.16}$)
84 *saya sahassa nayutāṅga* = 1 *nayuta* = $(84\times10^5)^{24}$ = 152301038780983553895924756542673273316819599 36 ×10^{120}	(=$10^{166.08}$)
84 *saya sahassa nayuta* = 1 *cūlikāṅga* =$(84\times10^5)^{25}$ = 127932872576026185272576795495845549586128463462 4 ×10^{125}	(=10^{173})
84 *saya sahassa cūlikāṅga* = 1 *cūlikā* = $(84\times10^5)^{26}$ = 107463612963861995628964508216510261652347909308416×10^{130}	(=$10^{179.92}$)
84 *saya sahassa cūlikā* = 1 *śīrṣaprahelikāṅga* = $(84\times10^5)^{27}$ = 902694348896440763283301869018686197879722438190694 4 ×10^{135}	(=$10^{186.84}$)
84 *saya sahassa śīrṣaprahelikāṅga* = 1 *śīrṣaprahelikā* = $(84\times10^5)^{28}$ = 758263253073010241157973569975696406218966848080183296×10^{140}	(=$10^{193.76}$)

Table 3.1: Scale of 84

The aforementioned time-scales consists of two orders: power of 10 and of 84 ×10^5. Power of 10 goes up to *sayasahassa* (10^5) and then shifts to the power of 84×10^5 which finally stops at *śīrṣaprahelikā* (84 ×10^5)^{28} the so-called end of the countable number.

śīrṣaprahelikā in the notation of modern mathematics can be written as,

$(84\times10^5)^{28} = (84)^{28}\times10^{140}$

=758,263,253,073,010,241,157,973,569,975,696,406,218,966,848,080,183,296×10^{140}.

Though in any of the canons we don't find the above value of this number, yet it's sure that they were aware of this number. This is 194 digits number as accepted in *Māthurī* council (300 A.D.) while the *Vallabhī* council (300 A.D.) accepted it as a 250 digits number with 70 digits and 180 zeros.

i.e. $(84\times10^{5})^{36}$ =

$$187955179550112595419009699813430770797465494261977747657257345718681 6\times10^{180}$$

Abhayadeva Sūrī, a scholar-saint belonging to 11th C. A.D., calculates this number and expresses it by giving the digits in this number from left to right as "seven, five, eight,.. ……, two, nine, six, followed by one hundred and forty zeroes."

According to commentary (*cūrṇi*) of ADS the number from *antarmuhūrta* (intermediate period from 2 *samaya* to 48 min. less 1 *samaya*) to *pūrvakoṭi* were used to calculate the life span of humans and animals and from *truṭita* to *śīrṣaprahelikā* was being used to count the lifespan of hellish beings, mansion-dwelling (*bhavanapati*) and forest-dwelling (*vāṇamantara*) deities[1].

What if something exists in number beyond *śīrṣaprahelikā.* Then *Āgama* says- *etāva tāva gaṇie, etāva tāva gaṇiassa visae, teṇaṁ pare ovamie*[2] (BS 6.7.132) (JDP, 2.24) - Numbers till *śīrṣaprahelikā* is the subject of counting (arithmetic) and beyond this measurement is done by using simile (analogy). In other words, after *śīrṣaprahelikā* counting is beyond the capacity of human brain. If we study Jaina history, its time cycle, the life of living beings may it be human or deities the number is always beyond our counting. How would they count? We will discuss number beyond limits of counting in chapter four.

3.10.1 A Comparative View of *śīrṣaprahelikā*

Number scale differs in *Āgamic* works of Śvetāmbara tradition and Digambara tradition also, which can be compared through following table:

[1] *Bhagavaī,* Vol. II. pp. 287-288.

[2] *Jambūdvīpaprajñapti Sūtra*, pp. 27.

Table 3.2: Comparative Number Scale

S. No.	BS(6.7.132, p.286-287), JDP(2.24, p.27), ADS(6.129, p.142), JJ(3.178b, pp.50-51)	JK (62-71, p.38-39)	JS (3.114-115, P.88-89)	LV (5.127-136, p.96-97)	TP (296-312, p.84-89)	TSB (4.15, p.209-10)	Rājavārtika (RV, 3.38, p 209)
1.	*pūrvāṅga*	*pūrvāṅga*	*pūrvāṅga*	*pūrvāṅga*	*pūrvāṅga*	*pūrvāṅga*	*pūrvāṅga*
2.	*pūrva*	*pūrva*	*pūrva*	*pūrva*	*pūrva*	*pūrva*	*pūrva*
3.	*truṭitāṅga*	*latāṅga*	*nayutāṅga*	*parvāṅga*	*parvāṅga*	*ayuta*	*parvāṅga*
4.	*truṭita*	*latā*	*Nayuta*	*parva*	*parva*	*kamala*	*parva*
5.	*aḍaḍāṅga* (*aṭaṭāṅga, ADS*)	*mahālatāṅga*	*nalināṅga*	*nayutāṅga*	*nayutāṅga*	*nalina*	*niyutāṅga*
6.	*aḍaḍa* (*aṭaṭa, ADS*)	*mahālatā*	*nalina*	*nayuta*	*nayuta*	*kumuda*	*niyuta*
7.	*avavāṅga*	*nalināṅga*	*mahānalināṅga*	*kumudāṅga*	*kumudāṅga*	*tuṭi*	*kumudāṅga*
8.	*avava*	*nalina*	*Mahānalina*	*kumuda*	*kumuda*	*aḍaḍa*	*kumuda*
9.	*hūhūkāṅga*	*mahānalināṅga*	*padmāṅga*	*padmāṅga*	*padmāṅga*	*avava*	*padmāṅga*
10.	*hūhūka*	*mahānalina*	*padma*	*padma*	*padma*	*hāhā*	*padma*
11.	*utpalāṅga*	*padmāṅga*	*kamalāṅga*	*nalināṅga*	*nalināṅga*	*hūhū*	*nalināṅga*
12.	*utpala*	*padma*	*kamala*	*nalina*	*nalina*		*nalina*
13.	*padmāṅga*	*mahāpadmāṅga*	*kumudāṅga*	*kamalāṅga*	*kamalāṅga*		*kamalāṅga*
14.	*padma*	*mahāpadma*	*kumuda*	*kamala*	*kamala*		*kamala*
15.	*nalināṅga*	*kamalāṅga*	*truṭitāṅga*	*tutyāṅga*	*tutyāṅga*		*tutyāṅga*
16.	*nalina*	*kamala*	*truṭita*	*tutita*	*tutita*		*tutita*
17.	*arthanipurāṅga*	*mahākamalāṅga*	*aṭṭāṅga*	*aṭaṭāṅga*	*aṭaṭāṅga*		*aṭaṭāṅga*
18.	*arthanipura*	*mahākamala*	*aṭṭa*	*aṭaṭa*	*aṭaṭa*		*aṭaṭa*
19.	*ayutāṅga*	*kumudāṅga*	*avavāṅga*	*amamāṅga*	*amamāṅga*		*amamāṅga*
20.	*ayuta*	*kumuda*	*avava*	*amama*	*amama*		*amama*
21.	*prayutāṅga* (*nayutāṅga, JDP, ADS*)	*mahākumudāṅga*	*hūhūkāṅga*	*hāhāṅga*	*hāhāṅga*		*hāhāṅga*
22.	*prayuta* (*nayuta, JDP, ADS*)	*mahākumuda*	*hūhūka*	*hāhā*	*hāhā*		*hāhā*
23.	*nayutāṅga* (*prayutāṅga , JDP, ADS*)	*truṭitāṅga*	*ayutāṅga*	*hūhū-aṅga*	*hūhū-aṅga*		*hūhū-aṅga*
24.	*nayuta* (*prayuta, JDP, ADS*)	*truṭita*	*ayuta*	*hūhū*	*hūhū*		*hūhū*
25.	*cūlikāṅga*	*mahātruṭitāṅga*	*prayutāṅga*	*latāṅga*	*latāṅga*		*latāṅga*
26.	*cūlikā*	*mahātruṭita*	*prayuta*	*latā*	*latā*		*latā*
27.	*śīrṣaprahelikāṅga*	*aḍaḍāṅga*	*śīrṣaprahelikāṅga*	*mahālatāṅga*	*mahāāṅga*		*mahāāṅga*
28.	*śīrṣaprahelikā*	*aḍaḍa*	*śīrṣaprahelikā*	*mahālatā*	*mahālatā*		*mahālatā*
29.		*mahāḍaḍāṅga*		*śirahprakampita*	*śrīkalpa*		*śrīkalpa*
30.		*mahāaḍaḍa*		*hastaprahelita*	*hasta-prahelita*		*hasta-prahelita*
31.		*ūhāṅga*		*acalātmaka*	*acalātmaka*		*acalātmaka*
32.		*ūha*					
33.		*mahāūhāṅga*					
34.		*mahāūha*					
35.		*śīrṣaprahelikāṅga*					
36.		*śīrṣaprahelikā*					

If we compare the scale mentioned in BS and ADS, few differences can be perceived.

- We can observe the change in order of the names: The order found in BS is *prayutāṅga, prayuta, nayutāṅga, nayuta* and ADS mentions it as *nayutāṅga, nayuta, prayutāṅga, prayuta*. Yet it does not affect the value of *śīrṣaprahelikā*.
- JDP (2.24) follows BS and ADS and JS with minor differences in order of the names, while JJ perfectly follows BS. They all have the same value of *śīrṣaprahelikā*.
- JK follows Vallabhī council (with 250 digits), while BS follows Māthurī (with 194 digits).
- In PS though we do not find the aforementioned scale up to *śīrṣaprahelikā* like BS and ADS, yet counting scale found to measure the numbers show that decimal system was established by that period. The higher counting goes as *sahassāṅaṁ* (thousand), *saya-sahassāṅam* (*lākh*,10^5) up to *koḍākoḍī* (10^{14}) etc. SCP is completely astronomical text. We do not find such scale here in SCP and SK also.
- It's so surprising that SBT has very small scale and the highest number is *hūhū* i.e. $(84\times10^5)^{11}$.
- *Lokavibhāga* is a non-*Āgamic* work, here also the great difference can easily be observed from the table. There is no time scale, but the number scale starts from *pūrvāṅga* and one *pūrvāṅga* is equal to 8400000 years. It means the decimal system was of course in use but the author did not feel necessity to explain it in the text. Therefore he started from the order of 8400000.
- The highest number in BS is *śīrṣaprahelikā* while in Digambara tradition (LV/TP) it is *acalātmaka*. The value of highest number *śīrṣaprahelikā* in BS is $(84\times10^5)^{28} = (84)^{28}\times10^{140} \approx 10^{193.76} = 75826325307301024115797356997569640621896684808018329 6\times10^{140}$

In Digambara tradition the scale moves differently. Each term is not multiplied by 84×10^5, but alternatively. For example, *pūrva*×84 = *parvāṅga; parvāṅga*×84×10^5 = *parva.* Thus the value of highest number *acalātmaka*

$= (84)^{31}\times10^{90} = 10^{150.76}$

$= 84^{31}\times10^{90} =$

$= 449,425,663,149,385,461,975,295,566,818,875,162,751,606,526,724,516,960,272,384\times10^{90}$

Hence the total number of digits in *acalātmaka* is 151 whereas *śīrṣaprahelikā* has 194 digits.

3.11 Large Numbers in Vedic Tradition

An extensive number system existed in Vedas. *Yajurveda* (1200 to 1000 BC)[1] praises high numbers as :

Hail to a hundred, hail to a thousand, hail to *ayuta* [ten thousand], hail to *niyuta* [hundred thousand], hail to *prayuta* [million], hail to *arbuda* [ten million], hail to *nyarbuda* [hundred million], hail to *samudra* [billion], hail to *madhya* [ten billion], hail to *anta* [hundred billion], hail to *parārdha* [trillion], hail to the dawn, hail to the daybreak… hail to the world, hail to all (*Yajur-veda* 7.2.20).

Vedic *Taitiriya Saṅhitā* (7.12.20.) mentions a scale based on 10 with powers of ten designated as *eka* (10^0), *daśa* (10^1), *śata* (10^2), *sahasra* (10^3), *ayuta* (10^4), *niyuta* (10^5), *prayuta* (10^6), *arbuda* (10^7), *nyarbuda* (10^8), *samudra* (10^9), *madhya* (10^{10}), *anta* (10^{11}), *parārdha* (10^{12}), *usas* (10^{13}), *vyuṣita* (10^{14}), *deśyata* (10^{15}), *udyata* (10^{16}), *udita* (10^{17}), *svarga* (10^{18}) and *loka* (10^{19})[2]. In Mahābhārata (1st century A.D.) 2nd chapter of *Ādiparva* (*sūtra* 19-26) explains about one *akṣauhini army* which 21870 chariots, 21870 elephants, 65610 cavalry and 109350 infantry (i.e. in the ratio 1:1:3:5). *Kaurava*-s army was of 11 *akṣauhiṇī* (2405700 personnel) and *Paṇḍava*-s army of 7 *akṣauhiṇī* (1530900 personnel). Total army of both was 3936600.

3.12 Large Numbers in *Rāmāyaṇa*

The terminology of different powers of ten continued with some changes and designations till 9th C. A.D. *Vālmīki Rāmāyaṇa* (1st C. A.D.) contains a scale[3], in *gāthā* 33-24 of *sarga* 28 of *yuddha kaṇḍa* (8th), going from *śata-sahasra* (10^5) to *koṭi* (10^7) and thereafter in multiples of 10^5 as *śaṅku* (10^{12}), *mahāśaṅku* (10^{17}), *vṛnda* (10^{22}), *mahāvṛnda* (10^{27}), *padma* (10^{32}), *mahāpadma* (10^{37}), *kharva* (10^{42}), *mahākharva* (10^{47}), then thousand times is *samudra* (10^{50}), and again 10^5 times *ogha* (10^{55}), and lastly *mahaugha* (10^{60}). In Rāmāyaṇa, the great Indian epic, there is a description of two armies facing, each other. The Rāvaṇa[4] had 10,000 elephants, 10,000 chariots, 20,000 horses and 1,00,00,000 foot soldiers (demons) and each demon had 10,00,000 demons in his army according to

[1] Plofker, Kim, *Mathematics in India*, Princeton University Press, New Jersey, 2009, p. 14.

[2] *Prācīna Bhāratīya Gaṇita kī Eitihāsika va Sānskṛtika Jhalakiyāṅ*, pp. 28-29.

[3] *Rāmāyāṇā* by Vālmīki, trans. by Rāmānārāyaṇadatta Śāstrī, Gita Press, Gorakhpur, 9th edn., 1993, p. 717.

[4] *Ibid*, p. 730.

Rāmāyaṇa. Therefore, the total army of Rāvaṇa is 10,000+10,000+20,000+($10^7 \times 10^6$)= 1,000,000,040,000. while the Rāma, the hero of the epic in opposition has the larger army of *vānara* (monkeys)[1], which is counted as,

10^{10}(1000 *koṭi*)+10^{14} (100 *śanku*)+10^{20} (1000 *mahāśanku*)+10^{24} (100 *vṛnda*)+10^{30} (1000 *mahāvṛnda*)+10^{34} (100 *padma*)+10^{40} (1000 *mahāpadma*)+10^{44} (100 *kharva*)+10^{52} (100 *samudra*)+10^{57} (100 *mahaugha*)+10^{62} (crore *mahaugha*)+5= 100,001,000,010,000,000,100,010,000,010,001,000,001,000,100,000,100,010,000,000,005. It's a 63 digits number.

3.13 Large Number in Buddhist Literature

The love for high numbers in Buddhist tradition can also be seen in *Lalitavistara Sūtra* (3rd C. A.D.). In this text Buddha (*Bodhisatva*) is said to be a man of mathematics. During his education in childhood Bodhisattva[2] at the age of eight, sage Viśwāmitra explains the counting of the number to the child as shown below with the numeric value.

Table 3.3: Numbers in Buddhism

Lākha	10^5
Koṭi	10^7
Nahuta	10^9
Ninnahuta	10^{11}
Khamba	10^{13}
Viṣkhamba	10^{15}
Ababa	10^{17}
Aṭṭaṭa	10^{19}
Kumuda	10^{21}
gundhika	10^{23}
Utpala	10^{25}
Puṅḍarīka	10^{27}
Paduma	10^{29}

[1] *Rāmāyāṇa* , pp. 717-718.
cf. The Crest of The Peacock, p. 340.

[2] *The Universal History of Numbers*, p. 422.

Another illustration we find here is the context of his marriage where he has to show his mastery in arts among 500 *śākya* which included mathematics of high number. The questions starts and in his reply, Bodhisattva had given the following table[1]:

Table 3.4: Number scale of *Bodhisattva*

1 *ayuta*	100 *koṭi*	10^{9}
1 *niyuta*	100 *ayuta*	10^{11}
1 *kaṅkara*	100 *niyuta*	10^{13}
1 *vivara*	100 *kankara*	10^{15}
1 *akṣobhya*	100 *vivara*	10^{17}
1 *vivāha*	100 *akṣobhya*	10^{19}
1 *utsaṅga*	100 *vivāha*	10^{21}
1 *bahula*	100 *utsaṅga*	10^{23}
1 *nāgabala*	100 *bahula*	10^{25}
1 *tiṭilambha*	100 *nāgabala*	10^{27}
1 *vyavasthānaprajñapati*	100 *tiṭilambha*	10^{29}
1 *hetuhila*	100 *vyavasthānaprajnapati*	10^{31}
1 *karaphu*	100 *hetuhila*	10^{33}
1 *hetvindriya*	100 *karaphu*	10^{35}
1 *samāptalambha*	100 *hetvindriya*	10^{37}
1 *gaṇanāgati*	100 *samāptalambha*	10^{39}
1 *niravadya*	100 *gaṇanāgati*	10^{41}
1 *mudrābala*	100 *niravadya*	10^{43}
1 *sarvabala*	100 *mudrābala*	10^{45}
1 *visaṅjñāgati*	100 *sarvabala*	10^{47}
1 *sarvasaṅjña*	100 *visaṅjñāgati*	10^{49}
1 *vibhutaṅgamā*	100 *sarvasaṅjña*	10^{51}
1 *tallakṣaṇa*	100 *vibhutaṅgamā*	10^{53}

[1] *Lalitavistara*, trans. by Śāstrī, Śāntibhikṣu, Uttarpradesha Hindi Sansthana, Lucknow, 1984, pp. 286-287. *cf.* Ifrah, p. 423.

The counting does not stop here. In *Lalitavistara* higher counting is also described, which is quoted by Ifrah (2000, p. 423) also. He says *dhvajāgravati* system is also made up of twenty-four terms, and its first term is the *tallakṣaṇa* (the largest number in the preceding system, that is 10^{53}). Since its progression increases geometrically by a ratio equivalent to one hundred, therefore the value of its final term is:

$$1\ dhvajāgravati = (10^{7+46\times1}) \times (10^2)^{23} = 10^{7+46\times2} = 10^{99}.$$

As this is the last term in the preceding system, it becomes the first in the following one, that is to say the third system, the *dhvajāgraniśāmaṇi*, the final number of which being equal to:

$$1\ dhvajāgranishāmaṇi = (10^{7+46\times2}) \times (10^2)^{23} = 10^{7+46\times3} = 10^{145}.$$

Step by step, we thus arrive at the ninth counting system, of which the name of the last term has the value:

$$(10^{7+46\times8})\times(10^2)^{23} = 10^{7+46\times9} = 10^{421}.$$

It may be interesting to observe that even in contemporary Buddha literature very large numbers are found. Buddhist scriptures also reveal the knowledge of decimal system of numeration. Enumeration begins with 1 and keep on increasing by multiple of 10 and the names of counting is the same as we know today, i.e. *dasa, śata, sahasra, dasasahasra, śatasahasra, dasaśatasahasra*, and *koṭi* (10^7); and from here onwards the scale goes on increasing by a factor of 10^7. Ifrah (p. 422) lists very large numbers as *pakoṭi* (10^{14}), *koṭippakoṭi* (10^{21}), *nahuta* (10^{28}), *ninnahuta* (10^{35}), *akhobhinī* (10^{42}), *bindu* (10^{49}), *abbuda* (10^{56}), *nirabbuda* (10^{63}), *ahaha* (10^{70}), *ababa* (10^{77}), *aṭaṭa* (10^{84}), *soganṅdhika* (10^{91}), *uppala* (10^{98}), *kumuda* (10^{105}), *puṇḍarīka* (10^{112}), *paduma* (10^{119}), *kathāna* (10^{126}), *mahākathāna* (10^{133}), and *asaṅkhyeya* (10^{140}).

3.14 Large Numbers in Jaina *Āgama*

The world of numbers in Jaina scriptures is very wide using extremely large numbers of higher orders than those in other civilizations. The Greeks could not go beyond ten thousand. Romans had specific name for numbers up to hundred thousand only. The French introduced the word 'million' in their vocabulary in 13th century. Jaina saint scholars even before Christian era went up to tremendously large numbers.

The incredible speculations by Jainas are appreciated by Ifrah (pp. 425) as :

There are other, even older Jaina *texts, where numbers containing eighty or even hundred orders of units are described as 'minuscule' in comparison with those under speculation:*

these numbers are as high as, or greater than ten to the power of 250, which we would write today ad 1 followed by at least two hundred and fifty zeros.

3.14.1 Wisdom of Gargantuan Numbers

Ample references in Jaina *āgamic* literatures reveal that decimal system, place value system as well as large numbers were part and parcel of their philosophy. Plenty of examples enumerate the number, life-span, height, and hundreds of aspects related to living beings are explained in large numbers. Cosmology is the subject of almost all canons. Cosmography of entire universe from every subtle aspect and angle is explained in great detail. For illustration, the area and circumference of *Jambūdvīpa, Lavaṇasamudra,* dimension of Mountain Meru and numerous other mountains, the number of rivers, mountains etc. are described elaborately.

a) *Bhagavatī Sūtra* mentions very large numbers. In *Āgama* use of numbers thousands and hundred thousand are very common and they are very small numbers. The life span of hellish being of first hell (*ratnaprabhā*) who has come from five-sensed developed animal-species devoid of mind (*paryāpta-asañjī-pañcendriya-tiryagyonika*) is *pūrvakoṭi* (i.e. $(8400000)^2 \times 10^7$) = 7056×10^{17} years]. In the same chapter, for the same kind of living beings the life-span is explained in detail from various perspectives. Explaining the location of the *sabhā* (palace) of a deity *Vairocanendra Vairocanarāja* Bali[1]. *Camarcañca,* the residence of *Asurendra* Camar is located at diagonally *chakkkoḍisae paṇṇapannaṁ ca koḍīo paṇatīsaṁ sayasahassāiṁ, pannāsaṁ ca sahassāiṁ*..i.e. six *koḍisata* (10^8), fifty five crore, thirty five lakh and fifty thousand (6,55,35,50,000) *yojana* away from *Aruṇodaya* ocean (13.6.96). Aruṇodaya Sea is 18^{th} among the islands and seas encircling JD. Hence the distance of Camara from the center of JD (or Mt. Meru) is: $(1/2 + 2 + 4 + 8 + \ldots + 2^{17}) \times 10^5 + 6553550000 = (2^{18} - 1) \times 10^5 - 50000 + 6553550000 = 32767800000$ *yojana.*

b) Like PS, ADS, SK also discusses the problem of determining the population of developed human beings in *khaṇḍa* (volume) 2, *adhikāra* (chapter) 5, *sūtra* 29. It states that it lies in the range from *koḍākoḍākoḍī* (10^{21}) to *koḍākoḍākoḍākoḍī* (10^{28}). As per practice then range *koḍākoḍākoḍākoḍī* was taken upto 10 *koḍākoḍākoḍākoḍī* – 1. This is then a 29 digit number. It further states in the same *sūtra* that this

[1] *Bhagavaī* , Vol. II, p. 150.

number also lies between 6th successive square of 2 (= 2^{64}) and 7th successive square of 2 (= 2^{128})[1].

c) *Jambūdvīpapaṇṇatti* (JDP), an *Āgamic* work, describes about the geographical features and dimensions in respect of *Jambūdvīpa* (JD, our earth). The diameter and circumference of JD, chords, bows and arms of continents and continental mountains, lengths of orbits of sun, moon, planets, asterisms and stars etc. are expressed in decimal place value notation. The biggest number encountered in JDP is the number of stars in JD which is 13,395, 000,000,000,000,000 which is expressed as one hundred thousand thirty-three thousand nine hundred and fifty–hundred trillion (*egaṁ ca sayasahassaṁ tettisaṁ ... sahassāiṁ ṇava ya sayā paṇṇāsā ... koḍīkoḍiṇaṁ*)[2] (JDP, 7/1).

The number of animals and humans in the army[3] of Cakravartī Bharata as mentioned in JDP (3.86) is very huge. It is said that he had 84×10^5 horses (*caurāsīie āsasaysahssṇaṁ*), 84×10^5 elephants (*caurāsīie daṁtiisaysahssāṇaṁ*), 84×10^5 chariots (*caurāsīie rahasaysahssāṇaṁ*) and 96×10^7 foot-soldiers in his army (*caṇṇauī manussakoḍīo…*), which is 985200000 in total. His other living and non-living assets were very large in number.

It's surprising that the number of *audārika śarīra* of human beings given here in JDP is same as the number given in ADS with the same process.

d) *Prajñāpanā Sūtra* is full of mathematics for explaining various metaphysical aspects. Along with the frequent use of *saṅkhyāta, asaṅkhyāta* and *ananta,* decimal system is used to enumerate very large numbers. The number *pūrva koḍi* is used several times. For example, how long a female animal can take birth in the same species [*sūtra* 18.1262 (2)][4]? A female animal can remain as the same in many birth for 3 *palyopama* and *pthaktvakoṭi* years. *Sūtra* 18.1366[5] says that maximum period

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[1] *Satakhandāgama* with Dhavalā commentary by Vīirasena, ed. by Hiralal Jain, 16 Parts, Jaina Sahityoddharaka Fund Karyalaya, Amaraoti, 1st edn., 1939-1959. Reprint by Jaina Sanskrti Sangha, Solapur, Vol. I, 1940.

[2] *Jambūdvīpapaṇṇṇatti* (*Uvaṅgasuttāṇi*), Vol. II, p. 552.

[3] *Jambūdvīpaprajñapti Sūtra*, p. 176.

[4] *Prajñāpnā Sūtra,* Vol. II, p. 327.

[5] *Ibid,* p. 361.

of *kevalī āhāraka* is little less than *koḍi–pūrva* where *koḍi–pūrva* is equivalent to $7056 \times 10^{17} = 705600000000000000000$.

e) *Jīvājīvābhigama* is an encyclopedia of detailed classification and explanation of living being (*jīva*) and inanimate matter (*ajīva*). In Jaina philosophy not only humans but animals and other forms of living beings are also discussed for the spiritual purpose. For example, the maximum life-span of aquatic animals[1] is *pūrvakoḍi* (*sūtra* 1.38). Its value is already discussed in the previous paragraph. Similarly, the life of reptiles (1.39, *ura-parisarpa and bhuja-parisarpa*) is also *pūrvakoḍi*[2]. The maximum life of human-female (2.47.2) is *tiṇṇi paliovamāiṁ* (3 *palyopama*), aqua animals-female (2.47.1) is *pūrvakoḍi*, and many more are very large numbers. Explaining the *Puśkaradvipa* (*sūtra* 3.176.a) it is said that the number of stars in it is 964440000000000000000 as mentioned in the following verse :

chaṇṇui sayasahassā cattālīsaṁ bhave sahassāiṁ
cattāri sayā pukkharavara tārāgaṇakoḍīkoḍīṇaṁ[3].

i.e. ninety six hundred thousand, forty four thousand, four hundred *koḍākoḍī* (10^{14}).

f) *Jyotiṣa Karaṇḍaka* (before 3rd C. A.D.) is regarded as composed prior to ADS. It contains numerous instances of numeration with decimal place-value system. While describing the family of a single moon it says in *sūtra* 124 that it contains 6697500000000000000 stars which is put in words as*:*

chāvaṭṭhiṁ ca sahassa nava ya sayā pañcasattarā honti
ega sasī parivaro tārāgaṇakoḍakoḍīṇaṁ[4]

(sixty-six thousand nine hundred seventy-five ... *koḍākoḍī*), where *koḍākoḍī* = $10^7 \times 10^7 = 10^{14}$.

g) In *Bṛhatkṣetra Samāsa, Ācārya* Jinabhadragaṇi has made numerous calculations to give dimension of the geographical features. For example, number of stars[5] in *Kālodadhi* ocean mentioned in *gāthā* 579 is 2812950 *koḍikoḍiṇaṁ* which equals to

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[1] *Jīvāvīvābhigama,*Vol. I, p. 92.

[2] *Ibid*, p. 95.

[3] *Ibid*, vol. II, p. 40.

[4] *Jyotiṣkaraṇḍaka*, p. 71.

[5] *Bṛhatkṣetra samāsa,* Vol. II, p. 242.

2,81,295,000,000,000,000,000 (=281295×10^{15}). Volume[1] of *lavaṇasamudra* (LS) is 1693399155000000(=1.6×10^{14}) cubic *yojana* which written as :

solas koḍākoḍī, teṇaui koḍisayasahasāiṁ
ūyālīsa sahassā, navakoḍisayā ya pannarasā. (*gāthā* 481)

pannāsa sayasahasā joyaṇāṇaṁ bhave aṇūnāiṁ
lavansamuddassehiṁ joyaṇsaṅkhāi ghanagaṇiyaṁ (*gāthā* 482)

i.e. Sixteen *koḍākoḍī* (10^{14}), ninety three *koḍīsayasahassa* (10^{12}), fifteen hundred fifty thousand.

Chord and bows of various mountains and continents are also calculated in this text. In this work we find many instances of use of decimal place-value system, where he exhibits considerable proficiency with arithmetical operations of addition, subtraction, multiplication, division, fractions, extraction of square roots, etc.

He calculates surface area of LS as 99,611,715, 000 square *yojana*-s and expresses in *gāthā* 479 as '*navanauī koḍisayā egaṭṭhī koḍi lakkha sattarasa pannarassa sahassāṇi*'[2]. Also note that Jinabhadragaṇi reads numbers from left to right.

h) *Jivasamāsa* also introduces large numbers. According to *sūtra* 146 the minimum number of souls in 6th *gunasthāna i.e. pramatta saṅyata*[3] is *koḍi sahsra puhattam i.e.* between 1000 *koḍi* (=10^{10}) and maximum 9999 *koḍi* (=10^{11} -1).

3.15 Dyadic scale in *Āgama*

In Jaina canonical literature along with decimal system dyadic system is also found at few places. For illustration, ADS *sūtra* 490, and PS *sūtra* 12.921.1[4] while enumerating the number of bound corporeal body (*baddha audārika śarira*) of human beings makes following statements:

i) This number is with 29 digits and is equal to *saṅkhyāta koḍākoḍi* (*saṅkhejjāo koḍio egūntīsaṁ ṭhānaiṁ, tijamalpayassa, uvarim causamalpayassa heṭṭhā, ahava ṇaṁ chaṭṭho vaggo pañcamavaggapaḍuppaṇṇo, ahavā ṇaṁ chaṇṇuicheyaṇgadāyirāsī*).

[1] *Bṛhatkṣetra samāsa*, p. 168.

[2] *Ibid*, p. 168.

[3] *Jīvasamāsa*, p. 121.

[4] *Prajñāpanā Sūtra*, Vol. II, pp.113-114.

ii) The *sūtra* says that this number is greater than "*tiyamalapaya*" and less than "*cauyamalapaya*". "*yamalapaya*" means a number with 8 digits,"*biyamalapaya*" is the number with 16 digits, "*tiyamalapaya*" is the number with 24 digits, "*cauyamalapaya*" is with 32 digits, etc. Hence, statement in i) says that number of digits lies between 24 and 32.

iv) "*yamalapaya*" also stands for second successive square of 2 i.e. 2^{2^2}, "*biyamalapaya*" is then 4th successive square of 2, i.e.2^{16}, "*tiyamalapaya*" is 6th successive square of 2, i.e. 2^{64}. *cauyamalapaya* is then 8th successive square of 2, i.e.2^{256}.

Let's say the number of humans is H. According to the section i) the number of human beings $2^{32} \times 2^{64} = 2^{96}$.

The first successive square = $2^2 = 4 = 2^{2^1}$

The 2nd successive square $4^2 = 16 = 2^{2^2}$

The 3rd successive square = $16^2 = 256 = 2^8 = 2^{2^3}$

The 4th successive square = $256^2 = 65536 = 2^{16} = 2^{2^4}$

The 5th successive square = $65536^2 = 4{,}294{,}967{,}296 = 2^{32} = 2^{2^5}$

The 6th successive square $(4{,}294{,}967{,}296)^2 = 18{,}446{,}744{,}073{,}709{,}551{,}616 = 2^{64} = 2^{2^6}$

v) This number can be obtained by multiplication of 6th successive square of 2 with 5th successive square of 2 (as per section i) stated in the *sūtra)*, i.e.

$2^{64} \times 2^{32} = 2^{96}$ = 18,446, 744,073,709,551,616×4,294,967,296

= 79,228,162,514,264,337,593,543,950,336 (a number with 29 digits, as per section ii) which is less than with digits 32 and greater than with digits 24.)

Jaina scholars were also aware of the use of **dyadic scale** and the relation between dyadic and decimal scale. As in the above example, we see that to get the desired number they are saying to multiply the number in dyadic scale .i.e. $2^{64} \times 2^{32} = 2^{96}$ and the resulting number is of 29 digits in decimal scale.

3.16 Human Brain and Large Numbers

- The largest number with *any* real world application or, in this case, real **world's** application – is probably $10^{10^{10^7}}$, which is one recent estimate of the number of universes in the multiverse. That number is so huge that the human brain would be literally unable to perceive all those different universes, as the mind is only capable of roughly $10^{10^{16}}$ configurations.

- In 1933, Stanley Skewes proved that the upper bound for when the prime-counting function first becomes an underestimate is $10^{10^{10^{34}}}$. It's hard to really comprehend in even the most abstract sense what a number like that actually is, and to that point it was easily the largest number ever used in a serious mathematical proof. Since then, mathematicians have been able to reduce the upper limit to the relatively tiny figure of about 10^{316}, but the original figure remains known as Skewes' number of type I.

3.17 Decimal Scale Separation (*puhatta/ pṛthaktva*)

A unique concept found frequently (hint not explained) in Jaina *āgamic* canonical texts and other works also is the concept of decimal scale separation or *pṛthaktva***.** *puhatta* (*pṛthaktva*) means separation which was defined by later *Ācārya*-s and commentators as 2 to 9 and not as 1 to 9, as 1 was not regarded as a number.

puhatta or *pṛthaktva* is basically a place separator. It tells about shift of place or value of the number. For example, *pṛthaktva* means 2-9 which is a range of the numbers having unit place value or the number with one digit. The meaning of *dasapuhatta* is interpreted as 10-90 which need to be revised. However from etymological considerations it should rightly be: *dasapuhatta* should be any number from 10 to 99 (= 100–1). It holds the place value of 'tens' (10) i.e. numbers with two digits. Because if we consider its range 10-90, then the numbers from 91-99 will be left. Similarly *sayapuhatta* should be any number from 100 to 999 (= 1000 – 1), which the range of the numbers of having place value of 'hundreds' i.e. numbers with three digits; *sahassapuhatta*, should be any number from 1000 to 9999 (= 10000 – 1) i.e. numbers with four digits; *koḍipuhatta*, any number from 10^7 to $10^8 – 1$ i.e. numbers with eight digits; *sāgarapuhatta* any number between 1 *sāgara* and 10 *sāgara*–1. This concept of *puhatta* (that *puhatta* separates numbers according to places of multiples of tens) is found mentioned everywhere in *Āgamic* and karma literature.

It is widely used to enumerate the range of time, counting, life-span and linear dimension of various objects living and non-living. BS (21.1.7)[1] enumerates that the duration of rice, wheat and the like in several consecutive lives as the same is maximum *pṛthaktva* (2-9) years and there maximum height (length) can be *pṛthaktva dhanuṣa*[2] (*sūtra* 21.1.3). Answering the question how long a female can remain female in several births[3], it is said here (PS, *sūtra* 18.1327) that there are five options from various perspectives. In each option the cumulative life-span may be *pūrvakoṭi pṛthaktva* along with various length of *palyopama.* Maximum life of deities of *sarvārthasiddhi* is *dohiṁ vāsapuhattehiṁ* and 33 *sāgaropama*[4] (BS, 24.24.358). The maximum number of bound *āhāraka śarīra* is *ukkoseṇaṁ sahassapuhattaṁ*[5] (PS, 12.910.3) i.e. between 1000 and 9000 (9999).

All the later commentators, scholars and *Ācārya*-s defined *dasa-puhatta* (ten-separator) as a number between 10 and 90, *sayapuhatta* (hundred-separator) as a number between 100 and 900, *sahassapuhatta* (thousand-separator) as 1000 and 9000, *koḍipuhatta* (10^7-separator) as a number between 1×10^7 and 9×10^7, *sāgarapuhatta* as a number between 1 *sāgara* to 9 *sāgara*-s. *sayasāgarapuhatta* , a number between 200-900 *sāgaropama*.

3.18 Conclusion

The decimal system which we use to write numbers is based on 10, probably because we have ten fingers and thumbs –digits. Other bases are also possible as we have seen in case of ancient cultures. There is a gradual development of the number system what we use today. The ancient pictorial numerals, the word numerals and base 20 and base 60 systems were not meeting the need of the developing cultures. Another problem was of place value system. In many of the numeral system place value was not clearly assigned, which again would create ambiguity in understanding the numbers. The necessity of the era was fulfilled by the genius of India. Zero the incomparable gift to the world of mathematics is most probably given by Jaina saint scholars as per the evidence available till now.

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[1] *Bhagavaī* (*Āṅgasuttāṇi*), Vol. II, p. 836.

[2] *Ibid*, p. 835.

[3] *Prajñāpaṇā sūtra,* Vol. II, p. 344.

[4] *Bhagavaī* (*Āṅgasuttāṇi*), p. 900.

[5] *Prajñāpaṇā sūtra,* Vol. II, p. 98.

Chapter-4

Number Theory (2): Transfinite Numbers

Mathematicians are not satisfied because they know there are no solutions up to four million or four billion. They really want to know that there are no solutions up to infinity.

~ Andrew Wiles ~

Infinity is a floorless room without walls or ceiling.

~ Author Unknown ~

Any object and entity having finite magnitude in the world can be subject of measure or counting. Incredible human brain may not be able to count the things up to extremely large level, but can comprehend them or tame them as has been discussed in the last chapter. Now the question is- what is beyond the finite world? Does the world of infinite really exist? Is it possible to count the infinite things? Is it an abstract concept? Is there any way to figure out transfinite and infinites? In the light of Jaina *Āgama* the number beyond countable will be discussed in this chapter. For, Jaina's fascination about numbers does not stop at *śīrṣaprahelikā*.

Gauss Carl Friedrich (1777-1855) says, in ordinary conversation, infinite means something that is very great in comparison with everyday things. In mathematics, however, infinity is not a number but a concept of increase beyond bound.[1] **Infinity** is an abstract concept describing something *without any bound* or *larger than any number*.

Although the concept of infinity is difficult to grasp, we may simply define it as 'not finite', where finite means something that – at least in theory – is completely determinable by counting or measurement.

[1] Gullberg, Jan, *Mathematics* : *From the birth of Number* ,W.W. Norton & Company, New York, 1997, p. 29.

4.1 Analogy measures (*upamā māna*)

In Jaina canons enumeration goes up to *śīrṣaprakelikā* = $(84^{28}) \times (10^{140})$, a number with 194 digits. In ADS and BS there is a significant remark that counting (*gaṇita*) ends here and further enumeration can be understood only through some sort of simile-measures (*uvamiye*), i.e. these numbers can be comprehended by similes only.

Time unit which is measured or illustrated through simile is known as *aupamika kāla.* When it was said that beyond *śīrṣaprakelikā* measures are not possible then Gautama *gaṇadhara* asked, '*se kim taṁ ovamiye?*' The question is, if we cannot count the number then what is the simile (*upamā,* analogy) measure? Here we enter the phase of analogy. The answer given in *Bhagawaī sūtra* 6.7.133 is *uvamiye duvihe paṇṇatte, taṁ jahā paliovame ya, sāgarovame ya,*[1] replies Bhagawāna Mahāvīra. The answer given is, it is twofold - *palyopama* and *sāgaropama.*

These two terms are unique but very significant to understand Jaina philosophy. Life span of deities and hellish beings and the duration of karma are measured through this scale. BS, ADS, JDP, TP and JS define *palyopama* with an illustration based on the principle of one-to-one correspondence.

4.1.1 Understanding *palyopama*

The word *palyopama* is derived from the word '*palya'. palya* is a round shaped bowl or any container, which was being used in ancient time to measure the quantity of grain. The time period explained using analogy of *palya* is known as *palyopama.* In this simile (analogy) *palya* stands for a pit. Dimension of the pit is one *yojana* in width, length and height. In other words it's a right circular cylindrical pit with the diameter and depth of 1 *yojana* each. Furthermore, it is said to be stuffed fully and compactly with the hair-tips (*bālāgra*) of a new born infant of one to seven days old of *Devakuru* or *Uttarakuru*, sub-continents of the continent *Mahāvideha.* Here it says about number of hair-tips which is *bālagga koḍīṇaṁ* i.e. crores of hair-tips (1 crore = 10^7). The pit is crammed so compactly that hair-tips cannot be taken out or destroyed by any means like fire, air etc. After every one hundred years one hair-tip is to be taken out till the pit is absolutely exhausted. The length of the time taken for exhausting the pit in this manner is known as *palyopama.*

[1] *Bhagavaī*, Vol. II, p. 288.

4.1.2 Why the infants of *Devakuru* or *Uttarakuru*?

In the table (4.3) we can see that the length of the hair of humans of all the regions vary and is said that hair of *Devakuru* or *Uttarakuru* are much finer and soft than others. *Bālāgra* of infant of *Devakuru* and *Uttarkuru* are 8^4 times smaller than that of *pūrva* or *apara videha.* Perhaps the *palya* would be better stuffed with those hair tips. Probably for this reason the authors have preferred the hair-tips of *Devakuru* or *Uttarakuru.*

Ācārya Mahāprajña in his commentary of ADS (*sūtra* 10.424) writes about the size of hair-tips from two perspectives[1] :

1. A hair-tip is innumerable-th part of or innumerable time smaller than a visible object by a human eye.

2. From spatial viewpoint a hair-tip occupies the space innumerable times less than the body of fungus (*panak*). i.e. a hair-tip is innumerable times smaller than a fungus.

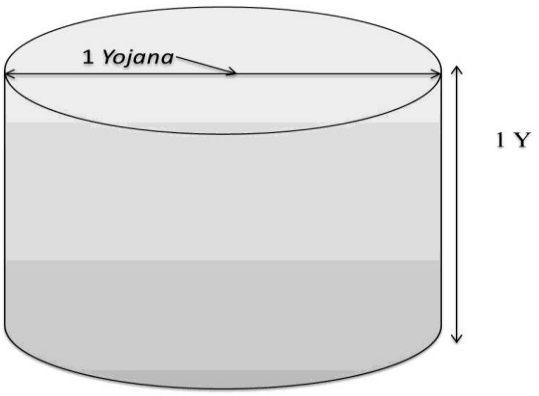

Fig 4.1: Palya or pit

In *cūrṇi* of ADS it is mentioned as equivalent to the size of gross earth-bodied living being.

4.2 Height and Distance Scale

To calculate this value of *palyopama* let's first calculate *yojana* in terms of hair-tips. The analogical time starts with *paramāṇu.*

[1] *Anuogadārāiṁ*, p. 281.

The length scale given in BS(6.7.134),[1] and JDP(2.25)[2] is as follows:

Infinite *paramāṇu* = 1 *utślakṣaṇaślakśaṇika*

8 *utślakṣaṇaślakśaṇika*= 1 *ślakṣaṇaślakśaṇika*

8 *ślakṣaṇaślakśaṇika* = 1 *ūrdhvareṇu*

8 *ūrdhvareṇu* = 1 *trasareṇu*

8 *trasareṇu* = 1 *rathareṇu*

8 *rathareṇu* =1 *devakuru-uttrakuru manuṣya bālāgra*

8 *devakuru-uttrakuru manuṣya bālāgra*=1 *harivarṣa-ramyakvarṣa manuṣya bālāgra*

8 *harivarṣa-ramyakvarṣa manuṣya bālāgra*=1 *haimavata-hairaṇyavata manuṣya bālāgra*

8 *haimavata-hairaṇyavata manuṣya bālāgra* = 1 *pūrva videha-apara videha manuṣya bālāgra*

8 *pūrva videha-apara videha manuṣya bālāgra* = 1 *likṣā*

8 *likṣā* = 1 *yukā*

8 *yukā* = 1 *yavamadhya*

8 *yavamadhya* = 1 *aṅgula*

6 *aṅgula* =1 *pāda*

12 *aṅgula* = 1 *vitasti*

24 *aṅgula* = 1 *ratni*

48 *aṅgula* = 1 *kukṣi*

96 *aṅgula* = 1 *daṇḍa (dhanuṣa, yuga, nālikā, akṣa* or *musal)*

2000 *daṇḍa* = 1 *gavyūta* (*kośa*)

4 *gavyūta* = 1 *yojana*

[1] *Bhagavaī*, Vol. II, pp. 288-289.

[2] *Jambūdvīpaprajñapti Sūtra*, p. 29.

From above scale we find that

1 *yojana* = 8000 *dhanuṣa*

1 *dhanuṣa* = 96 *aṅgula* (*a̤*) (here *aṅgula* is *utsedha aṅgula*)

1 *yojana* = 96 × 8000 = 768000 *a̤*

i. Let's calculate the **number of hair-tips** (as per **Śvetāmbara** tradition) taking the value from aforementioned measure-scale:

1 *a̤* = 8^7 *Devakuru bālāgra*

1 *yojana* = 768000 *a̤*

= 768000 × 8^7 *bālāgra*.

Let's say, Volume of *palya* is V_p, which is equivalent to number of *bālāgra* stuffed in the pit.

$V_p = \pi r^2 h$ cubic *yojana*; (r = radius; h = height of the pit)

where, $\pi = 19/6$; r = ½ *yojana*; h =1 *yojana;*

$V_p = \frac{19}{6} \times (1/2)^2 \times 1$ cubic *yojana*

$= 19/24 \times (768000)^3 \times (8^7)^3$ cubic *bālāgra*

$= 19/24 \times (768)^3 \times (8^{21}) \times 10^9$

$= \frac{79,382,905,019,175,010,127,906,340,864 \times 10^9}{24}$

= 3,307,621,042,465,625,421,996,097,536,000,000,000 (figure actually calculated by Malayagiri of 11th C.A.D.)

$\approx 3.31 \times 10^{27} \times 10^9$

$\approx 3.31 \times 10^{36}$ cubic *bālāgra*

Using logarithm,

$= 10^{36} \times 10^{\log 3.31} = 10^{36} \times 10^{.5198} = 10^{36.5198}$ cubic *bālāgra*

The corresponding **Digambara** value will come to (by taking $\frac{19}{24}$ cubic *pramāṇa yojana* as volume and 1 *pramāṇa yojana* =500 *y*)

$\frac{19}{24} \times (768)^3 \times 8^{21} \times (500)^3 \times 10^9$

= 413 452 630 308 203 177 749 512 192 000 000 000 000 000 000

$\approx 4.13 \times 10^{44}$

$\approx 10^{\log(4.13)} \times 10^{44} \approx 10^{.6} \times 10^{44}$

$\approx 10^{44.6}$ cubic *bālāgra*

ii. Estimating *palyopama* and *sāgaropama*

Now it's clear that the *palya* is stuffed with $10^{36.51}$ cubic *bālāgra.* According to the illustration one hair-tip is extracted after every hundred years. Then the time taken in emptying the *palya* is *palyaopama.*

1 *palyopama* = $10^{36.51} \times 100$

= $10^{38.51}$ years (time spent to take out all the hair tips according to Śvetāmbara tradition)

Probably the capacity for such larger calculation was developed earlier, which is revealed by works like *Lokavibhāga* and *Bṛhatkṣetra samāsa* and *Tiloyapaṇṇatti*.

The value of π was assumed as 19/6 instead of √10. The correct value of π is about 3.1415. By assuming this value the number of hair-tips in the pit is $\approx 3.28 \times 10^{36}$. Then

palyopama = $3.28 \times 10^{36} \times 100 = 10^{\log 3.28} \times 10^{38}$

= $10^{.5158} \times 10^{38} = 10^{38.5158}$ which is almost equal to the value calculated by Malayagiri

i.e. $10^{38.51}$ years.

Calculating *sāgaropama:* Assuming aforementioned value [section 4.3.(ii)], $10^{38.51}$ of *palyopama,*

sāgaropama = 10^{15} *palyopama*

= $10^{15} \times 10^{38.51}$

= $10^{53.51}$years

iii. The number of oceans and islands (say N_0) in *Madhya loka* (middle cosmos)

Islands and oceans exist only in middle cosmos. The question is how many islands and oceans are there in *loka* (cosmos)? In BS, JDP and PS there is no mention of any quantity of islands and oceans. But ADS (10.426) gives this value as, *jāvaiyā ṇaṁ aḍḍhāijjāṇaṁ uddhārasāgarovamāṇaṁ uddhārasamayā evaiyā ṇaṁ diva samuddā udhhāreṇaaṁ paṇṇattā.*[1]

JS (3.124) answers as,

jāvaio uddhāro aḍḍhāijjāṇa sāgarāṇa bhave

tāvaiyā khalu loe havaṅti dīva samuddā ya

i.e. The number of islands and oceans in the *loka* is same as the number of *samaya* required for exhaustion of two and a half *sāgaropama*.

The total number of oceans and islands is equivalent to 2.5 *sāgaropama*

$= 2.5\times10^{15}$ *palyopama*

$= 2.5\times10^{15}\times 10^{38.51}$

$=2.5\times10^{53.51} = 10^{53.51}\times10^{\log 2.5}$

$\approx 10^{53.51}\times10^{.39}$

$\approx 10^{53.90} \approx 10^{54}$

iv. Diameter of the last ocean, say $D_{N0} = 2^{N_0+1} - 3$ (in the unit of 1 *lakh yojana*)

There are *asaṅkhyāta* islands and oceans in middle-cosmos (*madhya loka*) and the first one is *Jambūdvīpa* (JD) and the last ocean N_0^{th} is *Swayambhūramaṇa* with diameter *d*. The width (W_1) of the first island *Jambūdvīpa* is 1 *lākh yojana* (ϒ) and the width of succeeding ocean or island gets doubled of the preceding one. The width of *Lavaṇasamudra* will be 2ϒ and of next *Dhātakīkhanḍa* will be 4ϒ.

$W_1 = 1ϒ$ (ϒ= 10^5 *yojana*)

$W_2 = 2^{2-1} = 2ϒ$

$W_3 = 2^{3-1} = 4ϒ$

$W_4 = 2^3 = 8ϒ$

$W_{N_0} = 2^{N_0-1}$ ϒ (The width of the last ocean)

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[1] *Anuogadārāiṁ*, p. 260.

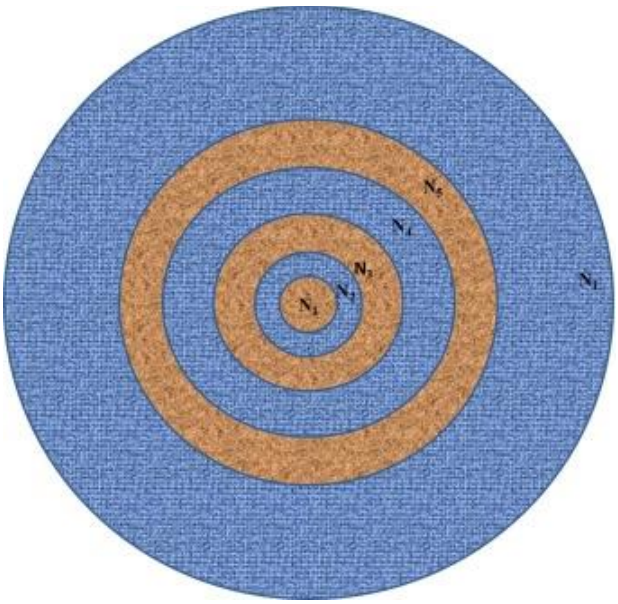

Fig 4.2: Islands and Oceans

For, the subsequent island and oceans are in annular shape. The diameter of the subsequent island or ocean will be calculated as,

Diameter of 1st island $D_1 = 1\ \Upsilon$

External diameter of 2nd (next) ocean $D_2 = 1+2.2 = 5\Upsilon = 2^3 - 3$

External diameter of 3rd island $D_3 = 1+2.2+2.4 = 13\Upsilon = 2^4 - 3$

External diameter of 4th ocean $D_4 = 1+2.2+2.4+2.8 = 29\Upsilon = 2^5 - 3$

External diameter of N_0th island or ocean $D_{N_0} = 1+2.2+2.4+2.8+2.16+\ldots\ldots..2.2^{N_0-1}$

$= 2^{N_0+1} - 3\ \Upsilon$ (diameter of *Swayambhūramaṇa*)

In view of enormity of N_0, we can approximate the above equation as (by ignoring exponent 1 and the number 3)

$D_{N_0} = 2.2^{N_0}$

$= 2.2^{10^{54}}$ {from the section (4.2.(iii)) total number of island and oceans is $2.2^{10^{68}}$}

$= 2.10^{10^{54}\times\log 2} = 2.10^{10^{54}\times.3010}$

$= 2.10^{53\times 10\times.3010} = 2.10^{10^{53}\times 3} \approx 2.10^{10^{53}\times 10^{log 3}}$

$= 2.10^{10^{53}\times 10^{.47}} = 2.10^{10^{53.47}}\ \Upsilon$

$= 2.10^{10^{53.47}}\times 10^5$ *yojana*

$\approx 10^{\log 2}\ 10^{10^{53.47}} \times 10^5$

$\approx\ 10^{10^{53.47}} \times 10^{5+.30103}$

$\approx\ 10^{10^{53.47}} \times 10^{5.3}$

$\approx\ 10^{(10^{53.47}+5.3)}$

$\approx\ 10^{10^{53.47}}$ *yojana*

Since, 1 *rajju* (ṛ) = D_{N_0} (the width of middle cosmos is 1 *rajju* and the middle cosmos is occupied by island and oceans and the last one is *Svayambhūramaṇa*. In other words, it can be said that *Svayambhūramaṇa* is the boundary of the middle cosmos. Thereafter starts *aloka*) :

1 *rajju* (ṛ) = $10^{10^{53.47}}$ *yojana* (as per Śvetāmbara tradition)

4.3 Estimating Time-cycle (*kāla cakra)*

Jaina-s believe that time is infinite, without any beginning or end. Time is divided into infinite equal time cycles (*kāla cakra*-s). Each time cycle is further sub-divided in two equal halves. The first half is the progressive cycle or ascending order, called *ussappiṇi* (*utsarpiṇī*). The other half is the regressive cycle or the descending order, called *ossappiṇi* (*avasarpiṇī*). Every *ussappiṇi* and *ossappiṇi* is divided into six unequal periods called *ārā*-s (*ārā* literally means a spoke; thus, the 6 subdivisions are the spokes of wheel of time). During the *ussappiṇi* half cycle, progress, development, environmental situations, happiness, strength, age, body, religious trends, etc., go from the worst conditions to the best. During the *ossappiṇi* half cycle, progress, development, environmental situations, happiness, strength, age, body, religious trends, etc. shift from the best conditions to the worst. Presently, we are in the fifth *ārā* of the *ossappiṇi* phase. When the *ossappiṇi* phase ends, the *ussappiṇi* phase begins. This *kālacakra* keeps repeating and continues forever.

ADS, *sūtra* 219 describes the total time scale beginning with smallest (ultimate) unit of time *samaya* and going up as *āvalī*, *prāṇa*, ... , *muhutta*, day, month, year, 10 years, 10^2 years, ... , 10^5 years, 84×10^5 years (*puvvaṅge*), $(84 \times 10^5)^2$ years (*puvva*), ... , $(84 \times 10^5)^{28}$ years (*sīsapaheliyā*), ... , *paliovame* (pit-measure), *sāgarovame* (sea-measure), *ossappiṇi* (ascending-cycle epoch), *ussappiṇi* (descending-cycle epoch), *poggalapariyaṭṭe* [(*pudgalaparāvartana* or matter-cycle epoch) (*pp*)], *tītaddhā* (past period), *aṇāgataddhā* (future period), and *savvaddhā* (all-time). *sūtra* 616 (ADS) states about infinity of time by

mentioning the existence of aforementioned time scale in all-time which includes past period and future period, these (past and future) include matter-cycle epochs, matter-cycle epochs in turn include ascending and descending cycle epochs. They (ascending and descending cycle epochs) include *sāgarovama*-s and *sāgarovama*-s include *paliovama*-s and so on and finally this chain goes up to *samaya*. In BS and JDP *poggalapariyaṭṭe* and beyond do not feature.

BS (6.7.134) and JDP (2.25) describe time-span of both *ossappiṇi* and *ussappiṇi* in detail with six epochs of each. 6 epochs of *ossappiṇi* (*avasarpiṇī*) are:

1. *susama-susamā* - 4×10^{14} *sāgarovama*-s
2. *susamā* - 3×10^{14} *sāgarovama*-s
3. *susama-dusamā* - 2×10^{14} *sāgarovama*-s
4. *dusama-susamā* -1×10^{14} *sāgarovama*-s less 42,000 years
5. *dusamā* -21,000 years
6. *dusama –dusamā*- 21,000 years

In *ossappiṇi* the epochs get reversed. In total time measure of two cycles become 20 *koḍākoḍī sāgaropama*-s. Today we are supposed to be in the fifth epoch (*ārā*) of current *ossappiṇi* where the epoch is supposed to have started after three and a half years of the emancipation of Bhagawāna Mahāvīra.

Although these terms are used frequently but ADS neither defines nor explains the terms *ossappiṇi*, *ussappiṇi*, *poggalapariyaṭṭe*, etc. Perhaps at the time of composition of ADS, these terms were quite prevalent and widely known. Therefore, author did not feel requirement of explaining these terms. JDP talks in detail about *ossappiṇi* and *ussappiṇi*.

Accordingly, 10^{15} *sukṣma-addhā-sāgarovama* (subtle temporal-sea-measure) = 1 *ossappiṇi* (ascending cycle epoch) = 1 *ussappiṇi* (descending cycle-epoch); 1 *ossappiṇi* + 1 *ussappiṇi* = *kalpa* (full-cycle epoch) = 2×10^{15} *sukṣma-addhā- sāgaropama*-s.

ADS does not give any details about *poggalapariyaṭṭe* although it is referred frequently in it as well as in previous *āgamic* literature (including SK of Digambara tradition).

$$\textit{ussappiṇi} = \textit{ossappiṇi} = 10\ \textit{koḍākoḍī sāgaropama}$$

$$= 10^{53.51}\times10^{15} = 10^{68.51}\text{ years} \approx 10^{68}\text{ years}$$

one *kāla cakra* (*ussappiṇi* + *ossappiṇi*)= 2×10^{68} years

$= 10^{68.3}$ years

Even these figures are unimaginably high.

We have calculated the value of *palyopama* and time-cycle as per the treatment of BS and JDP, which is $10^{38.51}$ years [section 4.3. (ii)]. Probably, subsequent *Ācārya*-s found out that this figure falls too short of *śīrṣaprahelikā* [$(84)^{28}\times10^{140}$]. *śīrṣaprahelikā* they found went up to the order of 10^{193} whereas *palyopama* was $10^{38.51}$ years, which is much less than *śīrṣaprahelikā* . *palyopama* should be much greater than *śīrṣaprahelikā.* Therefore, probably they had to device other *palyopama* measures.

Theoretically it is said in *Āgama* that *śīrṣaprahelikā* is the last numerable number and thereafter analogy measures start which cannot be counted. But practically the calculated value of *palyopama* is smaller. To find out the way perhaps, they came up with new concept of pit-filling, which is found in ADS and Karmagrantha, JS, JK etc. in the form of classification of *palyopama*.

4.4 Classification of *palyopama*

In BS[1] (first revised in 500-400 BC, in first council) and JDP (*sūtra* 2.25) [300 BC, but first revised in 300 AD, second council] there is no any classification of *palyopama*, whereas in ADS *palyopama* is classified in many categories. This clearly shows that JDP was composed few centuries before ADS, and even at the time of 3rd conference at Vallabhī, this updating was not done in BS and JDP but of course other older canonical works like TS, etc. are updated.

Therefore, ADS, 10.419-440 explains 3 types of *palyopama* (and *sāgaropama*), viz. *uddhāra, addhā* and *khetta,* and 2 subdivisions of each of them as *vyavahāra* (practical) and *sukṣma* (subtle). They have mentioned that there is no direct application of *vyavahāra palyopama.* It is introduced just to calculate the subsequent measures. The classification goes as:

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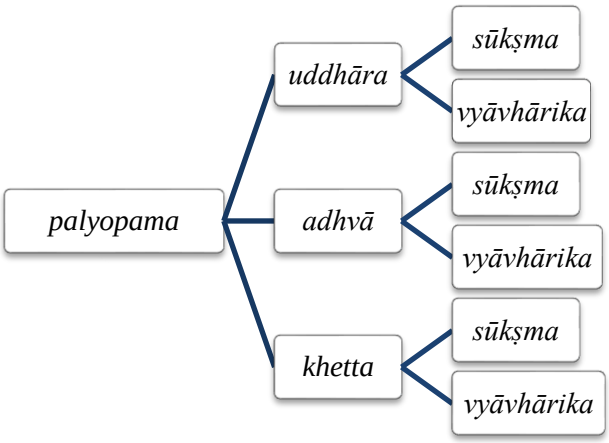
[1] *… tao ṇaṁ vāsasae- vāsasae gate egamegeaṁ vālaggaṁ avahāya jāvatiyeṇaṁ kāleṇaṁ se palle khīṇe nirae nimmale niṭṭhie nilleve avahaḍe visuddhe bhavai. se ttaṁ paliovame.*

–Bhagavaī, Vol. II, p. 289, *sutra,* 6.7.134.



Types of Palyopama

Let's first understand the three words *uddhāra, addhā* and *khetta* according to ADS. Out of them first two classifications are based on number of time taken in emptying the pit and the last one from spatial occupancy.

i) *vyāvhārika uddhāra palyopama*: Here the pit is stuffed with hair-tips without dividing it further and if one hair-tip is removed per *samaya* (the smallest unit of time), then time elapsed in emptying the pit is known as *vyāvhārika uddhāra palyopama* (say P).

ii) *sūkṣma uddhāra palyopama*: If these hair-tips are further divided in *asaṅkhyāta* (innumerable) pieces, say A, and each such part is removed per *samaya* then the time required to empty the pit is called "*sukśma uddhāra palyopama*" = P × A.

iii) *vyāvhārika addhā palyopama:* If one hair tip is removed after every 100 years, then time required to empty the pit will be known as *vyāvhārika addhā palyopama.*

iv) *sūkṣma addhā palyopama:* If hair-tips are further divided into *asaṅkhyāta* pieces and then one piece is removed after every hundred years then the time passed in making emptying the pit is *sūkṣma addhā palyopama.*

v) *vyāvhārika khetta palyopama:* The space points occupying the pit will be far larger than the number of hair-tips. Each hair-tip will be occupied by a large number of space points. If each space point is removed per *samaya* then time required to exhaust the pit is known as *vyāvhārika khetta palyopama.*

vi) *sūkṣma khetta palyopama:* If hair-tips are further divided into *asaṅkhyāta* pieces then the pit will be exhausted at the rate of 1 space point per 100 years. The time required to exhaust the space points in the pit is known as *sūkṣma khetta palyopama.*

The author of ADS says that there is no purpose or use of *vyāvhārika-uddhāra-addhā-khetta palyopama* (*sūtra* 423, 430, 437) they are just introduced to construct further units. If we analyze the definition then it seems that in fact, ADS has expanded the idea of *palyopama*. In BS the hair-tips are not divided but are removed after every 100 years. Since, this figure falls very much short of *śīrṣaprahelikā* therefore in ADS and other post-*āgamic* works, the hair-tips are divided into *asaṅkhyāta* parts and exhausted at the rate of one part per 100 years to prolong the time-measure of *palyopama*.

We have already discussed above that, if these hair-tips are divided in *asaṅkhyāta* (innumerable) parts, say A, and each part is removed per *samaya* then the time required to exhaust the pit is called *sukśma uddhāra palyopama* = P × A. As we shall see later, *asaṅkhyāta* is a variable quantity in Jaina mathematics and when it is not specified we may take it as the least of *asaṅkhyāta* called *jaghanya paritāsaṅkhyāta*, which we will denote by A hereafter.

If *vyavahāra/sukśma uddhāra palyopama* is multiplied by *samaya*-s in 100 *saṁvatsara*-s (years of 360 days each), then we get corresponding *vyavahāra/sukśma addhā palyopama*. Now for defining *samaya*, ADS gives the following scale:

asaṅkhyāta samaya-s = 1 *āvalīkā*

saṅkhyāta āvalīkā = 1 inspiration (or expiration)

3773 inspiration (expiration) = 1 *muhūrta*

30 *muhūrta*-s = 1 *divasa* (day)

360 days = 1 *saṁvatsara* (year).

Here, however, *asaṅkhyāta* is not the minimum one (*jaghanya paritāsaṅkhyāta*) mentioned above. In fact, ADS and BS specify it as *asaṅkhyāta* of higher order called *jaghanya yuktāsaṅkhyāta* which is equal to A^A. Also *saṅkhyāta* (S) in the above table is any number between 2 and A-1 (both inclusive).

Thus, number of *samaya*-s in 100 years will be :

H =100 × 360 × 30 × 3773 ×S × A^A where S is the variable *saṅkhyāta* defined above.

If the above *vyavahāra/sukśma uddhāra/addhā palyopama* is multiplied by *dasa koḍākoḍi* (=$10 \times 10^7 \times 10^7 = 10^{15}$), we get respective *vyavahāra/sukśma uddhāra/addhāsāgaropama*. All the above can be summarized as per table below[1] :

a. *vyavahāra uddhāra-palyopama* = P

b. *sukśma uddhāra-palyopama* = P × A

c. *vyavahāra addhā-palyopama* = P × H

d. *sukśma addhā-palyopama* = P × A × H

e. *vyavahāra uddhāra-sāgaropama* = $P \times 10^{15}$

f. *sukśma uddhāra-sāgaropama* = $P \times A \times 10^{15}$

g. *vyavahāra addhā-sāgaropama* = $P \times H \times 10^{15}$

h. *sukśma addhā-palyopama* = $P \times A \times H \times 10^{15}$.

4.5 Evaluating various *palyopama*

4.5.1 *Vyavahāra uddhāra- palyopama* (*vup*) (practical exhaustion-pit-measure)

The spatial measures are given in *sūtra* 399 (ADS). According to it, 8^7 (= 2,097,152) hair-tips of *devakuru* or *uttarkuru* human beings = 1 *utsedha aṅgula* (measure-finger), 768,000 *utsedha aṅgula* = 1 *utsedha yojana* (as per scale described in section 4.3). To determine *vyavahāra-uddhāra palyopama*, we have to construct a pit of the shape of a right-circular cylinder with diameter and height 1 *yojana* each and fill it compactly with hair-tips described above.

The volume of this pit, say *p*, is then = $\pi \times (1/2)^2 \times 1$ cubic *yojana*.

We have already described in section 4.3.(i) that Malayagiri (12^{th} A.D.) has calculated the number of hair-tips in a *palya* of diameter and depth of 1 *yojana* as $10^{36.52}$ hair-tips.

If each hair-tip is removed per *samaya* the time required to empty the pit is called *vyavahāra-uddhāra- palyopama* (practical exhaustion-pit-measure) and is equal to $10^{36.52}$ *samaya*. Malayagiri rightly remarks that this time span is far less than even required for a blink. This is because innumerable (*asaṅkhejja*) *samaya*-s make one *prāṇa* (respiration = $\frac{1}{3773}$ *muhūtta* = $\frac{1}{3773 \times 30}$ day = $\frac{24 \times 60 \times 60}{3773 \times 30}$ seconds ≈ 0.76 sec.) and *asaṅkhejja* is so

[1] Shah, R.S., *Lure (Lore) of Large Numbers,* Arhat Vacana, Vol. 21, No.4, Oct.-Dec., 2009, pp. 61-78.

enormous that even *vyavahāra uddhāra-palyopama* (*vup*) becomes infinitesimal. It is for this reason, ADS states this measure is only theoretical having no practical value, although ironically it is named as practical pit-measure. 10^{15} *vups* make 1 *vyavahāra uddhārasāgarovama* (*vus*). Even this *vus* is infinitesimal in view of enormity of *asaṅkhejja*.

In Śvetāmbara tradition, 1 *pramāṇa-aṅgula* = 400 *utsedha -aṅgula* and hence measure of *vup* $\approx (400)^3 \times 10^{36.52} \approx 2.12 \times 10^{44} \approx 10^{44.3}$

In Digambara tradition the pit has the diameter and height of 1 *pamāṇa-joyaṇa* (and not *utsedha-joyaṇa* as assumed by Malayagiri). According to Digambara tradition,

1 *pramāṇa-aṅgula* = 500 *utsedha -aṅgula* and

Hence the pit will contain $(500)^3 \times 10^{36.52} \approx 4.13 \times 10^{44} \approx 10^{44.616}$. (The actual value derived by Yati Vṛṣabha (6^{th} C. A.D.) is

413,452,630,308,203,177,749,512,192 x $10^{18} \approx 10^{44.616}$ (TP, Vol. I, p. 15).

However, these quantities ($10^{36.52}$ or $10^{44.3}$ or $10^{44.616}$) as *samaya*-s for *vup* are insignificant, again far less than time required for a wink. So also is the corresponding *vyavahāra-uddhāra-sāgarovama* (practical-linear-pit-measure ($10^{51.52}$ or $10^{59.3}$ or $10^{59.616}$).

4.5.2 *sukṣma-uddhāra- palyopama* (*sup*) (subtle-linear pit-measure)

This measure, derived from *vup* above, is used for determining the number of island (continents) and seas in the mid-cosmos. According to Jaina cosmography, our earth (*Jambudvīpa*) is at the center of the mid-cosmos and shaped like a circular disc with diameter of 100000 *joyaṇa*-s, surrounded alternately by innumerable annular seas and island-continents, and the diameter of the succeeding island or ocean becomes double of the preceding one. The number of these islands and seas is stated to be number of *samaya*-s in 25 *koḍākoḍī sukṣma-uddhāra- palyopama* (i.e. 25×10^{14} *sup* = $10^{15.4}$ *sup*).

The measure *sup* is obtained in following manner. Each of the hair-tips in the pit described in the section 4.3.(i) is to be divided in innumerable (say $Å_1$) parts. If each such part is picked up per *samaya*, then the time elapsed in exhaustion of the pit is called *sukṣma-uddhāra- palyopama* (ADS *sūtra* 10. 424).

Thus, *sup* = *vup* × $Å_1$ = $10^{36.52}$ × $Å_1$ *samaya*-s.

However, from the temporal scale mentioned in *sūtra* 417,

1 year = 12 × 30 × 30 × 3773 × $\mathring{A}_2$ *samaya*-s, where $\mathring{A}_2$ is also an innumerable quantity.

Hence, *sup* = ($10^{36.52}$ × $\mathring{A}_1$) / (12 × 30 ×30 × 3773 × $\mathring{A}_2$) ≈ 10^{29} × ($\mathring{A}_1$ / $\mathring{A}_2$) years. Malayagiri asserts that this is an enumerable quantity equal to enumerable *koḍī* (= 10^7) years. This means $\mathring{A}_1$ / $\mathring{A}_2$ is an enumerable quantity. If we take it as 1, then *sup* as temporal measure is an enumerable quantity in units other than *samaya*. 10^{15} times *sup* equals *sukṣma-uddhāra-sāgaropama* or *sus* (subtle linear sea-measure) ≈ 10^{44} years (if $\mathring{A}_1$ = $\mathring{A}_2$).

This measure, however, is not used as a temporal measure but is used to determine the number of island-continents and seas in the middle world. As stated earlier this number is equal to *samaya*-s in 25 *koḍākoḍī sup* = 2.5 *sus* = 25 × 10^{14} × $10^{36.52}$ × $\mathring{A}_1$ ≈ 10^{52} × $\mathring{A}_1$ which is also an innumerable quantity.

4.5.3 *vyavahāra-addhā-palyopama* (*vap*) (practical temporal-pit-measure)

If each of hair-tip in the pit described above is picked up every 100 years, then the time elapsed in exhausting the pit is *vyavahāra-addhā-palyopama*. Thus, *vap* = 100 × *vup* years = $10^{38.52}$ years. 10^{15} times this measure (*vap*) is *vyavahāra-addhā-sāgarovama* (*vas*). Thus, *vas* = 10^{15} × $10^{38.52}$ years = $10^{53.52}$ years. These measures are of theoretical value and serve no purpose. However, they serve as base for subsequent measures. (ADS, *sūtra* 10.428).

4.5.4 *sukṣma-addhā-palyopama* (*sap*) (subtle temporal-pit-measure)

If each hair-tip in the pit is divided mentally in innumerable parts, and each piece is picked up after every 100 years then the time elapsed in exhausting the pit is called *sukṣma-addhā-palyopama* (*sap*). It is clear that,

sap = 100 × *vap* years = 100 × $10^{36.52}$ × $\mathring{A}_1$= $10^{38.52}$ × $\mathring{A}_1$ years.

10^{15} times *sap* is *sukṣma-addhā-sāgaropama*, i.e. is equal to $10^{53.52}$ × $\mathring{A}_1$, which again is an innumerable quantity. The measures *sap* and *sas* are employed in expressing life spans of living beings and period of bondage of karmas [ADS, *sūtra* 431].

4.5.5 *vyavahāra-khetta-palyopama* (*vkp*) (practical space-pit-measure)

Each hair-tip in the pit is in contact with innumerable space-points. If every such space point is exhausted per *samaya*, then the time required to exhaust all those space-points is

called *vyavahāra-khetta-palyopama* (*vkp*). Assume that each hair-tip is in contact with $Å_3$ space-points, where $Å_3$ is an innumerable number. Then $vkp=10^{36.52} \times Å_3$. In fact, this is the number of space-points contained in the pit. ADS does not specify this. However, in a contemporary work *Jīvasamāsa* of unknown author, it is stated that this period is equivalent to innumerable *ussappiṇi* -s (ascending cycle epochs) and *ossappiṇi* -s (descending cycle epochs). [JS, 131] This is explained by Jinabhadragaṇī Kṣamāśramaṇa (6th C. A.D.) by quoting an ancient verse (*gāthā*) in his *Viśeṣāvaśyakabhāṣya* thus: "time is subtle (fine), but space is finer. The space-points contained in a length of one *aṅgula* are equivalent to time-points (*samaya*-s) in innumerable ascending and descending cycle epochs."

suhumo ya hoti kālo tattoo suhumatarayaṁ havai khettaṁ

aṅgulaseḍhīmette osappiṇīo asaṅkhejjā. [VB, 617]

An ascending cycle epoch = descending cycle epoch = *dasakoḍākoḍī* (*sukṣma-addha*) *sāgarovama* $= 10^{15} \times 10^{53.52} \times Å_1 = 10^{68.52} \times Å_1$, which is an innumerable number.

Thus, $vkp = 10^{36.52} \times Å_3$ *samaya* (Here $Å_3$ is *yuktāsaṅkhyāta samaya*, ADS, 10.590), which is again an innumerable number. [ADS, *sūtra* 436] Then, as in all earlier cases, *vyavahāra-khetta-sāgaropama* (practical space-sea-measure) or $vks = 10^{15} \times vkp = 10^{51.52} \times Å_3$ *samaya* [ADS, *sūtra*, 436, 437].

4.5.6 *sukṣma-khetta-palyopama* (*skp*) (subtle space-pit-measure)

Imagine that each of the hair-tip in the pit is divided in innumerable parts, then each such part is in contact with innumerable space-points. If each of these space-points is exhausted per 100 years, then the time required to exhaust all the space-points is the measure of *sukṣma-khetta-palyopama* (*skp*). i.e. $skp = 100 \times 10^{36.52} \times Å_3$ years $= 10^{38.52} \times Å_3$ years. 10^{15} times *skp* is *sukṣma-khetta-sāgaropama* (subtle space-sea-measure) or $sks = 10^{53.52} \times Å_3$ years. These measures are used for estimating matter as described in *Dṛṣṭivāda* (12th *aṅga*). [ADS, 10. 438, 440].

In the later (post-*āgamic*) Śvetāmbara literature, the prefix '*vyavahāra*' is replaced by '*bādara*' (gross), i.e. the measures are designated as *bādara*/ *sukṣma*, *uddhāra*-/ *addhā*-/ *khetta*-, *palyopama*/ *sāgarovama* (gross/ subtle, linear-/ temporal-/ space-, pit-/ sea-, measures).

4.6 Analogy in Digambara Tradition

In Digambara literature, analogy measures are of 3 types, viz. (1) *vyavahāra palya* (practical pit-measure), (2) *uddhāra palya* (linear pit-measure), and (3) *addhā palya*. There is no further subdivision like *vavahāra* (*bādara*, gross) and *suhuma* (*sūkṣma*, subtle) under each of these measures. Further, as remarked in a (i) above, the dimensions of pit are 1 *pramāṇa* (standard) *yojana* (= 500 *yojana*-s) as diameter and 1 *pramāṇa yojana* as depth of the pit.

4.6.1 *vyavahāra palyopama* (*vp*) (practical pit-measure)

In section 4.6.1 above, we have seen that number of hair tips that can be accommodated in the pit (with both the diameter and depth of 1 *pramāṇa joyaṇa*) is $10^{44.616}$. Each of these hair-tips is to be divided in innumerable parts. Then if each of those parts is exhausted every 100 years, then the time required to exhaust the practical pit is the *vyavahāra palyopama* or *vp* (practical pit-measure).

In terms of notation in section 4.6 above, this time-span will be: $100 \times 10^{44.616} \times Å_1$ years = $10^{46.616} \times Å_1$ years. Thus, it will be clear that *vp* of Digambara tradition is similar (if not equivalent) to *sap* of Śvetāmbara tradition (= $10^{38.52} \times Å_1$ years as stated in section 4.6.4). 10^{15} times of *vp* is *vyavahāra sāgarovama* or *vs* = $10^{15} \times 10^{44.616} \times Å_1 = 10^{59.616} \times Å_1$ years[1] (*gāthā* 125).

4.6.2 *uddhāra palyopama* (*up)* (exhaustion pit-measure)

If each of the parts of hair-tips in the practical pit is further sub-divided in as many sections as there are *samaya*-s in innumerable *koḍī* (10^7) years (of course mentally) and each of those sections are exhausted per *samaya*, then the *samaya*-s required in exhausting that pit is termed *uddhāra palyopama* (*up)* (practical pit-measure). Thus,

up = $10^{44.616} \times Å_1 \times 10^7 \times Å_3$ *samaya*-s = $(10^{51.616} \times Å_1 \times Å_3) / (12 \times 30 \times 30 \times 3773 \times Å_2)$ years $\approx (10^{51.616} / 10^{7.61}) (Å_1 Å_3 / Å_2)$ years $\approx 10^{44} \times Å_1 Å_3 / Å_2)$ years = $Å_4$ say, an innumerable quantity. 10^{15} times *up* is *uddhāra sāgarovama* (*us*) = 10^{15} $Å_4$. The number of island-continents and seas is stated to be 2½ *uddhāra sāgarovama* = $2.5 \times 10^{15} \times Å_4$, again an innumerable number[2] (*gāthā* 126, 127)

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[1] *Tiloyapaṇṇatti*, Vol. I, p. 28.

[2] *Ibid*, p. 29.

In *Śvetāmbara* tradition number of islands and oceans are calculated as $10^{52} \times Å_1$ which is an innumerable quantity different values can be reconciled by different values for $Å_1$ and $Å_4$.

4.6.3 *addhā palyopama* (*ap*) (temporal pit-measure)

Each of the fragments of the parts of the hair-tips described under *uddhāra palyopama* is (mentally) divided further in innumerable sub- fragments. If each such sub- fragment is removed per *samaya*, then the time required to exhaust all the sub- fragments is *addhā palyopama* (*ap*). Thus, $ap = up \times Å_5 = Å_4\ Å_5$ years. As described earlier, *addhā sāgaropama* or $as = 10^{15} \times Å_4 \times Å_5$. These measures are used for determining the duration of *karma* of all the living beings.[1]

4.7 Analogy in Other Ancient Tradition

It appears that the concept of analogy measures was prevalent in pre-Mahāvīra days also. *Bhagavatī Sūtra*[2] (13.101) narrates a dialogue between Mahāvīra and Gośālaka – a follower of Ājīvaka sect who became disciple of Bhagawāna Mahāvīra for few years and then left him. Once during conversation Gośālaka mentions analogy measures to Mahāvīra such as *sāgaropama* – derived as exhaustion of sand-particles of river *Gangā-s* in the same manner as the analogy measures found in *Āgama*. According to Gośālaka, the dimensions of the *Gangā-s* are:

The *Gangā* is 500 *yojana* in length, ½ *yojana* in width and 500 *dhanuṣa* ($1/16^{th}$ *yojana*) in depth.

7 *Gangā-s*	=	1 *Mahāgangā*
7 *Mahāgangā-s*	=	1 *Sādīna Gangā*
7 *Sādīna Gangā-s*	=	1 *Mṛta Gangā*
7 *Mṛta Gangā-s*	=	1 *Lohita Gangā*
7 *Lohita Gangā-s*	=	1 *Āvatī Gangā*
7 *Āvatī Gangā-s*	=	1 *Paramāvatī Gangā*
∴ 1 *Paramāvatī Gangā*	=	7×7×7×7×7×7 = 1,17, 649 *Gangā-s*

[1] *Tiloyapaṇṇatti*, Vol. I, p. 29.

[2] *Bhagavaī*, Vol. III, pp. 288-289.

So, the volume of *Gangā-s* $500 \times 1/2 \times 1/16$ cubic *yojana* By taking, sand particles equals to *rathareṇu* we have,

1 *aṅgula* = 8^9 *rathareṇu* and 1 *yojana* =768000 *aṅgula*,

we get the volume of *Gangā* as,

$$\frac{500 \times (768000)^3 \times 8^{27}}{16 \times 2} = \frac{500 \times (768)^3 \times 8^{27} \times 10^{11}}{32} = \text{cubic } rathareṇu$$

volume of *Paramāvatī Gangā-s* $= \frac{500 \times (768)^3 \times 8^{27} \times 10^{11} \times 7^6}{32}$ cubic *rathareṇu*

Further it is divided in 2 classes as gross (*bādara*) and subtle (*sukṣma*) as is described in ADS. Subtle is of no use and the under gross *sāgaropama* one sand particle is removed after every 100 years and the time elapsed to empty *Paramāvatī Gangā is* 1 *sara*.

1 *sara* = $\frac{500 \times (768)^3 \times 8^{27} \times 10^{11} \times 7^6}{32} \times 100$

$= \frac{500 \times (768)^3 \times 8^{27} \times 10^{13} \times 7^6}{32}$

= 201 336 064 379 438 797 156 525 607 802 529 382 400 000 000 000 000

$\approx 2.013 \times 10^{50}$ *years*

We can see this is comparable with *sāgaropama*.

Further, 1 *mahākappa* = 3×10^5 *sara*

= 604 009 964 323 162 595 695 269 466 465 280 597 164 032 000 000 000 000 00

$\approx 6.04 \times 10^{55}$

1 *mahāmānasa* = 8400000 *mahākappa*

= 507 368 370 031 456 580 384 026 351 830 835 701 617 786 880 000 000 000 000 000 000

$\approx 5.07 \times 10^{62}$ years

Further he says that during the cycle of rebirths prior to Gośālaka he was born as a deity having an age of 10 *sāgaropama.* It shows that the same time unit with different value was prevalent in other traditions also.

4.8 Number Field in Jain *Āgama*

Jaina mathematician have calculated various quantities relating to living, non-living beings, geography, astronomy etc. They were very passionate to know '*how much*' and '*how many*' in context of every existing object of the world living and non-living. This curiosity pushed them to device the number field. In Jaina philosophy numbers are associated with either time or space (distance). This classification of number is the uniqueness of number theory of Jaina philosophy where innumerable and infinite both are put into different domain whereas both are beyond human counting.

It is already mentioned that Jaina number field does not start with 0 or1. When it was asked (ADS, 10.574), '*se kiṁ taṁ gaṇaṇāsaṅkhā?*'[1] *ekko gaṇaṇā na uveī, duppabhii saṅkhā, taṁ jahā-saṅkhejjāe, asaṅkhejjāe, aṇantae i.e.* What is then the *saṅkhā* (number) as counting? The answer given was: number 1 is not considered for counting purpose. Numbers for enumeration start from 2 and are three-fold: (1) *saṅkhejjāe* (enumerables), (2) *asaṅkhejjae* (innumerables), and (3) *anantae* (infinities). They all are further classified.

Jaina philosophy has always been interested in usage of large numbers to enumerate the length, height, time-span of the living beings etc. The number field explained by Jaina saint-scholars had been the field of positive integers, i.e. $Z^+ \equiv \{2, 3, ...\}$. They showed great proficiency in tackling uncountably high numbers and some pristine ideas on infinity. In this respect, treatment of such unmanageably monster numbers is parallel to modern treatment of large numbers and has remote resemblance to Cantor's treatment of transfinite numbers and more so with his theory of hierarchy of infinities. ADS displays quite clear and advanced ideas on these issues.

4.8.1 Numerable, Innumerable, Infinite

In *Āgama* the countable decimal scale of numeration goes up to *śīrṣaprahelikā* and thereafter subsequent measures are mentioned as simile measures, innumerable and infinite. In BS, JDP, PS we do not find sub-classification of these scales like ADS. But the fundamental three measures are used abundantly. For an example, there are *asaṅkhyāta* souls in the roots, stem, skin, branch and leafs of multi-seeds trees (PS, *sūtra* 1.41). ADS (*sūtra* 574 to 603) deals with number field Z^+ in great detail.

[1] *Anuogadārāiṁ*, p. 309.

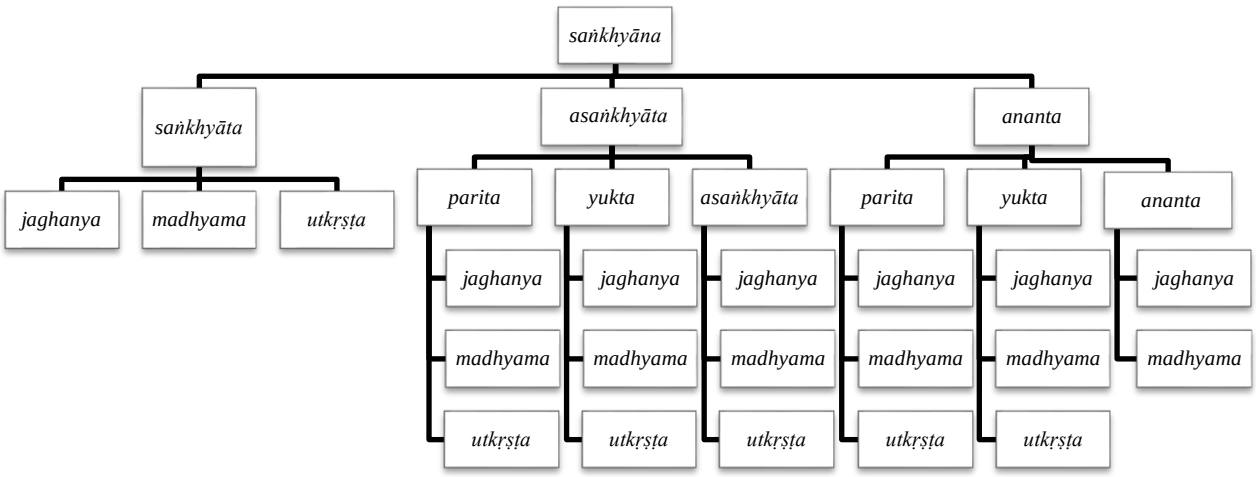


Fig 4.3: Types of Number Field in ADS

Numbers are divided in three classes: 1) *saṅkhejja* (*saṅkhyāta*, numerable or enumerable or countable or measureable), 2) *asaṅkhejja* (*asaṅkhyāta*, innumerable or innumerable or uncountable or in modern parlance incomputable 3), and *ananta* (literally meaning endless, i.e. infinite).

1. **Numerable** are further divided in three: minimum (*jaghanya*) numerable, intermediate (*ajaghanya-anutkṛṣṭa*) which literally means neither minimum nor maximum (post-ADS scholars have used the term '*majjhima'* for intermediate numerable), and maximum (*utkṛṣṭa*) numerable.

2. **Innumerable** are firstly divided in three classes: *parittāsaṅkhejja* (*paritāsaṅkhyāta,* lower order innumerable – *paritta* literally means 'reached'), *juttāsaṅkhejja* (*yuktāsaṅkhyāta,* middle order innumerable- *yuktā* literally means 'proper'), and *asaṅkhejjāsaṅkhejja* (higher order innumerable - *asaṅkhejjāsaṅkhejja*) which literally means 'innumerable-innumerable'. These three classes are further sub-divided in 3 parts, minimum, intermediate, and maximum.

3. **Infinite** (*ananta*) is divided ,in first place, in 3 classes as *parittānanta* (lower order infinite), *yuktānanta* (middle order infinite), and *anantānanta* (higher order infinite) and each of lower and middle order infinites are further subdivided in 3 parts as minimum, intermediate, and maximum. *anantānanta* is divided in only two parts minimum and intermediate. According to ADS maximum *ananta* does not exist.

Thus, there are 3 + 9 + 8 = 20 divisions of number line. ADS asserts that field of middle order infinity is endless indicating indirectly that there is no maximum (*sūtra* 603). This is equivalent to modern concept of infinity.

4.9 Sets of Number Fields

1) *jaghanya saṅkhyāta* (minimum numerable): 2
2) *ajaghanyānutkṛṣṭa saṅkhyāta* (intermediate numerable): 3,4,5,……., S – 1.
3) *utkṛṣṭa saṅkhyāta* (maximum numerable): S = A- 1
4) *jaghanya paritāsaṅkhyāta* (minimum lower order innumerable): S + 1 = A
5) *ajaghanyānutkṛṣṭa paritāsaṅkhyāta* (intermediate lower order innumerable): A+1, A+2,…., A^A – 2
6) *utkṛṣṭa paritāsaṅkhyāta* (maximum lower order innumerable): A^A - 1 or, B-1.
7) *jaghanya yuktāsaṅkhyāta* (minimum middle order innumerable): A^A = B, say
8) *ajaghanyānutkṛṣṭa yuktāsaṅkhyāta* (intermediate middle order innumerable) : A^A +1, A^A +2,… or, B+1, B+2…..C-2
9) *utkṛṣṭa yuktāsaṅkhyāta* (maximum middle order innumerable): $(A^A)^{A^A}$-1 = B^B-1= C-1
10) *jaghanya asaṅkhyātāsaṅkhyāta* (minimum higher order innumerable): $(A^A)^{A^A}$= B^B = C, say
11) *ajaghanyānutkṛṣṭa asaṅkhyātāsaṅkhyāta* (intermediate higher order innumerable): B^B+1, B^B+2,…. or, C+1, C+2…..D-2
12) *utkṛṣṭa asaṅkhyātāsaṅkhyāta* (maximum higher order innumerable): C^C -1 or D-1
13) *jaghanya paritānanta* (minimum lower order infinite): C^C = D, say
14) *ajaghanyānutkṛṣṭa paritānanta* (intermediate lower order infinity): D+1, D+2,……E-2
15) *utkṛṣṭa paritānanta* (maximum lower order infinity) : D^D -1 or E-1
16) *jaghanya yuktānanta* (minimum middle order infinity): D^D =E, say
17) *ajaghanyānutkṛṣṭa*. *yuktānanta* (intermediate middle order infinity): E+1, E+2…….. F-2
18) *utkṛṣṭa yuktānanta* (maximum middle order infinity): E^E -1 or F-1
19) *jaghanya anantānanta* (minimum higher order infinity): E^E = F, say
20) *ajaghanyānutkṛṣṭa anantānanta* (intermediate higher order infinity): F+1, F+2,……
21) *utkṛṣṭa anantānanta* (maximum higher order infinity): immeasurable.

4.10 Idea and Treatment of Numerable (*saṅkhejjā*)

1. ***jaghanya*** (minimum) ***saṅkhejjā***: Number 2 is regarded as the minimum of enumerable (countable) numbers. While describing numerable ADS rejects 0 and 1 under counting. It says that minimum of numerable (*jaghanya saṅkhejjā*) starts from number 2. 1 was not regarded as number for counting purposes. Commentator Haribhadrasūrī (8th C. AD) explains (ADS *Haribhadrīya vṛtti*, p. 109):

   ektvasaṅkhyāviṣyatvepi vā prāyoasaṁvyavhāryatvād āha dviprabhṛtih saṅkhyāh.

   i.e. in spite of being subject of number 1 is not useful from practical purpose and carries very small value therefore, counting starts from 2.

   Commentator Malayagiri (12th C. A.D.) justifies this by arguing that since on squaring 1 it remains same (i.e. 1× 1 = 1), it is not treated as countable number. Regarding 0 (zero), it is already discussed in chapter 3 that zero with place value must have come in usage in numeration round about 300 A.D. However, analytical theory about it must not have been developed by then and hence it did not find place in the number field Z^+.

2. ***ajaghanya-anutkṛṣṭa*** (neither minimum nor maximum, i.e. intermediate) ***saṅkhejjā***: Numbers between 2 (i.e. 3 onwards) and maximum enumerable constitute intermediate enumerable numbers. The last intermediate number is one less than maximum numerable number (*utkṛṣṭa saṅkhyāta*).
3. ***utkṛṣṭa*** (maximum) ***saṅkhejjā***: Although *utkṛṣṭa saṅkhyāta* is called enumerable, it can be estimated by analogy only. In fact, it is almost equal to A, since in view of enormity of A, A-1≈A.

4.10.1 Treatment For *Utkṛṣṭa Saṅkhyāta*

i) The treatment of *saṅkhyāta* has undergone changes from time to time. To make its value higher the analogy was changed and we see gradual development.

ii) In BS and JDP no treatment or illustration is found to calculate the *utkṛṣṭa saṅkhyāta* number. No further classification is mentioned in them.

iii) ADS illustrates an analogy to obtain the value of *utkṛṣṭa saṅkhyāta* through 2 pits *anavasthā* (variable) and *śalākā* (measure) whereas JS[1] in *sūtra* 4.139 mentions of

[1] *Jīvasamāsa*, p.100.

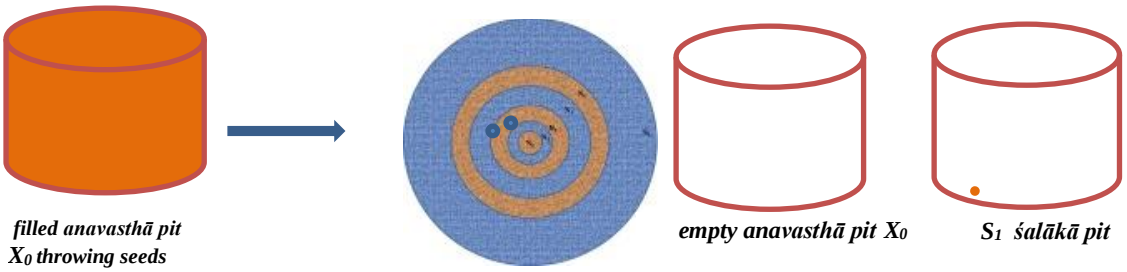
three pits (*kuṇḍa*-s), viz. (1) *anavasthā* (variable), (2) *śalākā* (S_1, first measure pit for *anavasthā kuṇḍa*), (3) *pratiśalākā* (S_2, second measure pit for *śalākā kuṇḍa*).

iv) The authors of Karmagrantha-s moved for higher value of A. Perhaps, therefore they imagined one more pit *mahāśalākā* (S_3, third counting unit for *pratiśalākā kuṇḍa*). Even *Ācārya* Malladhārī Hemacandra (12th C. A.D.) in his *vṛtti* (*sūtra*. 218, 219) as well as *Trilokasāra* (verse, 14-35), TP (4.310-312) and *Karmagrantha* (vol. IV, pp. 213-217) also talk of the same.

4.11 Analogy of Maximum Numerable Number

i) To start with, in ADS, *sūtra* 586, describes the procedure to arrive at this number. Here Jaina saint-scholars imagined two types of pits (*kuṇḍa*-s) –variable (*anavasthā*) pits and measure-pit (*śalākā kuṇḍa*). The first variable pit (X_0) and measure pit (S_1) are shaped like right-circular cylinder with the diameter of 100000 *yojana* (equal to that of our earth, *Jambudvīpa*) and 1000 *yojaṇa* deep. A very large number of variable pits X_0, X_1, X_2 ... are 1000 *yojaṇa*-s deep but their diameters go on increasing incredibly exponentially whereas measure-pit (*śalākā kuṇḍa*) is of constant dimension (like X_0).

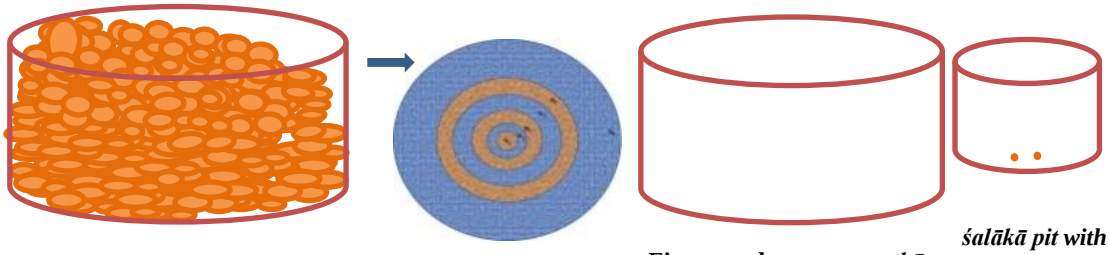
The *śalākā* measuring pit is equal in dimensions to the first variable pit. It can be seen in the following figure.



The first variable (*anavasthā*) pit X_0 is to be filled compactly with mustard seeds in such a manner that no more seeds can be accommodated in it.

Then some god with incredible speed will throw one seed in each island and ocean alternately. First seed is thrown in first [and central] island JD, second in the ocean *Lavaṇa Samudra* [encircling *Jambūdvīpa*], the third in the island *Dhātakī Khaṇḍa* [encircling *Lavaṇa* ocean] etc. so that by throwing seeds in islands and seas [encircling each other], in this manner, the first variable pit will be completely emptied after a while. As an indicator of emptying the first variable pit, a seed is

thrown in *śalākā* pit S_1. Now prepare another variable (*anavasthā*) pit X_1 with depth of 1000 *yojana* and of diameter equal to the diameter of the island or ocean where the last seed from the first variable pit X_0 was thrown. Now the second *anavasthā* pit X_1 is unimaginably bigger than the first one, as we will see later.



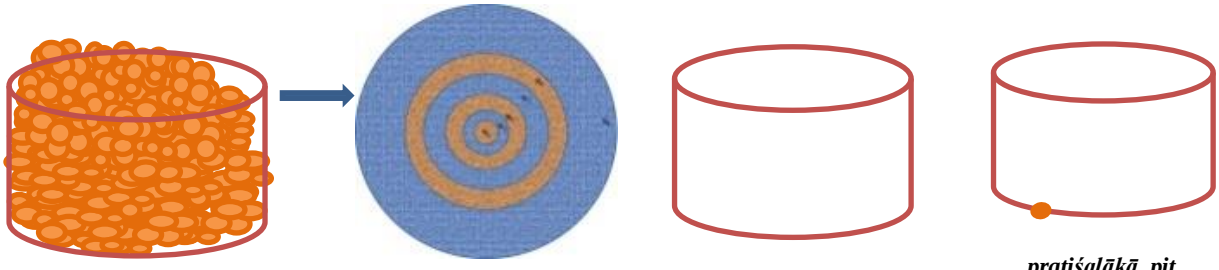
second anavasthā pit X_1

Fig: second empty anvasthā pit X_1

śalākā pit with 2 seeds

Then fill this second variable pit X_1 compactly with mustard seeds and let this second variable pit be emptied by the god in a similar manner by throwing one seed in each succeeding islands and oceans alternatively till the second variable pit is also emptied and as a symbol of it throw a second seed in *śalākā kuṇḍa* S_1. This process [of constructing ever increasing variable pits, filling them with seeds, emptying them in subsequent islands and oceans, and throwing seed in *śalākā kuṇḍa* as a symbol of emptying the variable pits] goes on till *śalākā* pit is also compactly and completely filled. Maximum numerable (*utkṛṣṭa saṅkhyāta*) u_s is one less than the total number of seeds in *śalākā kuṇḍa* S_1 and the seeds thrown in islands and oceans (together, N_0 in number).

ii) As per treatment in JS three pits are there to count the value of *utkṛṣṭa saṅkhyāta.* When S_1 is full by abovementioned process then empty the *śalākā* S_1 pit by throwing the seeds one by one in islands and ocean as earlier and as a token of emptying the *śalākā kuṇḍa* S_1 for first time throw a seed in *pratiśalākā kuṇḍa* (S_2).



śalākā pit S_1

empty śalākā pit S_1

pratiśalākā pit

Again fill the variable pit and continue this recursive process *ab initio* till the *pratiśalākā kuṇḍa* is full. To fill the *pratiśalākā kuṇḍa* completely *śalākā kuṇḍa*-s will be emptied as many times as the number of seeds can be accommodated in *pratiśalākā kuṇḍa*. Maximum numerable (*utkṛṣṭa saṅkhyāta*) is one less than the total number of seeds in both the pits S_1 and S_2 including the seeds thrown in islands and oceans.

iii) According to Karmagrantha-s the treatment goes for four pits by adding *mahāśalākā kuṇḍa*. The process starts with filling the variable pit, *śalākā kuṇḍa*-s, *pratiśalākā kuṇḍa*-s and then empting them. After emptying these three pits, the process is continued *ab initio* till the *mahāśalākā kuṇḍa* is filled completely. Then one less than the number of seeds in this *mahāśalākā kuṇḍa*, last *pratiśalākā kuṇḍa*, last *śalākā kuṇḍa*, last variable pit, and all the seeds thrown earlier in the seas and islands altogether constitute the maximum numerable (*utkṛṣṭa saṅkhyāta*). If in total A seeds are thrown then maximum numerable number will be u_s = A-1.

iv) There is one more difference – each of the variable pits and measure pits are to be filled to form a conical heap above them. It is further specified that height of the conical heap is 1/11th of the circumference of right circular cylindrical pits.

Note that under section i) there will be N_0 variable pits, while under ii) there will be N_0^2 and under iii) will be N_0^3 variable pits.

4.11.1 Development of Mathematical Treatment of *Asaṅkhyāta* in ADS

Pit X_0 is to be filled up compactly with mustard seeds in such a manner that no more mustard seeds can be accommodated therein. Then imagine a god of high powers emptying this pit by throwing the seeds one by one, starting with the island *Jambūdvīpa*, then second in *Lavaṇa* sea, third in *Dhātaki* island, and so on till the last seed falls in an island or a sea. As a token of emptying X_0, drop a seed in the pit S_1. Then construct another variable pit X_1 of diameter equal to the island or ocean where the last seed from X_0 was dropped. Fill X_1 compactly with the seeds and start throwing seeds, one by one, in subsequent seas and islands, till X_1 is also emptied. As a symbol of emptying X_1, put one more seed in S_1. Then construct third variable pit X_2, with diameter equal to the island or sea where the last seed from X_1 was dropped and fill it with seeds compactly. This process of filling the variable pits with the seeds, throwing those seeds in subsequent islands or seas till the pits are emptied, throwing a seed in S_1 as a sign of emptying the variable pits etc. should be

continued till the *śalākā kuṇḍa* (measure-pit) S_1 is filled up compactly. Then total number of seeds thrown since beginning, i.e. number of seeds in all the variable pits and the seeds in *śalākā kuṇḍa* (measure-pit) S_1 constructed less one is the maximum numerable (*utkṛṣṭa saṅkhyāta*). If one is added to this we get minimum lower order innumerable (*jaghanya paritāsaṅkhyāta*). The wordings of available original text of ADS are not very clear and in detail. We have gone through this treatment on the basis of version by commentators.

Let's put it into mathematical notation. Say the first *anavasthā kuṇḍa* X_0 contains N_0 mustard seeds so compactly that there is no room for more seeds. The measuring pit also has the dimension same as the first variable pit. Therefore it can accommodate N_0 seeds. To count total numbers of seeds we will have to see how many times all the pits are filled and emptied.

First of all analyze the variable pit. Seeds from X_0 are thrown in the alternate ocean and island till it is exhausted. And at the moment of exhaustion one seed will be put in to S_1 *śalākā.* Let's calculate the volume and diameter of the variable pits X_0, X_1, X_2 …. and the volume of the seed ?

$D_{N_0} = \{1 + 2\ (2 + 4 + 8 + ... + 2^{N_0-1})\}\ \Upsilon = (2^{N_0+1} - 3)\ \Upsilon.$

When the first variable pit is empty, the second pit X_1 is constructed and filled compactly with the seeds say N_1 in number and its diameter will be equal to the diameter of N_0^{th} island or ocean i.e. $(2^{N_0+1}-3)\ D_{N_0}$. Now the question is how many seeds can be accommodated in the first or the subsequent variable pits.

4.11.2 Number of seeds in X_0

In the times of ADS, it was not even possible to compute the number of seeds in first variable pit X_0. But Nemicandra Siddhāntacakravartī (10^{th} C.), Abhayadeva Sūrī (11^{th} C.), and Ācārya Malayagiri (12^{th} C.) made bold attempt to calculate the number of seeds in X_0, say N_0.

Now, the volume of the pit X_0 with diameter 100000 *yojaṇa* and depth 1000 *yojaṇa* is:

Let the volume of pit X_0 is V_0 and volume of one seed is V_m then

$V_0 = \pi r^2 h = \pi\ (10^5/2)^2 \times 10^3 = \pi \frac{10^{13}}{4}$ cubic *pramāṇa yojana*

$Vm = (4/3)\boldsymbol{\pi} r^3$, where r = (1/2) *sarṣapa*

1 *yojana* = 4×2000 *daṇḍa*

= 8000×96 *aṅgula*

= $8000\times96\times500$ *utsedha aṅgula* (as per **Digambara** tradition)

= $96\times4\times10^6\times64$ *sarṣapa*

= 24576×10^6 *sarṣapa*

= $6\times8^4\times10^6$ *sarṣapa*

radius of the seed (as per spatial scale given in ADS is) = $1/(2\times6\times8^4\times10^6)$ *yojana*

N_0 (Number of seeds in X_0) = V_0 / V_m [assuming the radius of the seed as ½ *sarṣapa* seed which is equal to $1/(2\times6\times8\times8\times8\times8\times10^6)$ *yojana* according to Nemicandra Sidhhānta Cakravartī (10^{th} C.A.D.)][1]

Then, the number of seeds in X_0 is:

$N_0 = V_0 / V_m = 4294967296 \times 256 \times 10^{31} = 19791209299968 \times 10^{31} \approx 1.979 \times 10^{44}$.

$N_0 = V_0 / V_m = \frac{\pi r^2 h}{\frac{3}{2}\pi r^3} = \frac{\pi\times 10^{13}\times2\times2^3\times6^3\times8^{12}\times10^{18}}{\pi\times4\times3} = 6^2 \times 8^{13} \times 10^{31} = 19{,}791{,}209{,}299{,}968$

(by using wrong formulae for volume of sphere as $\frac{3\pi r^3}{2}$ instead of correct as $\frac{4\pi r^3}{3}$)

$= 1.97 \times 10^{44}$

$\approx 2 \times 10^{44}$

$\approx 10^{log\,2} \times 10^{44}$

$\approx 10^{.30103} \times 10^{44} = 10^{44.30103}$ seeds ……(i)

$N_0 = 10^{44.30103} = 10\uparrow\uparrow2\ (44.3)$ [In Knuth's up arrow notation]

R.C Gupta claims that the formulae used in above calculation is not correct because there $\pi = 3$ and the formulae is also different. [Gupta, pp. 33-34]

Prof. R.C. Gupta[2] has obtained this value by using the correct formula $(4/3) \times \pi \times (\text{radius})^3$

$N_0 = V_0 / V_m = 2.34228 \times 10^{45} \approx 10^{45.37}$

Similarly, we may determine this value as per **Śvetāmbara** tradition by taking correct formula

$4/3\ \pi r^3$ and 1 *pramāṇa yojana* = 400 *aṅgula*

1 *pramāṇa yojana* = $768000\times$ 400 *aṅgula*

= $768000\times 400\times64$ *sarṣapa* (64 *yukā* (*sarṣapa*) = 1 *aṅgula*)

= $196{,}608 \times 10^5$ *sarṣapa*

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[1] Shah. R.S., *Mathematics of Anuyoadwāra sūtra,* Gaṇita Bhāratī, Vol. 29, 2007, pp. 81-100.

[2] Gupta, R.C., *Ancient Jain Mathematics,* Jain Humanities Press, Canada and USA, p. 190.

$= 6\times8^5\times10^5$ *sarṣapa*

$N_0 = V_0 / V_m = \frac{\pi r^2 h}{\frac{4}{3}\pi r^3}$ =

$= \frac{\pi\times 10^{13}/4}{\pi\times\frac{4}{3}\times\left(\frac{1}{2}\times\frac{1}{400\times768000\times8^2}\right)^3}$

$= \frac{10^{13}\times3\times2^3\times400^3\times768000^3\times8^6}{4\times4} = 96\times768^3\times8^6\times10^{28}$

= 113 997 365 567 815 680 000 000 000 000 000 000 000 000 000

$\approx 1.13\times10^{44}$

$= 10^{log\,1.14}\times10^{44} = 10^{.05}\times10^{44} = 10^{44.05}$ seeds (Śvetāmbara tradition)

The number of seeds in Digambara tradition $10^{44.30}$ is comparable with the value $10^{44.05}$ of Śvetāmbara tradition which is almost equal.

Once the pit X_0 gets exhausted, one seed will be dropped into *śalākā* pit S_1 and the second pit X_1 with diameter same as of the N_0^{th} ocean/island is constructed with the same depth of 1000 *y*. The *anavasthā* pit X_1 is also to be filled up compactly with mustard seeds. Let N_1 be the number of seeds in X_1. Now continue this process of throwing the seeds from X_1 starting with $(N_0+1)^{th}$ ocean/island till X_1 becomes empty and the last seed will fall into $(N_0+ N_1)^{th}$ ocean/island. Now the third pit X_2 will be constructed with the diameter equal to $(N_0+ N_1)^{th}$ ocean/island with the same depth. And the second seed will be dropped in the same *śalākā* pit. The process will goof filling X_2 compactly (with say N_2 seeds) then emptying it as described above, then throwing 3rd seed in S_1 etc. be continued till *śalākā* pit becomes completely full. Thus there will be N_0 *anavasthā* pits in total. The N_0th pit will have the diameter

$=\{\left(2^{N_0+N_1+N_2+N_3\ldots\ldots+N_{N_0-1}}\right)-3\}.N_0$

The number of seeds in X_{N_0} pit is N_{N_0}

$N_{N_0} =\{\left(2^{N_0+N_1+N_2+N_3\ldots\ldots+N_{N_0-1}}\right)-3\}^2.N_0$

The number of seeds in the pit X_1 is N_1.

$N_1 = \frac{\pi(\frac{2^{N0+1}-3}{2})^2\times1000}{\pi(\frac{1}{2})^2\times1000} = (2^{N_0+1}-3)^2.N_0$; Here 1 from exponent of 2 and 3 in bracket can be ignored as it is so small before 2^{N_0}

$\approx 2^{2N_0}.\ N_0$

By putting the value of N_0 from the equation (i)

$N_1 \approx 2^{2N_0} \times N_0 = 2^{2\times 10^{44.3}} \times 10^{44.3} = 2^{10^{log2}\times 10^{44.3}} \times 10^{44.3}$

$= 2^{10^{.30103}\times 10^{44.3}} \times 10^{44.3} = 2^{10^{44.6}} \times 10^{44.3} = 10^{10^{44.6}(\log 2)} \times 10^{44.3} = 10^{10^{44.6}\times 0.3} \times 10^{44.3} =$

$= 10^{10^{44.6}\times 10^{\log(0.3)}} \times 10^{44.3} = 10^{10^{44.6}\times 10^{-0.522}} \times 10^{44.}$

$= 10^{10^{44.07}} \times 10^{44.3} = 10^{10^{44.07}}$

$10^{44.3}$ is very negligible before$10^{10^{44.07}}$, so it gets ignored. Therefore, resultant value of number of seeds N_1 in the second variable pit X_1

$N_1 = 10^{10^{44.07}} = 10\uparrow\uparrow 3\ (44.07)$ [In Knuth's up arrow notation]

$N_2 = 2^{2N_1} \times N_1 = 2^{2\times 10^{10^{44.07}}} \times 10^{10^{44.07}}$

$= 10^{2\times 10^{10^{44.07}}} \times 10^{10^{44.07}}$

$= 10^{(2\times 10^{10^{44.07}})(\log 2)} \times 10^{10^{44.07}}$

$\approx 10^{(.6\times 10^{10^{44.07}})} \times 10^{10^{44.07}}$

$\approx 10^{(10^{10^{44.07}}-0.22)} \times 10^{10^{44.07}}$

(-0.22 in exponent and factor $10^{10^{44.07}}$ can be ignored as per rules of operations of large numbers.)

$\approx 10^{10^{10^{44.07}}}$

$N_2 = 10^{10^{10^{44.07}}} = 10\uparrow\uparrow 4\ (44.07)$ [In Knuth's up arrow notation]

Similarly, we can prove $N_3 \approx 10^{10^{10^{10^{44.07}}}} = 10\uparrow\uparrow 5\ (44.07)$ [In Knuth's up arrow notation]

$N_{N_0} = 10^{10^{10^{10^{\cdot^{\cdot^{\cdot^{10^{44.07}}}}}}}}$ $= 10\uparrow\uparrow N_0+2(44.07)$ [In Knuth's up arrow notation]

Where the height of tower of 10 is nearly 10^{44}.

When the last *anavasthā kuṇḍa* and *śalākā* are filled up the total number of seeds thrown so far will be known as *jaghanya paritāsaṅkhyāta* (A).

Therefore A= $N_0 + N_1 + N_2 + N_3 \ldots\ldots + N_{N_0}$

N_{N_0} is so huge that the sum of earlier terms ($N_0 + N_1 + N_2 + N_3 \ldots\ldots\ldots + N_{N_0-1}$) can be ignored. Hence we may write A ≈ N_{N_0} = $10\uparrow\uparrow N_0+2(44.07)$

Let's compare A with Skewes (1933) type 1 number of modern mathematics

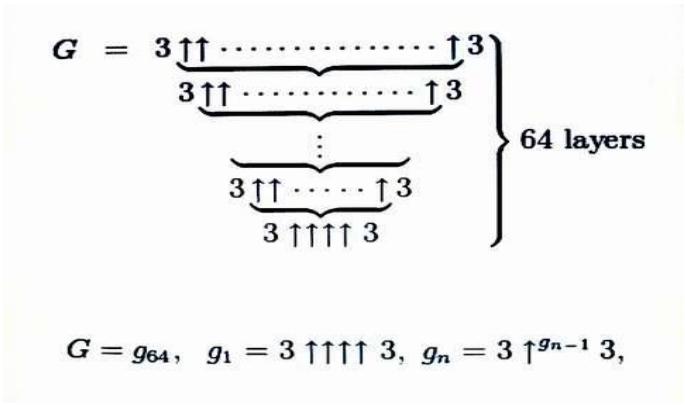
i.e. $e^{e^{e^{79}}} < 10^{10^{10^{34}}}$

Later in 1955 he proposed number of type 2

i.e. $e^{e^{e^{e^{7.705}}}} < 10^{10^{10^{964}}}$

We can see that the A $\left(10^{10^{10^{10^{\cdot^{\cdot^{\cdot^{10^{44.07}}}}}}}} \right)$ is much higher than even second Skewes number $\left(e^{e^{e^{e^{7.705}}}} < 10^{10^{10^{964}}}\right)$. In fact, Skewes' numbers are negligibly miniscule compared to A.

In fact, it may be comparable with Graham's number $G\,(g_{64})$ which can be diagrammatically represented as using Knuth's up-arrow notation



This is an unimaginably, mind-bogglingly large number, which cannot be even grasped by human brain.

4.11.3 *asaṅkhyāta* in Post-canonical Literature

Subsequent Jaina saint-scholars further refined the recursive process and arrived at a very large value of A.

We find difference in the analogy for *jaghanyaparitāsaṅkhyāta*. ADS envisions 2 types of pits *anavasthā* and *śalākā*, JS and TP adds one more *pratiśalākā* and Karmagrantha and *Pañcasaṅgraha* adds one more pit as *mahāśalākā*. It brings enormous difference in the value of A.

In the beginning there are two pits- *anavasthā kuṇḍa* X_0, and *śalākā* S_1.In the later versions the process of exhaustion (called *uddhāra*) of *anavasthā* (variable) pits $X_{N_0}$is to be continued even after S_1 is filled up. When *śalākā* S_1 is filled up fully then one seed is dropped in, *pratiśalākā* S_2 as a token. S_1 is then emptied and process of exhaustion described in previous section continued further till the pit S_1 is filled up again, when second seed is put in S_2. In this way when S_2 is also filled up fully, a seed is to be put in S_3. S_2 is then emptied and the process of exhaustion of X_{N_0}, emptying of S_1, then of S_2, is continued till the last pit S_3 (*mahāśalākā*) is filled up. Thus there will be N_0^3 *anavasthā kuṅḍa*.

It can be concluded that the value of A is much larger than ADS. According to ADS, $A \approx 10\uparrow\uparrow N_0$. In JS A becomes $10\uparrow\uparrow N_0^2$; whereas in later texts it becomes as high as $10\uparrow\uparrow N_0^3$.

Thus number A of ADS is so large that it is beyond computation through computer also then we can imagine the enormity of A in post-canonical literatures. This is *jaghanya parita asaṅkhyāta*. Thus *utkṛṣṭa* (maximum) *saṅkhyāta* (innumerable), which is A-1, is also equally unfathomable.

4.11.4 Numerical Symbolism of A

We have seen above that the value of A is beyond comprehension of human beings and even computers. However just to illustrate the enormity we start by assuming A= 4 (notionally).

b) A (*jaghanya paritāsaṅkhyāta*)= 4

c) B (*jaghanya yuktāsaṅkhyāta*) = $A^A = 4^4 = 256$ (say, B)

d) C (*jaghanya saṅkhyātāsaṅkhyāta*) = $B^B = 256^{256}$ =

323170060713110073007148766886699519604441026697154840321303454275246551388678908931972014115229134636887179609218980194941195591504909210950881523864482831206308773673009960917501977503896521067960576383840675682767922186426197561618380943384761704705816458520363050 4

28875758915410658086075523991239303855219143333896683424206849747865645694948561760353263220580778056593310261927084603141502585928641771167259436037184618573575983511523016459044036976132332872312271256847108202097251571017269313234696785425806566979350459972683529986382155251663894373355436021354332296046453184786049521481935558536110595962306560

$\approx 3.2317\times10^{616}$

e) D (*jaghanya paritānanta*)= $C^C = (256^{256})^{256^{256}} = 256^{256^{257}} = 10^{10^{619.29}}$

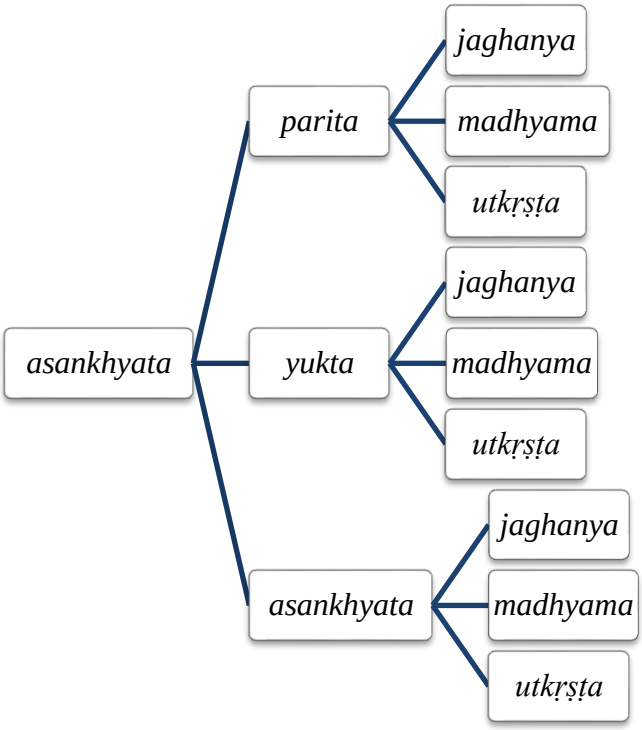
f) E (*jaghanya yuktānanta*) $=10^{10^{10^{619.29}}}$

g) F (*jaghanya anantānanta*) = $10^{10^{10^{10^{619.29}}}}$

This shows that with A= 4, digits of C only can be evaluated through computer. D, E and F are beyond its grasp. It can represent it only in the form of tower of 10. This is with A = 4. One can only imagine what will happen if actual value of A is taken.

4.12 Domain of Innumerable (*asaṅkhyāta*)

To understand Jaina philosophy it is necessary of being acquainted with innumerables. (Buddhist literatures too have a term *asaṅkhyeya* $=10^{140}$ which shows that it is not an innumerable number, *Paddhitakalpa*, *sūtra* 52). ADS describes various types of innumerables. *asaṅkhyāta* is basically divided in three and each of them are further classified in three classes as :



As we have already discussed that innumerables are beyond our measure, yet Jaina philosophy conceives of such number field with nine-fold order as mentioned in ADS (587-594):

i. *jaghanya parita asaṅkhyāta* (minimum lower order innumerable): This field contains only one member. It is the number obtained by adding one to u_s or *utkṛṣṭa saṅkhyāta* (ADS 587). In other words A is $10^{10^{10^{10^{10^{\cdot^{\cdot^{\cdot}}}}}}}$ as it is already mentioned. It is unimaginably monstrous number which can be compared probably with Graham number of modern mathematics. (Its enormity will be explained shortly.)

ii. *ajaghanyānutkṛṣṭa paritāsaṅkhyāta* (intermediate lower order innumerable): If *jaghanya parita asaṅkhyāta* is denoted by A, then it will contain the numbers A+1,A+2,…., $A^A - 2$.

iii. *utkṛṣṭa paritāsaṅkhyāta* (maximum lower order innumerable): It is identified as one less than minimum of next innumerable i.e. *jaghanya yuktāsaṅkhyāta.* A^A - 1 or, B-1.

iv. *jaghanya yuktāsaṅkhyāta* (minimum middle order innumerable): According to ADS when *jaghanya parita asaṅkhyāta* is raised to its own power then it becomes *jaghanya yuktāsaṅkhyāta* . i.e. A^A = B(say). This process of raising a number to its own power is known as *anyanyobhyāsa* or *yukta.* It is also specified in ADS that number of *samaya* in one *āvalikā* is equal to *jaghanya yuktāsaṅkhyāta.*

v. *ajaghanyānutkṛṣṭa yuktāsaṅkhyāta* (intermediate middle order innumerable): As we have denoted A^A by B, therefore it forms the set of the numbers B+1, B+2…..B^B-2

vi. *utkṛṣṭa yuktāsaṅkhyāta* (maximum middle order innumerable): This is one less minimum of next order of *asaṅkhyāta* = B^B-1.

vii. *jaghanya asaṅkhyātāsaṅkhyāta* (minimum higher order innumerable): $(A^A)^{A^A} = B^B$ =C . Similarly, this number again is raised power of power.

viii. *ajaghanyānutkṛṣṭa asaṅkhyātāsaṅkhyāta* (intermediate higher order innumerable): B^B+1, B^B+2,…. or, C+1, C+2…..D-2 i.e.

ix. *utkṛṣṭa asaṅkhyātāsaṅkhyāta* (maximum higher order innumerable): C^C -1 or D-1.The maximum of higher order on innumerable is just one less than the entrance of the realm of infinity. Here the power of power is raised three times = $[(A^A)^{A^A}]^{[(A^A)^{A^A}]}$-1. This number is the extreme boundary of the innumerable.

4.12.1 Various sets of Innumerable

The treatment of *asaṅkhyāta* is not merely a flight of imagination but Jaina mathematicians have identified the various sets of innumerable and even ordered them appropriately,

Ācārya Malayagiri in the commentary[1] mentions few sets of *asaṅkhyāta* as stated below:

1. space points of *loka-ākāśa* (cosmos)
2. space points of *dharmāstikāya* (medium of motion)
3. space points of *adharmāstikāya* (medium of rest)
4. space points of one soul
5. corporeal bodies of all *nigoda* (lowest form of living beings)
6. *pratyeka sarīrī* organisms (one soul in one body, plants)
7. states of soul (*adhyavasāya*) which cause *anubhāga* (intensity of *karma*)
8. *adhyavasāya* which cause *sthiti* (duration of *karma*)
9. Total number of ultimate units of strength of mental (*mana*) vocal (*vacana*) and physical (*kāya*), activity (*yoga*)

[1] *Anuogadārāiṁ*, p. 33.
cf. Bṛhatkṣetra Samāsa, p.10.

10. No. of *samaya* in one *ussappiṇi* and *ossappiṇi*.

4.13 The World of Infinite Numbers

What is infinity? It's not big. It's not huge. It's not tremendously large. It's endless. Infinity is the idea of something that has no end. Philosophers have speculated about the nature of the infinite, notably Zeno of Elea, who proposed many paradoxes involving infinity, and Eudoxus of Cnidus, who used the idea of infinitely small quantities in his method of exhaustion. Modern mathematics uses the concept of infinity in the solution of many practical and theoretical problems, notably in calculus and set theory, and the idea also is used in physics and the other sciences. L.C. Jain[1] states:

It is said that the concept of infinity in mathematics was conceived for the first time by Brahmgupta (628 C.A.D) while in the western countries this credit goes to Bernhard Bolzano of the 19th century A.D. A study of infinity in mathematics as a mature concept was however taken up by Bhāskarācārya (1150 C.A.D) who deduced the value of quotient a/0 mathematically where 'a' is a finite quantity. But the description of infinity as endless or countless can be traced in Ṛgveda and many other ancient works including those by Jain and Buddhist scholars. An elaborate classification and philosophical explanation of infinity is found in the Jain canonical text as old as 300-400 B.C. like Ṭhāṇaṁ and Uttarādhyayana where infinity of even ten types is mentioned.

In Jaina canonical text the idea of infinity is combined with that of dimensions. *Ṭhāṇaṁ* (10.66)[2] also mentions that ten different kinds of infinities from various perspectives (counting, spatial, temporal, direction etc.) are recognized. Out of them five are mentioned here: infinite in one direction, infinite in two directions, infinite in area, infinite everywhere, and infinite with regard to eternity. Joseph[3] says that it was quite a revolutionary idea in more than one way:

1. The Jaina-s were the first to discard the idea that all infinities were the same or equal, an idea still generally accepted in Europe until the work of Georg Cantor in the late nineteenth century.

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[1] *Basic Mathematics*, p. 12 (Forward).

[2] *Thāṇaṁ*, p. 915.

[3] *The Crest of The Peacock*, p. 351.

2. The highest enumerable number (i.e., N) of the Jainas has some resonance with another concept developed by Cantor, aleph-null (the cardinal number of the infinite set of integers 1,2,…, N), known as the first transfinite number. It was Cantor who defined the concept of a sequence of transfinite numbers and devised arithmetic of such numbers.

3. In Jaina works on the theory of sets, two basic types of transfinite number (i.e., the cardinal numbers of infinite sets) are distinguished. On both physical and ontological grounds, a distinction is made between *asaṅkhyāta* and *ananta*, between rigidly bounded and loosely bounded infinities. With this distinction, the way was open for the Jaina-s to develop a detailed classification of transfinite numbers and mathematical operation for handling transfinite numbers of different kinds. However, unsurprisingly, they did not do so given their limited technical and symbolic compass. (For further details see Jaina, L.C. [1973, 1982] and N. Singh [1987])

4.13.1 Classification of Infinity

Infinity is the fascinating concept of Jaina philosophy and used very frequently in entire *Āgama*. Jaina-s did not put all infinities under one category. There are many types of infinity, as it is already mentioned earlier. This class of infinity is beyond our computation. The domain of Jaina infinity starts just after *utkṛṣṭa asaṇkhyātāsaṅkhyāta*. See also the chart in the section (4.10). The classification of infinity in ADS goes as:

i. *jaghanya paritānanta* (minimum lower order infinity): ADS (*sūtra,* 11.596) says that *jaghanya paritānanta* is obtained by raising *jaghanaya asaṇkhyātāsaṅkhyāta* to its own power. As we see in the section 4.10 that *jaghanaya asaṇkhyātāsaṅkhyāta* is C. Therefore, *jaghanya paritānanta* $C^C = D =$

$$\left\{(A^A)^{(A^A)}\right\}^{\left\{(A^A)^{(A^A)}\right\}}$$

In other words it is equal to one more number to *utkṛṣṭa asaṇkhyātāsaṅkhyāta* .

ii. *ajaghanyānutkṛṣṭa paritānanta* (intermediate lower order infinity): If *jaghanya paritānanta* is denoted by D then D+1, D+2,…… D^D -2 form the members of this set (ADS, 11.597).

iii. *utkṛṣṭa paritānanta* (maximum lower order infinity): When *jaghanya paritānanta* is raised to its own power (*aṇṇamaṇṇabbhāso*) then one less than the resultant number is *utkṛṣṭa paritānanta*. Since D is *jaghanya paritānanta*, therefore, member of this order will be D^D -1, which is one less than *jaghanya yuktānanta* (ADS, 11.598).

iv. *jaghanya yuktānanta* (min. middle order infinity): When *jaghanya paritānanta* is raised to its own power i.e. $D^D = E =$

$$\left(\left\{(A^A)^{(A^A)}\right\}^{\left\{(A^A)(A^A)\right\}}\right)^{\left(\left\{(A^A)^{(A^A)}\right\}^{\left\{(A^A)(A^A)\right\}}\right)}$$

It is *jaghanya yuktānanta*. In other words it is one more than *utkṛṣṭa paritānanta*. One significant remark ADS (11.599) gives is the number of *abhavya* (souls not having potentiality of getting liberation) souls are *jaghanya yuktānanta* in number.

v. *ajaghanyānutkṛṣṭa yuktānanta* (intermediate middle order infinity): The numbers between *jaghanya yuktānanta* and *utkṛṣṭa yuktānanta* is *ajaghanyānutkṛṣṭa yuktānanta* i.e. E+1, E+2…….. E^E -2 will be the members of this set, where E is *jaghanya yuktānanta.* (ADS, 600).

vi. *utkṛṣṭa yuktānanta* (maximum middle order infinity): When *jaghanya yuktānanta* is raised to its own power than one less than the resultant number is *utkṛṣṭa yuktānanta* i.e. E^E -1 (ADS, 601).

vii. *jaghanya anantānanta* (minimum higher order infinity): As per ADS, 602 When *jaghanya yuktānanta* is raised to its own power it results in *jaghanya anantānanta* and it is one more than the last number of its previous order *utkṛṣṭa yuktānanta* i.e. $E^E = F$

$$\left\{\left(\left\{(A^A)^{(A^A)}\right\}^{\left\{(A^A)(A^A)\right\}}\right)^{\left(\left\{(A^A)^{(A^A)}\right\}^{\left\{(A^A)(A^A)\right\}}\right)}\right\}^{\left\{\left(\left\{(A^A)^{(A^A)}\right\}^{\left\{(A^A)(A^A)\right\}}\right)^{\left(\left\{(A^A)^{(A^A)}\right\}^{\left\{(A^A)(A^A)\right\}}\right)}\right\}}$$

viii. *ajaghanyānutkṛṣṭa anantānanta* (intermediate higher order infinity): ADS in *sūtra* 603 mentions that all the numbers beyond *jaghanya anantānanta* (E^E) fall under the class of *ajaghanyānutkṛṣṭa anantānanta.*i.e. F+1, F+2,……..

ix. *utkṛṣṭa anantānanta* (maximum higher order infinity): ADS clearly states that this class does not exist.

This is the classification of infinities found in ADS. This classification was the effort of around 2nd or 3rd century AD. In fact, by stating that there is nothing like maximum higher order infinity, ADS has clearly indicated to the modern concept of infinites or transfinite numbers.

4.13.2 Does ADS Envision Infinity of Cantor?

The treatment of *saṅkhyāta*-s and *ananta*-s reminds us of Georg Cantor (1845-1918)'s treatment of ordinals and cardinals. Designating the magnitude (power) or order of an infinite set, the theory of transfinite numbers was introduced by Georg Cantor in 1874.

The mathematics of infinity is very intriguing. It was noticed by Galileo (1564-1642) that there are as many squares as there are natural numbers. For example,

Consider the sequence of numbers n (natural numbers) and n^2 (their perfect squares) shown below:

1	2	3	4	5	6	7	8	.....
↓	↓	↓	↓	↓	↓	↓	↓	
2	4	9	16	25	36	49	64	.....

Since every natural number is the square root of exactly one perfect square, it would seem that there are just as many perfect squares as natural numbers. In other words, there is a one-to-one correspondence for every number in both sequences.

Similarly this logic can be extended to any finite powers of whole or natural numbers or their multiples or any infinite subset of set of whole numbers. This led Cantor to state that all infinite subsets of infinite set of whole numbers have the same numbers of member i.e. any part of infinite set has the same cardinality as of the whole set.

Cantor denoted the infinity of whole numbers by symbol, $\aleph_0$ (aleph null) then even to his surprise he found that a $\aleph_0$, $\aleph_0/a$, $\aleph_0^a$ etc., where 'a' is a finite whole number, have the same cardinality and he called all these as countable sets. For this he employed a very powerful technique of one-to-one correspondence (which we will see employed by Jaina *Ācārya*).

In his quest for higher order infinities he found that $\aleph_0^{\aleph_0}$ or for that reason even $2^{\aleph_0}$ constitutes another infinity, which he denoted by $\aleph_1$. He asserted that this can continued further i.e. $\aleph_1^{\aleph_1}=\aleph_2$, $\aleph_2^{\aleph_2}=\aleph_3$, etc. Thus, there can be infinities of infinities and each having an order above the earlier infinity.

i.e. $\aleph_0 \subset \aleph_1 \subset \aleph_2 \subset \aleph_3 \ldots\ldots$

We have seen in the treatment arriving at A, the number of seeds in *anavasthaā kuṇḍa* form the series

N_0 ,

$N_1 = 2^{2N_0} \times N_0 \ (\approx {N_0}^{2N_0})$,

$N_2 = 2^{2N_1} \times N_1 \ (\approx {N_1}^{2N_1})$,

$N_3 = 2^{2N_2} \times N_2 \ (\approx {N_2}^{2N_2})$,

⋮

$N_{N_0} = 2^{2(N_{N_0-1})} \times N_{N_0-1} \ (\approx {N_{N_0-1}}^{2N_{N_0-1}})$

Thus the Ns give relations similar to that of Ns of cantor (although Ns are not strictly infinities).

Cantor also envisages infinities of ordinals (where order in which number increases is important). He says counting number 1, 2, 3… etc. goes up to infinity which it denoted by ω, then :

1,2, ….. ω| (ω is the first infinity.)

ω+1, ω+2, …. 2ω, …. 3ω, ….. ω^2, …. ω^3, ….. ω^ω | (ω^ω is the second infinity.)

ω^ω+1, ω^ω+2 …. $(\omega^\omega+2)^2$…. ω^{ω^ω} | (ω^{ω^ω} is the third infinity.)

ω^{ω^ω}+1, ω^{ω^ω}+2… …. $\omega^{\omega^{\omega^\omega}}$…….. where there are ω omegas and this number (ω↑↑ω) was called ε_0 by Cantor.

One simply wonders as to how about 2000 years before Cantor, saint-scholars in India were working with similar ideas, although with far less accuracy or rigor.

The similar relationship can be found in Jainas number field where different categories of *asaṅkhyāta* (A) and *ananta* (I) form a sequence:

2, 3…… A-1, A (is first innumerable) ,

$A+1 \ldots A^A-1$, $A^A = B$ (is second innumerable),

$B+1, B+2 \ldots\ldots B^B-1, B^B$ [$B^B = C = (A^A)^{A^A} = A^{A^{A+1}} \approx A^{A^A}$ (is third innumerable)]

$C+1, C+2 \ldots\ldots C^C-1, C^C$ [$C^C = D \approx A^{A^{A^A}}$ (is first infinity)]

$D+1, D+2, D+3 \ldots\ldots D^D$ [$D^D = E \approx A^{A^{A^{A^A}}}$ (is second infinity)]

$E+1, E+2 \ldots\ldots E^E$ = [$E^E = F \approx A^{A^{A^{A^{A^A}}}}$ (is third infinity)]

$F+1, F+2, F+3 \ldots\ldots\ldots$ (towards higher infinities).

We can see the striking similarities in the Jaina treatment of A's and I's (where A stands for *asaṅkhyāta* and I for *ananta*), with Cantors infinities of ordinals (ωs).

Dr. A.N. Singh, in his article "*Mathematics of Dhavalā*" in respect of a much later work of 10th C. AD remarks: "In the classification of numbers I see a primitive attempt to evolve a theory of infinite cardinal numbers. But there are some serious defects in the theory. These defects would lead to contradictions. One of these is the assumption of the number c-1, where c is infinite and a limiting number of a class. On the other hand the Jaina conception that *vargita-saṁvargita* (successive square) of a cardinal c, i.e. c^c, would lead to a new number is justifiable. … Any such attempt at such an early stage is bound to be a failure. The wonder is that the attempt was made at all".

4.13.3 Few Infinite Sets

Jaina-scholar *Ācārya*-s have introduced many types of infinities, therefore there are many infinite sets. Some members of such sets of *ananta*, according to *Karmagrantha*, are illustrated in BKS[1] are as below:

1. *abhavya jīva* falls under 4th category i.e. *jaghanya yuktānanta*.
2. The number of liberated souls and the souls, who have fallen from right faith (*samyaktva-braṣṭa*) is *madhyam yuktānanta*. (some saint-scholars are of the view that the liberated souls fall in eighth category i.e. *madhyama anantānanta*)
3. The following 22 types fall under the 8th class of *ananta* i.e. intermediate (*madhyama*) *anantānanta*
   i) Gross developed plant (*bādara aparyāpta vanaspati*)

[1] *Bṛhatkṣetra Samāsa,* Vol. I, p. 11.

ii) Gross developed soul (*bādara parypāta*)

iii) Gross undeveloped plant (*aparyāpta bādara vanaspati*)

iv) Gross undeveloped soul (*bādara aparyāpta*)

v) Gross beings (*bādara jīva*)

vi) Subtle undeveloped plant (*sukṣma aparyāpta vanaspati*)

vii) Subtle undeveloped souls (*sukṣma aparyāpta*)

viii) Subtle undeveloped plants (*sukṣma aparyāpta vanaspati*)

ix) Subtle developed soul (*sukṣma paryāpta jīva*)

x) Subtle souls (*sukṣma jīva*)

xi) Souls having potentiality of liberation (*bhavī*)

xii) *Nigoda* (smallest form of the living organism)

xiii) Plants

xiv) One-sensed being

xv) Animal kingdom (*tiryañca*)

xvi) Perverted-faith souls (*mithyādriṣṭi*)

xvii) Non-restrained souls (*avirati*)

xviii) Souls endowed with passion (*sakaṣāyī*)

xix) Non-omniscient (*chadmastha*) souls

xx) Souls endowed with physical, mental and verbal activities (*sayogī*)

xxi) Mundane (*sansārī*) souls

xxii) All living beings

Besides these there are numerous bigger sets. E.g. it is stated that the set of *pudgala*-s is infinite times the set of *jīva*-s.

4.14 Infinitesimal

In mathematics, **infinitesimals** are things so small that there is no way to measure them. The insight with exploiting infinitesimals was that entities could still retain certain specific properties, such as angle or slope, even though these entities were quantitatively small.

Infinitesimal, in mathematics, a quantity less than any finite quantity yet not zero.[1]

An **infinitesimal** is some quantity that is explicitly non-zero and yet smaller in absolute value than any real quantity. The understanding of infinitesimals was a major roadblock to the acceptance of calculus and its placement on a firm mathematical foundation.

The Greek mathematician Archimedes (c. 287 BC–c. 212 BC), in *The Method of Mechanical Theorems*, was the first to propose a logically rigorous definition of infinitesimals.

As Jaina *Ācārya*-s were dealing with very large numbers in terms of innumerable and infinities, they were also equally concerned with concepts of very small, i.e. infinitesimals. They came across very small size of the living beings and the space occupied by them which is very small part of the existing measure-scale. For example *sūtra* 499 of ADS states as *khettao asaṅkhejjāo saeḍhīo payarassa asaṅkhejaibhāgo…*it means from spatial viewpoint, number of bound transformational (*vaikrya*) bodies of astral gods is equal to number of space-points in innumerable world-lines lying in innumerableth part of world-plane.

In Jaina canonical literature there are plenty of illustrations of living and non-living quantity, quality or size described at ultimate micro level and minute level.

In modern science three basic dimensional units are explained. Similarly the field of infinitesimals mentioned in Jaina canons and post-*āgamic* literatures can broadly be classified under three:

1. Time
2. Length
3. Mass.

1. Time

In the field of time '*samaya*' is the ultimate indivisible unit of time and the largest mentioned unit is *pudgalaparāvartana* but time is even beyond. Time is infinite but '*samaya*' is infinitesimal.

[1] *Encyclopædia Britannica Online*, Encyclopædia Britannica Inc., 2016, Web. 23 Apr. 2016 http://www.britannica.com/topic/infinitesimal.

Its measure is described through two analogies:

i) **Lotus petals :** the minimum unit of time called *samaya* (instant) is inferable from the perforation of a large number of petals of lotus placed one above another in a single stock by one stock of a needle. It appears that all the petals are perforated at one time, but this is by its nature impossible. The successive perforation of the plurality of petals implies successive instants.[1]

ii) **Tearing the old cloth :** youth takes a cotton or silk piece of cloth and very quickly tears a strip of length one hand (*hātha*) from it. The pupil asks: "Is the time taken in tearing the cotton or silk of one *hātha* one *samaya*?" (Master replies): "No! It is not so". (Pupil): "Why?" (Master explains): "A cloth-piece is made by collection, union and confluence of enumerable threads, and unless the upper thread is cut, the lower thread cannot be cut. The upper thread is cut in different *samaya*, than the lower thread and for this reason the operation of tearing cannot be done in one *samaya*". On this explanation from the master, the pupil further asks: "Is then the time taken by the son of darner in separating upper thread one *samaya*?" (Master responds): "No! It is not so". (Pupil): "Why?" (Master again explains): "A thread is collection, union and confluence of enumerable fibres and unless upper fiber is cut, lower fibre cannot be cut. If upper fibre is cut in a particular *samaya*, the lower fibre is cut in different *samaya*".

Let's see the *samaya* in terms of second. From the time-scale mentioned in chapter 3, we find that,

3773 *prāṇa* = 1 *muhūrta* (48 min)

= 48 × 60 sec.

1 *prāṇa* = $\frac{48 \times 60}{3773}$ = .76 sec

According to Ācārya Siddhasenagaṇi (9th C. A.D.) says, in commentary on TSB (4.15)[2]

2^{24} *āvalikā* = 1 *muhūrta* (48 × 60 sec)

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[1] *Jaina Siddhānta Dīpika* of Gaṇādhipati Tulsī, English translation '*Illuminator of Jaina Tenets* by Satkari Mukherjee, ed. by Nathmal Tatia , Muni Mahendra Kumar, Jain Vishva Bharati, Ladnun, 2nd edn., 1995. p.14.

[2] *Sabhāṣyatattvārthādhigama Sūtra*, p. 292.

$\therefore$ 1 *āvalikā* = $\frac{48 \times 60}{2^{24}} = \frac{48 \times 60}{16,777,216} \approx .0001716\ldots$

= 1.7×10^{-4} sec. ($\approx$ 172 micro second, where 1 micro second = 10^{-6} sec, millionth part of the second)

From above we get 3773 *prāṇa* = 2^{24} *āvalikā*

Hence, 1 *prāṇa* = 2^{24}/ 3773 *āvalikā* = $4446\frac{2458}{3773}$

$\approx$4446.65 *āvalikā* (we do not find anywhere in *Āgama* that 2^{24} *āvalikā* constitutes a *muhūrta.* Probably, Malaygiri got this from ancient source. In *Āgama* it is only stated that a *prāṇa* = *saṅkhyāta āvalikā*)

1 *āvalikā* consists of *asaṅkhyāta samaya* (ADS, 10.417) and according to ADS (10.590) the number of *samaya* in *āvalikā* is *yuktāsaṅkhyāta*. Therefore,

1 *samaya* = $\frac{1.7 \times 10^{-4}}{A^A}$ (which is unimaginably small number)

It is so infinitesimally infinitesimal that the smallest time unit possible as per quantum physics, Planck time ($\approx$ 5.4×10^{-44} second), will look like innumerable eons against *samaya*. We have earlier seen while describing analogy measures that the number of hair-tips in the pit is the order of 10^{36} [section 4.3.(i)] *samaya*-s. Malayagiri has remarked very rightly that this time will be even much shorter than even a blink or wink.

It is said that innumerable '*samaya*-s' are gone in just one blink of eye. Through this illustration we can understand the micro level of the time. Although *samaya* is negligibly minute yet, Jaina scholars have given concrete examples of its operation.

Liberated soul arrives at '*siddhaśīlā'* (The permanent abode of the liberated soul) in one *samaya* through straight-line motion (*ṛju gati*); any soul for next-birth takes one *samaya* to reach if the next birth place is aligned with the earlier death-place; takes 2 *samaya* if there is one turn and takes 3 *samaya* if there are 2 turns (*ghumāva*).[1]

The null (*viraha-kāla*)[2] period, when no soul enters and exits from the hell, is 1 *samaya.* The maximum length of the time, when number of various beings (hellish beings, 1-5 sensed, deities etc.) neither grows nor diminishes or is stable in that realm is 1 *samaya* (BS, 5.8.213-220). Inanimate mega-aggregate (*acitta mahāskandha*) takes 4 *samaya* to

[1] Ācārya Mahāprajña, *Jīva Ajīva*, Jain Vishva Bharati, 20th edn., 2013, pp.4-5.

[2] *Samavāo*, p. 179.

spread over the entire *loka* (VB, 639). In omniscient inflation (*kevalī samudghāta*) space-points of soul takes 4 *samaya* for expansion throughout the *loka* and the equal *samaya* to contract in the original size. Jinabhadragaṇī in VB (377) states that the *paramāṇu* of the voice of a powerful speech takes 4 *samaya* to reach the extremities of *loka*.

2. Length

Spatial description is integral part of Jaina scriptures. The maximum measurable dimension mentioned is 14 *rajju* (VB, 428), whereas the smallest unit is *pradeśa* (space-points). Some other small units mentioned are *ą*/A, *ą*/I, etc., where *ą* is an *aṅgula* which we have shown as the smallest practical unit of length and where as per section 4.3,

1 *aṅgula* = 8^9 *utsalakṣaślakṣṇikā*

= 134217728 *utsalakṣaślakṣṇikā*

= 1.9 cm (ADS, 9.390, commentary by Ācārya Mahāprajña)

∴ 1 *utsalakṣaślakṣṇikā* = $\frac{1.9}{134217728}$ = 0.000000014 cm

They did not only coin the concept of infinitesimal but applied at large to explain the various philosophical aspects. Numerous illustrations of small units are found in the canons. Minimum height of any living being with corporeal body (*audārika śarira*) can be *ą*/A (SMA, p.160). Similarly size of grain (*śāli*), and rice (*vrīhi*) [BS, 22.1.1], developed five sensed souls without mind (*paryāpta asañjī pañcendriya*) [BS, 24.12.167], *āsāliyā* (type of worm) [PS, 1.82] and the human born through agglutination (*sammūrchim manuṣya*) [PS, 1.92] is *ą*/A.

Ancient Jaina scholars noticed the subtle growth or change in the cosmic world. The dimension of direction Indrā is increased by 2 *pradeśa* (BS, 13.4.52). Another example found in JJ (*sūtra* 3.170) is the depth of *Lavaṇsamudra* is increased by one *pradeśa* after 95-95 space-points from rampart (*vedikānta*).

3. Mass

The smallest indivisible unit of *pudgala* is *paramāṇu* which is the base of the entire visible world. The word '*paramāṇu'*, occurring frequently in Jaina, Baudha and Hindu scriptures, is generally translated as 'atom' by many scholars. But it is not so. BS (5.7.160) defines *paramāṇu* as one which cannot be bisected further (*anardha*), which has no middle (*amadhya*) and which does not occupy any space (*apradeśa*). As per Jaina

canon it is the ultimate particle of the matter. But when 2, 3, 4…… numerable (*saṅkhyāta*), innumerable (*asaṅkhyāta*) and *ananta paramāṇu*-s combine with each other, they form aggregate (*skandha*) and acquire spatial characteristics (*sapradeśatva*).

Dalton coined the term 'atom' as he thought it is not further divisible. However, with advances in science, scientists went further to fundamental particles, sub-particles and strings.[1]

anantānanta paramāṇu makes one *utsalakṣaślakṣṇikā* (section 4.3). Even if we take *anantānanta* as the lowest of infinities, the scale of *paramāṇu* will be 1/E *aṅgula*, where E is *jaghanya paritānanta.* This will make the smallest entity of modern science, 'string' (10^{-33} cms) or Planck length (10^{-34} cms) look like a monster of gigantic size.

A group of *paramāṇu*-s is called *vargaṇā*. Jinabhadragaṇi Kṣamāśramaṇa in VB explains it in detail. He says that *vargaṇā*-s may comprise of 2,3,4, ……, *saṅkhyāta*, *asaṅkhyāta* and *ananta paramāṇu*-s. The sequence of the *vargaṇā*-s of different kinds as per the increasing order of the number of *paramāṇu* goes as *audārika* (physical body), *vaikriya* (transformational body), *āhāraka* (assimilative body), *taijasa* (fiery-body), speech, respiration, mind and karma-s. These *vargaṇā*-s consist of *ananta paramāṇu*-s each, although the order of *ananta* may be different for these *vargaṇā*-s.

Thus, if we denote the number of *paramāṇu*-s by ω and multiplying factor (ananta) also by ω, then, the number of *paramāṇu*-s in the *varganā*-s of different bodies etc. would go on multiplying by a factor of ω exponentially. Thus, they will form a sequence:

In our notation $\omega^{\omega} = \omega\uparrow\uparrow 2$, $\omega\uparrow\uparrow 3$, $\omega\uparrow\uparrow 4$, $\omega\uparrow\uparrow 5$, $\omega\uparrow\uparrow 6$, $\omega\uparrow\uparrow 7$, $\omega\uparrow\uparrow 8$, $\omega\uparrow\uparrow 9$, $\omega\uparrow\uparrow 10$ are the number of *paramāṇu*-s in the respective *vargaṇā*-s (of *audārika, vaikreyika, āhāraka* etc.). Thus the size of each of the particles belonging to these categories of *vargaṇā*-s would be :

$$\frac{1}{\omega^{\omega}}, \frac{1}{\omega^{\omega^{\omega}}}, \frac{1}{\omega^{\omega^{\omega^{\omega}}}}, \ldots\ldots\ldots\ldots$$

These quantities go on diminishing exponentially. This is just to illustrate how Jaina mathematicians have gone to the utmost depths of infinitesimals as they have gone for utmost heights of infinities.

[1] Shah, R.S., *Jain Mathematics: Lore of Large Number,* Bulletin of the Marathawada Mathematical Society, Vol. 10, No.1, June, 2009, p. 58.

As regards living beings, the number of *paramāṇu*-s in the *vargaṇā* of a physical body (*audārika śarīra*) is least of all but *ananta* in number. This *ananta* is *ananta* times of the number of non-liberated souls (which is also *ananta*), but is one in *ananta* part of number of liberated souls (which again is *ananta*). But the number of *paramāṇu*-s increases exponentially in the succeeding *vargaṇā*-s.

4.15 Conclusion

The Jaina-s were not interested only in the enumeration of very large numbers, but dared to think beyond them by developing a very robust theory of infinities, which was intimately tied up with their philosophy of time and space. The world of transfinite numbers in Jaina philosophy holds significant place because most of the philosophical ideas and theories are explained through transfinite numbers.

For illustration, the intensity (*anubhāga*) of karma which is a qualitative aspect is also described in terms of its ultimate units (*avibhāga praticcheda*). There are innumerable stages in which the bondage of karma may take place and the number of stages is described as innumerable. It's further specified that this innumerable is equivalent to space points in innumerable universes (*asaṅkhyāta loka*). Further in each stage there will be infinite ultimate units of intensity of bondage of karma and it is specified that this infinite is equivalent to infinite times of all the living beings (*sarva jīva*) in the universe [SK, *Dhavalā*, book 12, pp.91-111]. Similarly, while discussing about *nigoda* (the primordial form of living beings) the *Ācārya*-s discussed quite mathematically.

What is striking is that about 2000 years back the Jaina saint-scholars could envision ideas on innumerables and infinities. What is more significant is that they imagined multitude of infinities. There recursive processes in arriving at these remind us of Cantor's enunciation of hierarchy of infinities – both cardinal and ordinal. In my explanation I have given many calculations which were probably beyond their competence then, but underlying processes of infinite sets and their infinite subsets was clearly laid out by them.

Chapter-5

Number Theory (3):
A Mathematical Solution of Metaphysical Problems

Mathematics is the only good metaphysics.

~ William Thomson Baron Kelvin of Largs ~

Number theory is the branch of mathematics concerned with properties of the numbers. Sometimes called "higher arithmetic", it is among the oldest and most natural of mathematical pursuits. Until the mid-20th century, number theory was considered the purest branch of mathematics, with no direct applications to the real world. The advent of digital computers and digital communications revealed that number theory could provide unexpected answers to real-world problems and it has applications in many branches of science.

The main goal of number theory is to discover interesting and unexpected relationships between different sorts of numbers and to prove that these relationships are true. It provides the solutions by playing with the numbers in various patterns. There may be multiples of questions like can we know the unknown if only few things are known? Can we know the things accurately? Is it possible to arrange the things in many ways, so that one can pick the arrangement of his choice? If something is not stable, keeps varying, then how can one know its status or result? Many of such problems gave rise to various operations of number theory, which was applicable in far past also. Therefore, we can say that **Number Theory** is eternal solution of the problems through its numbers. In previous chapters, number system, numbers at both extrema and many other such topics available in original Jain sources were introduced. Jain saint-scholars because of their requirements of the spiritual practices have always played with numbers and tried to reach the utmost depth of number theory. This chapter will focus on various patterns of number theory found in *Āgama* and how did number theory provide solutions to the philosophical queries.

In the pursuit of very large and very small the Jain saint-scholars made inroads into many other fields of mathematics, viz., indexation, binomial coefficients, exponentiation, combinatorics, divergent dyadic series, etc.

5.1 Rule of Three

'*Rule of Three*' is regarded as another important gift by India to the world of mathematics. It is such simple concept but was not known to the world for the long time. The *Rule of Three* is based on the technique that people tend to remember three things. In the history of transmission of mathematical ideas, the *Rule of Three* forms an interesting case. In simple it can be understood as 'If 10 men can dig a trench in 4 days, how long will 7 men take to dig a similar trench?'

A perfect illustration found in JDP (4.132)[1] BKS (*sūtra* 304-305)[2] verifies the use of *Rule of Three* in ancient Jain canonical texts. Gaṇadhara Gautama asked, 'O Lord! Where is the Mt. Meru located in *Mahāvideha Kṣetra*? The answer is an illustration of *Rule of Three*.

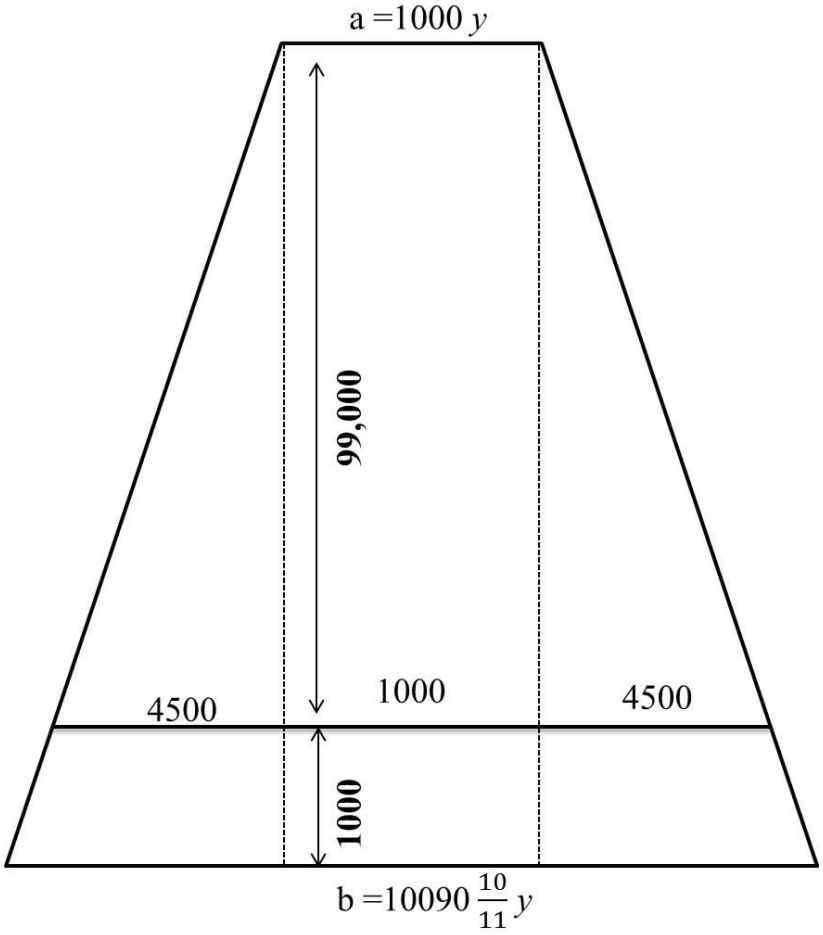

Fig 5.1: Mt. Meru

[1] *Jambūdvīpaprajñapti Sūtra*, p. 250.

[2] *Bṛhatkṣetra Samāsa*, p. 285.

According to Jain cosmography JD, is disc-shaped, with diameter 10,0000 *y*, thickness 1000 *y* , and at the centre of it is situated royal Mt. Meru of the shape of a frustum of cone. The height of Meru is 99000 *y* above the ground and penetrating 1000 *y* deep in the ground. Its diameter at the bottom on the ground is 10,000 *y* and keeps decreasing while moving up and remains 1000 *y* at top (see fig 5.1). Jinabhadragaṇi in BKS remarks that as we descend from top to bottom, the diameter will go on increasing at 1/11 of the descent.

Under the ground its width is $10090\frac{10}{11}$ and at ground level width is 10,000 *y* and at the top is 1000 *y*. So the increase in width from top to the ground level at the height of 99,000 *y* is

$$10000 - 1000 = 9000$$

Therefore, increase in width per *yojana* from the Rule of Three is

$$\frac{10000-1000}{99000} = \frac{1}{11}\ y$$

It can be concluded that as we descend from top to bottom, the diameter will go on increasing at $1/11^{th}$ of the descent.

Similarly, as we ascend from ground to upwards the diameter will go on decreasing by $1/11^{th}$ of ascent and is utilized to calculate the diameter of Mt. Meru in the foundation at depth 1000 *y*. It is thus,

$$1000 + \frac{100000}{11} = 1000 + 9090\,\frac{10}{11} = 11090\,\frac{10}{11}$$

or, alternatively, while going from bottom to top, the diameter at top at the height of

$$100000\ y = 10090\,\frac{10}{11} - \frac{100000}{11} = 1000\ y.$$

There are a number of mountains in Jain cosmography those are either of the shape of frustum of a cone or trapezoidal pyramid. Their diameters or widths and circumferences or perimeters at different heights are calculated by using rule of three as above. Similarly, the diameter of various hells and heavens are worked out on this basis.

ii) *Gautamadvīpa* (GD): Another example is quoted here from JJ[1] (3.161) and BKS (2.41). To get the height of GD, we need to understand the *Lavaṇasamudra* (LS). It can be seen in the fig 5.2 that this sea is a gigantic trench with vertical sections like 2 trapezoids kept one

[1] *Jīvājīvābhigama Sūtra*, Vol. II, p. 16.

above the other with their common faces of width 200000 *y* joined (as the diameter of LS is 200000*y* double of JD). The lower trapezoid has the depth of sea from the grounds of JD and *Dhātakīkhaṇḍa* (DK), which is gradually increasing to 1000 *y* in the span of 95000 *y* and height of waters above the grounds of the islands increasing to 700 *y* in the same span.

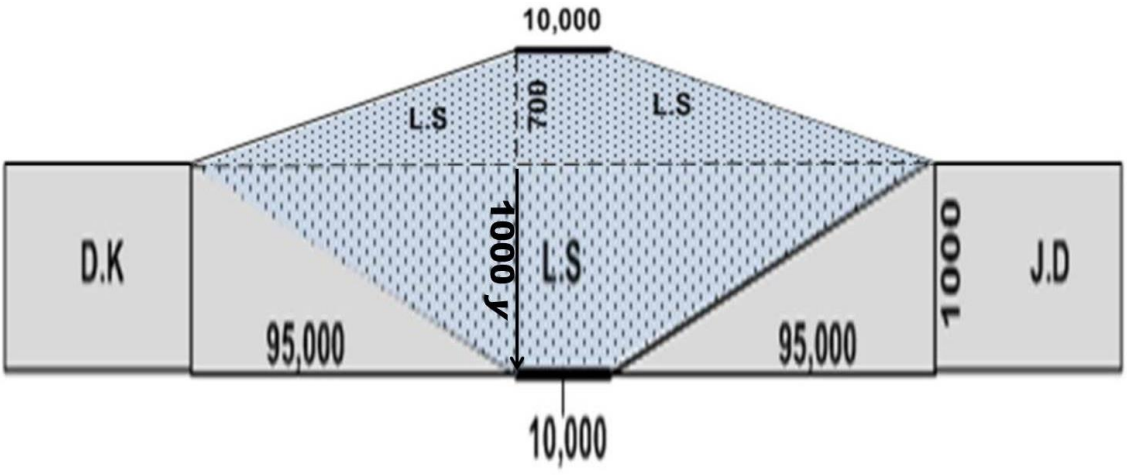

Fig 5.2: Lavaṇa Samudra

The increase in depth in 1 *y* is 1000/95000 (from Rule of Three) i.e. 1/95 *y*

The increase in height in 1 *y* is 700/95000 (from Rule of Three) i.e. 7/950 *y*

The land-front at the bottom and water-front at top is 10000 *y* each. BKS rightly remarks that rate of increase of depth (below ground-level) is 1/95 and rate of rise of waters (above ground-level) is 7/950 (by applying rule of three). He also states that the increment stated above are at the rate of innumerable-th part of an *aṅgula*, which gives clue of infinitesimal.

We can observe the application of rule of three in the measurements pertaining to Gautama Island (GD) in (BKS, *gāṭhā* 2.39-42).[1] (See figure 5.3). After entering LS horizontally for 12000 *y* from the edge of JD, there is GD which rises 2 *kośa* (*k*) (= ½ *y*) above the sea waters. Its horizontal span is 12000 *y*. Jinabhadragaṇi then calculates the height of GD above the ground-level of JD as:

Since the height of water is increased 700 *y* at the distance of 95000 *y* from JD, therefore increase in sea-level after 12000 *y* from JD will be

$$12000\times\frac{700}{95000}\,y$$

[1] *Bṛhatkṣetra Samāsa*, Vol. II, pp. 138-139.

$$= 88\frac{40}{95}y$$

Initially and after another 12000 *y* horizontal distance the rise is double, i.e. $176\frac{80}{95}$. JJ gives the value of the height of GD under LS as $88\frac{1}{2} \approx 88\frac{40}{95}y$ calculated by Rule of Three above.

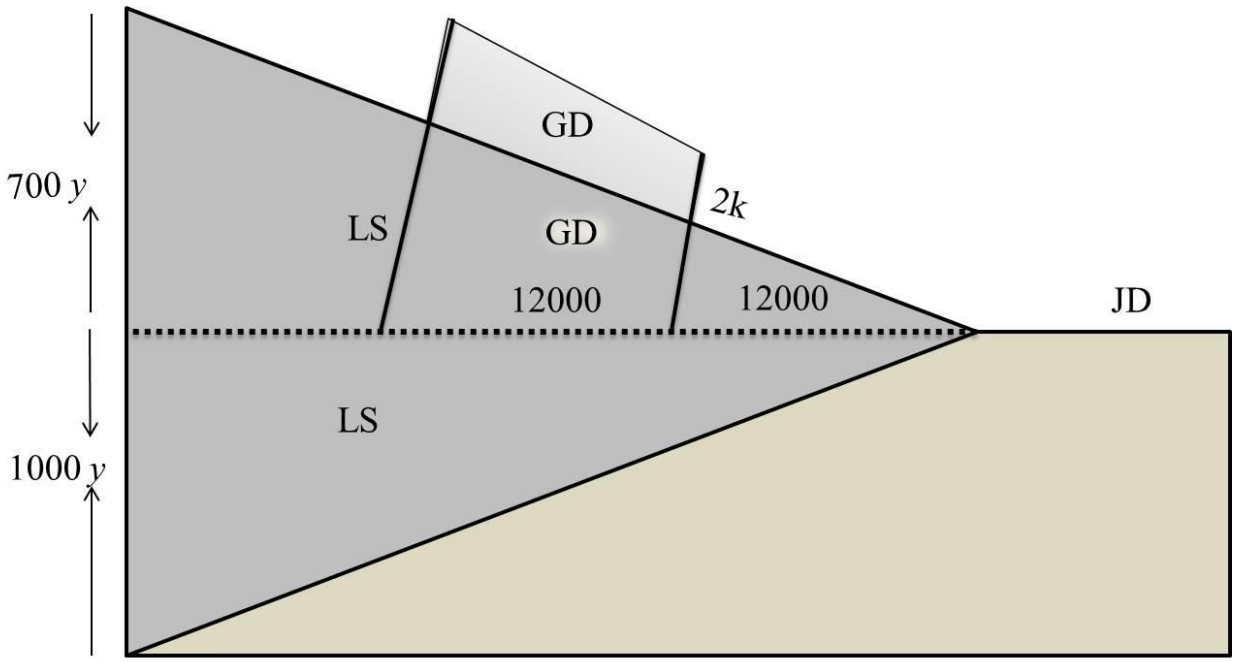

Fig 5.3:Gautam Dvīpa

5.2 Fraction

Jain *ācārya*-s were interested in knowing the dimension of the natural entities and the structure of the world they live in or can go in next lives. Therefore, perhaps Jain canons are full of geography of mountains, oceans, rivers, celestial air-ships, suns and moons etc. This curiosity led them towards minutest fractional values to achieve the accuracy. Numerous instances of total familiarity with operations involving fractions are found in *Āgama*.

a) The simple example is the length, width and circumference of the airship of the sun mentioned in JDP (7.163),[1] i.e. the length is 48/61 *y* (*aḍayālīsaṁ egsaṭṭhibhāe joaṇassa*), circumference (approximately) three times of the length (*taṁ tiguṇaṁ*) is $2\frac{22}{61}$ *y* and width is 24/61 *y* (*cauvīsaṁ egsaṭṭhibhāe joaṇassa bāhalleṇaṁ*), [i.e. $3\times\frac{48}{61} = \frac{144}{61}$ $= 2\frac{22}{61}y$]

[1] *Jambūdvīpaprajñapti Sūtra*, p. 321.

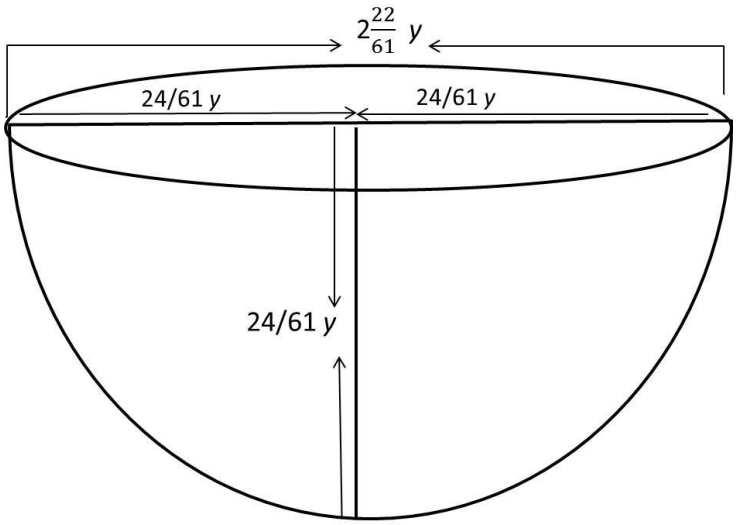

Fig 5.4: Sun's airship

Another example found for simple fraction is in chapter 2, *sūtra* 211 in *gāthā* 166 of PS.[1] While explaining the height of the liberated soul it says that the height of the liberated soul is equivalent to the 2/3rd (*tribhāgahīna*) of the height of the existing human body at that moment. The maximum height of human being can be 500 *dhanuṣa.* Therefore maximum height of liberated soul can be 333 $\frac{1}{3}$ (500× $\frac{2}{3}$) *dhanuṣa*. The medium height of liberated soul can be 4 $\frac{2}{3}$ *hātha* and the minimum can be 1 *hātha* and 8 *ą*. There are numerous such examples found in this text.

b) In *āgamic* literatures fractions were tackled very proficiently. In JDP north-south spans of continents and continental mountains involving fractions with denominator 19 are properly determined. Not only this, but chords of the continents and continental mountains as square root of quantities are fractions with denominator 19 are extracted with numbers as fractions with denominator 19 to a very close degree of approximation. e.g. northern chord of Mahāhimavanta =

$$2\sqrt{\frac{150000}{19}\ (100000\ -\ \frac{150000}{19})\}} = 53931.3198\ y.$$

The value in *sūtra* is $53931\frac{6}{19}$ = 53931.3158 (JDP, 4.96),[2] which is almost same. Sun can be seen at the distance of $47263\frac{21}{60}$ *y* in Bharat continent when it travels at the rate

[1] *Prajñāpanā Sūtra*, Vol. I, p. 191.

[2] *Jambūdvīpaprajñapti Sūtra*, p. 196.

of $5251\frac{29}{60}$ *y* in 1 *muhūrta* (JDP, 7.166).[1] Furthermore it is mentioned that while moving from innermost orbit to the second innermost orbit then it is seen at the distance of $47179\frac{57}{60} + \frac{19}{60\times61}$ *y*.

Chapter 7 of JDP also shows how the fractions with denominators 60, 61, 67 and 61 × 60 are successfully handled. E.g. it is stated that span of each of 183 lanes in which sun moves is $= \frac{510}{183} = 2\frac{48}{61}$ *y* where it can be seen that the fractions are simplified to lowest denominator [JDP, 7.164].

They have evaluated the complicated fractional computations also. For example, *sūtra* 7.180[2] mentions that the length of the second external orbit of moon is $100587 + \frac{9}{61} + \frac{6}{61\times7}$ *y*. They have very proficiently obtained this value through operations on fractions.

Diameter of the outermost orbit is 100660 *y* and moon crosses one circular orbit after another traversing an unobstructed distance of $\frac{15551}{427}$ *y*.

Therefore, diameter of the 2nd outermost orbit is,

$$100660 - 2\times\frac{15551}{427}$$

$$= 100660 - 72\frac{358}{427}$$

$$= 100588 - \frac{358}{427}$$

$$= 100587 + \frac{69}{427}$$

$$= 100581 + \frac{9}{61} + \frac{6}{61\times7}y$$

They have even worked with numerators and denominators having large numbers. Moon while exiting from the third inner orbit travels *pañca joaṇasahassāiṁ asīiṁ ca joaṇāiṁ egūṇavīse bhāgasae*[3]...i.e. $5080\frac{13319}{13725}$ *y* in 48 minutes (JDP, 7.181). This

[1] *Jambūdvīpaprajñapti Sūtra*, p. 326.

[2] *Ibid*, p. 344.

[3] *Ibid*, p. 346.

figure cannot be further simplified as the numerator (19×701) and denominator (3×3×25×61) are relatively prime to each other. To obtain this result they might have calculated such a big fractional number. There are plenty of such calculations of sun and moon and BS, SCP, JK, TS, SAM etc.

In SCP there are many references of fraction in astronomical calculations. In chapter (*prābhṛta*) 12 *muhūrta* (48 minutes) in various types of months are calculated. For example, number of *muhūrta*-s in moon month is $885\frac{30}{62}$ (*aṭṭhapañcasae muhuttae tettisaṁ ca bāsaṭṭhi bhāgā* , SCP 12.72.2(a). How this output is calculated? Then the author says,

tā egūṇatīsaṁ rāindiyāiṁ battīsaṁ bāsaṭṭhibhāgā rāindiyassa rāindiyagge ṇaṁ āhie tti vaejjā[1] (12.2b) i.e.

There are 1830 days and 62 moon month in a *yuga* (5 years). And one day and night has 30 *muhūrta*.

62 moon months = 1830 days

$\therefore$ 1 moon month = $\frac{1830}{62} = 29\frac{32}{62}$ days

30 *muhūrta* =1 day

Therefore the number of *muhūrta*-s in one moon month = $\frac{1830}{62} \times 30 = 885\frac{30}{62}$ *muhūrta*

c) Simplification of fractions by cancelling common factor from numerator and denominator was often followed. Here are few examples,

(i) Jinabhadragaṇi arrives at square-root of 58,545,048,750 as $241{,}960\frac{407150}{483920}$ while calculating the arc of north Bharat. He then states expressly that this reduces by cancelling zero from numerator and denominator to $241960\frac{40715}{48392}$ (*sunnamavaṇeu aṅsaṁ*) (BKS, *gāthā* 1.83).[2] [Note that 40175 (= 5×17×479) and 48392

[1] *Sūryaprajñapti Sūtra,* p. 144.

[2] *Bṛhatkṣetra Samāsa*, Vol. I, p. 108.

(= 2^3×23×263) are relatively prime). It shows they were aware that after that it cannot be further simplified.

(ii) In *gāthā* 1.88 (p.144), calculating the arc of Mt. Kṣullahimavanta, square-root of 150,000,000,000 is derived as $387298\frac{259196}{774596}$ and is simplified as $387298\frac{64799}{193649}$ by expressly stating that this is arrived at by dividing the numerator and denominator by 4 (*caukka uvaṭṭhie*). (Here the numbers 64799 and 193649 are relatively prime as 64799 = 7 × 9257 and 193649 is prime).

(iii) Chord of Mt. Vaitāḍhya is the square-root of 37,899,097,500 and is determined as $194676\frac{352524}{389352}$, which is further simplified by cancelling the factor 12 from numerator and denominator (*du cakko uvaṭṭhie*) to $194676\frac{29377}{32446}$. (BKS, *gāthā* 1.73,74, p.102). (Again, 29377 (=29×1013) and 32446 (=2×16223) are relatively prime).

Volume of Mt. Mahāhimavāna is arrived at as $883855\frac{338975}{1767710}$. Jinabhadragaṇi then says that the fractional part can be further simplified by dividing both the numerator and denominator by 35 (*paṇatīso vaṭṭhīe*), to $883855\frac{9685}{50506}$. He expresses this number as '*paṇapannā aṭṭhasayā tesisahassa aṭṭhalakkhā, ya*.....(BKS 1.100)

i.e. fifty five and eight hundred, eighty three thousand, eight lakh (to be read from right to left as 883855)

nava cha aḍapaṇa cheo, pañca ya sunna paṇa sunna chakko ya[1] .....(BKS 1.101)

i.e. in numerator nine, six, eight, five (=9685) and in denominator five, zero, five, zero, and six (=50506)'. This is a unique example. Here the integral part of number is stated in place-value manner but from right to left. However, the numerator and denominator of fractional part are recited by giving only digits from left to right. Further, 9685 (=5×13×149) and 50506 (=2×25253) are relatively prime. i.e. Jinabhadragaṇi knew that there are no further common prime factors in these 2 numbers. (BKS, g. 1.100-102).

[1] *Bṛhatkṣetra Samāsa*, p. 119.

(ii) Some special words were coined for certain fractions and like that are in use even today. E.g. $1\frac{1}{2}$ was mentioned as '*divaddhaṁ'* (two less half), $2\frac{1}{2}$ as '*aḍhāi'* or '*duaddha*' (two and half), $3\frac{1}{2}$ as '*addhuttha*. Higher numbers plus half were represented as: $15\frac{1}{2}$ is read as '*addhasolasa*' (sixteen less half) or $55\frac{1}{2}$ as '*addhachappanna*' (fifty-six less half). (BKS, *gāṭhā* 1.36).

(iv)(A) The shaded area of the figure 5.5 is the shape (segment of a circle) of Southern Bharata sub-continent of our earth JD with chord AB = c and height = h = $238\frac{3}{19}$ *y*.

Jinabhadragaṇi knew and used the formula $c^2=4h\ (d-h)$, where d = 100000 *y* is diameter of JD. To calculate c he takes 1/19 *y*=1 *kalā* , converts all units in *kalā* s by taking,

h = 238 *y*+ 3 *kalā* = 4525 *kalā* , d = 100000 *y* = 1900000 *kalā* and arrives at $\sqrt{4 \times 4525(1900000-4525)} = \sqrt{34308097500} = 185224\frac{167324}{370448}$ *kalā* .

Again he converts this in *yojana* s by dividing this figure by 19 and ignoring the fractional part as an approximation to arrive at the chord of Southern Bharata as 9748 $\frac{12}{19}$ *y* or 9748 *y* and 12 *kalā*.[1] (BKS, *gāṭhā* 1.37).

He even quotes the general formula for this:

jīvāvaggaṁ usuṇā, caurabbhattheṇa vibhaya jaṁ laddhaṁ

taṁ ususahiyaṁ jāṇasu, niyamā vaṭṭassa vikkhaṁbhaṁ.[2] (BKS, *gāṭhā* 1.38).

i.e. Take four times product of arrow (*h*) and excess of diameter over arrow (*d-h*). This is equal to square of the chord (c) of the circle:

$c^2 = 4h\ (d-h)$ (as shown in the fig. 5.5)

[1] *Bṛhatkṣetra Samāsa*, Vol. I, p. 70.

[2] *Ibid*, p. 70.

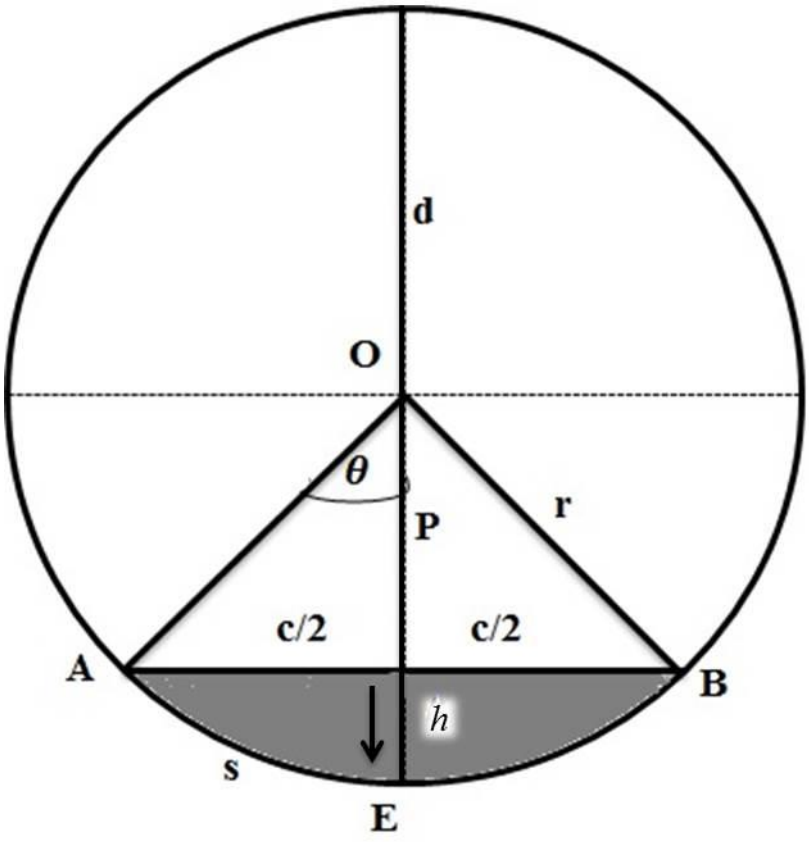

Fig 5.5: Bharat Continent

(B) In calculation of areas in cases like above, division by 361 will be required. This Jinabhadragaṇi accomplishes by a clever device. He calculates area of Mt. Himavanta as $7745960000\ \frac{1295960000}{193549}$ (*kalā*)^2. To get the area in y^2 he divides the above expression by 361 and obtains $\frac{7745966692\ \frac{80892}{193549}}{361}$ 361

$$\approx \frac{7745966692}{361} + \frac{\frac{80892}{193549}}{361}$$

$$\approx \frac{7745966692}{361} \text{ (by ignoring the } \frac{\frac{80892}{193549}}{361} \text{)}$$

$$\approx 21456971\frac{162}{361}$$

$$= 21456971\frac{8\frac{10}{19}}{19}$$

= 21456971+$\frac{8}{19}$ +$\frac{10}{19^2}$ y^2 and writes it in the form 21456971 *y*, 8 *kalā*, and 19th part of 10 *kalā*-s. Commentator Malayagiri uses the word *vikalā* for 1/19th part of *kalā* . (Jinabhadragaṇi spells this number as:

…igasattari navasaya chappanna sahassa caudassa ya (BKS, 1.91)

lakkhā du koḍi aṭṭha ya kalā u dase auṇavīsa bhagā[1]*…* (BKS, 1.92)

i.e. seventy one, nine hundred, fifty-six thousand, fourteen lakh, two crore, and 8 *kalā* exceeded by 19th part of 10 *kalā*. (Jinabhadragaṇi writes only *y* (*yojana*) and *kalā* for units of area y^2 ($kalā^2$).

From the discussion above it is clear that Jain saint scholars would tackle large numbers and fractions in arithmetical operations with great ease. *There is no mention of greatest common factor (GCF) or least common multiple (LCM) in BKS. However, as these are tackled by Āryabhaṭṭa who was almost a contemporary of Jinabhadragaṇi, we can assume that Jinabhadragaṇi was conversant with the same also*.

5.3 Extraction of Square Root

BS, JDP, ADS, JJ etc. are full of geographical details of six east-west mountains ranges dividing JD in 7 continents and the circumference of JD, chords, span of mountain ranges and arcual length of continents and mountain are worked out which involve extraction of square roots of very large numbers. They have worked out the dimensions with such fine accuracy that it leaves no doubt that they were fully conversant with procedure of extraction of square roots almost similar to what we employ today.

This procedure must have been in vogue some centuries before Jinabhadragaṇi since many *āgamic* works give the circumference of JD as 316227 y 3 *kośa* 128 *dhanuṣa* 13 ½+ *aṇgula*.

C = $\sqrt{10}$ d = $\sqrt{10d^2}$ = as it is clear that d = 100000 *y*

C = $\sqrt{10 \times 10^{10}} = \sqrt{10^{11}}$

[1] *Bṛhatkṣetra Samāsa*, Vol. I, p. 114.

Table 5.1: Sq. root of 10000000000

3	10000000000
3	9
61	100
1	61
626	3900
6	3756
6322	14400
2	12644
63242	175600
2	126484
632447	4911600
7	4427129
632454	484471

Hence, C = 316227 *y* $\frac{484471}{632454}$ *y*

Simplifying the fraction,

$$\frac{484471}{632454} \times 4 = \frac{1937884}{632454}$$

= 3 *kośa* $\frac{40522}{632454} \times 2000$ *dhanuṣa* = $\frac{81044000}{632454}$ = 128 *dhanuṣa* $\frac{89888}{632454} \times 96$ *ą*

=13 *ą* $\frac{407346}{632454}$ (where $\frac{407346}{632454}$ >1/2)

Hence the circumference of JD is 316227 *y*, 3 *kośa*, 128 *dhanuṣa*, $13\frac{1}{2}$ *ą* and little more which is exactly mentioned in the text.

BKS is full of calculations involving extraction of square-roots usually of large numbers. The work does not indicate any procedure for the same, but some *gāthā*-s give a clear indication of the procedure followed. Commentator Malayagiri, however, describes the procedure in detail. The procedure is similar to the modern one except a novel feature that the digits at every step need not be written or marked at each step. As an example, let us see as to how he arrives at sq.-rt. of 37899097500 (which is the mean of squares of northern and southern chords of Mt. *Vaitāḍhya* **.**

Table 5.2: Sq. rt. of 37899097500

1	3,78,99,09,7500
1	1
29	278
9	261
384	1799
4	1536
3886	26309
6	23316
38927	299375
7	272489
389346	2688600
6	2336076
389352	352524

Ācārya Malayagiri explains that the last divisor 389352 should be halved to give the root will be $194676\frac{352524}{389352} = 194676\frac{29377}{32446}$, Malayagiri quotes an ancient *gāthā* for halving the last divisor and Jinabhadragaṇi actually gives the numerator and denominator of the fractional part[1] :

……..donavatigasattasattaṅsā (numerator, BKS, 1.74)

..tiga duga cau cau, chakkā……(denominator, BKS, 1.75)

[1] *Bṛhatkṣetra Samāsa*, Vol. I, p. 102.

In Chapter 1 and 4 of JDP chords c of various continents and continental mountains are calculated by using the formula $c^2 = 4h (d-h)$ which involves extraction of square-roots. Also, as we have already seen above, even circumference C of JD is determined by extracting square-root of 10^{11}. Though in *Āgama* we do not find any description or hint for procedure of the extraction of square root, yet the figures they have given for various entities is not possible without applying the method of extracting the square root. Later, BKS gives the hint of it (5/6th C. A.D.) and in commentary by Malayagiri (12th C. A.D.).

Jinabhadragaṇi sometimes performs a trick for extraction of square roots of some large quantities with fractional parts. We have already mentioned in section [5.3(c) (iii) (B)] as to how he converts *yojana* in smaller units of *kalā* and extracts its square root and again the answer is obtained in the unit of *yojana*-s.

5.4 Combinatorics

Jain logic (*nyāya*) system is based on the foundation of two great tenets: seven-fold statements (*saptabhaṅgī)* and theory of conditional dialect (*syādvāda*). According to this, an object possesses an infinite number of modes (*paryāya*) and attributes (*guṇa*). Each attribute conveys different meaning of the object. Same object can be perceived by many persons from various aspects. If they all accept only themselves as right perceiver, then the truth is paralyzed. Consequently, the conflict starts taking shape. Thus the multi-dimensional nature of an object needs the concept of *anekānta* (theory of multi-dimensional truth) to explain it through multi-viewpoints.

i) *Saptabhaṅgī* is a unique principle to analyze and articulate any entity or event. It tries to explain any object in many possible ways. Basically there can be three types of statements – *is* (*asti*), *isn't* (*nāsti*), and *inexpressible* (*avaktavya*). By exploring the combinations of these three according to law of permutation and combination, seven combinations are possible. Therefore, there are seven statements in *saptabhaṅgī*.

$$\binom{3}{1} + \binom{3}{2} + \binom{3}{3} = 7.$$

Technically, the seven statements are :

1. *syāt asti* – It exists.
2. *syāt nāsti* – It does not exist.

3. *syāt avaktavya* – It is inexpressible.
4. *syāt asti, syāt nāsti* - It exists and it does not exist.
5. *syāt asti, syāt avaktavya* - It exists, and is inexpressible
6. *syāt nāsti, syāt avaktavya* - It does not exist and is inexpressible.
7. *syāt asti, syāt nāsti, syāt avaktavaya* - It exists, it does not exist and is inexpressible.

Here the word *syāt* is used in the meaning of 'from some perspective' which helps to keep the relativity of the statement. Exposition of how a thing can be done or described gave rise to permutations (*vikalpa*) and combinations (*bhaṅga*) which is known as *bhaṅga samutkīrtana*. In other words it can be said that the concept of combinatorics can be said as an outcome of the principle of *syādvāda*, which accepts the existence of multiple views and aspects of any object.

Sometimes seemingly contradictory aspects, may be viewed differently by different viewers and therefore may become cause of friction and discord. In fact, the principle of *syādvāda* or *saptabhaṇgī* is a corollary of the supreme principle *ahiṁsā* of Jainism which preaches non-injury through mind, speech or body (*mana, vaca, kāya*).

5.4.1 Permutation and Combination

Jain canons, especially BS, ADS etc. are replete with examples of permutations and combinations indicating the importance they gave to multitudinous aspects of various entities –physical and metaphysical. Thus multiplicity gave rise to multiple combination or choices. Permutations (*vikalpa*) and combinations (*bhaṅga*) are various ways to express the different possibilities. AS per modern mathematics these terms can be defined as:

Permutation: Permutation means arrangement of things. The word 'arrangement' is used, if the order of things is considered.

a) If *n* things are arranged in different ways, the total number of arrangements

$$n! = n\,(n\text{-}1)\,(n\text{-}2)\ldots\ldots 2.1$$

The basic formula for permutation is

b) Number of ways *r* things out of *n* can be arranged is given as

$$^{n}p_r = \frac{n!}{r!}$$

Numerous examples are available in many Jain canonical texts especially BS and ADS.

ii) In how many ways one can chant the names of 24 *tīrthaṅkara*? ADS (6.227-229)[1] uses three words *pūrvānupūrvī* (serial ordering), *paścānupūrvī* (reverse ordering) and *anānupūrvī* (all the arrangements excluding the other two).

ADS introduces the process to obtain *anānupūrvī* as: beginning with 1 form a sequence gradually increasing by 1 to obtain 24 terms (*gaccha*). Multiply mutually the terms of this sequence. The number so obtained reduced by 2 (serial ordering and reverse ordering) is the number of residual orderings i.e. 24! -2

Therefore, total possible arrangements to pray the name of 24 *tīrthaṅkara* can be :

pūrvānupūrvī + *paścānupūrvī* + *anānupūrvī*

24! = 620448401733239439360000

iii) ADS (5.179)[2] through examples give permutation of many numbers. e.g. for three types of *loka* then possible arrangement for *anānupūrvī* will be $3!-2 = 4$

iv) For 10 heavens it will be $10!-2 = 3628800 -2 = 3628798$. (ADS, 5.191)[3]

v) ADS, *sūtra* 150[4] says that the number of all possible arrangements of six substances (*dravya*-s) excluding the first (serial) and the last (reverse) is 718. This is 6! – 2 = 718. It gives the formula: *"eyāye chev egāiyāe eguttariyāe chhagachhagayāe seḍhiye aṇṇamaṇabhāso dūrūvūṇo*" i.e. it is the number obtained by mutual multiplication (*aṇṇamaṇṇabbhāso*) of the members of the series (*seḍhī*) less two i.e.

$(6\times5\times4\times3\times2\times1)-2$ (substances, basic elements of the universe, are six in number).

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[1] *Anuogadārāiṁ*, p. 143-144.

[2] *Ibid*, pp. 124-125.

[3] *Ibid*, p. 126-127.

[4] *Ibid*, p. 94.

Combination: Combination means selection of things. The word selection is used, when the order of things has no importance.

Plenty of examples are found where permutation and combination are applied to explain the numerous philosophical concepts about matter and living beings. While explaining about non-living, *pudgala* the idea of permutations (*vikalpa*) and combinations (*bhaṅga)* in chapter 10 (PS) shows that this notion was quite known to the author.

Few illustrations are being quoted here to show the combination. As per modern algebra if *r* things to be selected out of *n* things, then number of ways it can be done is :

$$^{n}C_r \text{ or } \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$= \frac{n\,(n-1)(n-2)\ldots(n-r+1)}{r\,(r-1)(r-2)\ldots 3.2.1}$$

It is clear that Jain scholars were aware of the formula :

$$\binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n-1} + \binom{n}{n} = 2^n - 1.$$

i) In *sūtra*-s 292-297,[1] while dealing with the number of ways in which the interplay of 5 dispositions (*bhāva*) will take place, when taken 2, 3, 4 and 5 at a time is given as 26. ADS lists all these combinations, further classifying them as 10 *dvik-saṅyogī* (taken 2 out of five), 10 *trik- saṅyogī* (taken 3 out of 5), 5 *chatuṣka-saṅyogī* (4 out of 5), and 1 *pañcaka-saṅyogī* (5 out of 5). This is equivalent to:

$$\binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5} = 10 + 10 + 5 + 1 = 26.$$

In fact, $\binom{5}{1} + \binom{5}{2} + \ldots + \binom{5}{5} = 5 + 26 = 31 = 2^5 - 1$

ii) While discussing as to how many combinations (*bhaṅga*) can be formed out of 4 passions when the soul is affected by 1,2,3,or 4 passions, the number given in is 15 (2^4-1). BS (1.5.245-247)

$$\text{i.e. } \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 4 + 6 + 4 + 1 = 15 = 2^4 - 1$$

iii) While discussing assimilation of food by the body, it considers 6 aspects: food which was assimilated, being assimilated, will be assimilated and its 3 negative

[1] *Anuogadārāiṁ*, pp. 169-175.

counterparts. Lists numbers of combinations by selecting 1, 2, 3, .. 6 options out of 6 as 6, 15, 20, 15, 6 and 1 and states that total number is 63, i.e.

$$\binom{6}{1} + \binom{6}{2} + \ldots + \binom{6}{6} = 6 + 15 + 20 + 15 + 6 + 1$$

$$= 2^6 - 1 = 63.$$

iv) It is clear that Jain mathematicians were aware of modern formula of

$$\binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \ldots + \binom{n}{n-1} + \binom{n}{n} = 2^n - 1.$$

It is in fact, quoted in SK as *saṅgrahaṇī sūtra*.

Śaṭakhaṅḍāgama[1] attempts to calculate the total number of words (*pada*) in canon by reasoning in this manner: There are 64 alphabets, symbols etc. in the language of *Āgama* and all the words that can be formed out of these can be obtained by considering all the combinations out of these 64 alphabets of 1, 2, ……, 64 letters and after giving the formula for $\binom{n}{r}$ is shown as equivalent to:

$$\binom{64}{1} + \binom{64}{2} + \binom{64}{3} + \binom{64}{4} + \ldots\ldots\ldots\ldots\ldots\ldots + \binom{64}{64}$$

$$= 2^{64} - 1 = 18,446,744,073,709,551,615$$

It means maximum vocabulary or number of words in Prākṛta dictionary will be the aforementioned number. How big the number is indicated in SK.

Number of words with one letter out of 64 can be formed as

$$\binom{64}{1} = 64 \text{ words}$$

Maximum possible number of words having two letters :

$\binom{64}{2} = \frac{64 \times 63}{2 \times 1}$ = 2016 etc. [11, part 9, p. 194] Vīrasena (9th C. A.D.) and later Nemicandra (10th C.A.D.) and Abhayadeva (11th C.A.D.) give the formula for :

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$= \frac{n\,(n-1)(n-2)\ldots(n-r+1)}{r\,(r-1)(r-2)\ldots 3.2.1}$$

[1] *Śaṭakhaṇḍāgama* with *Dhavalā* commentary, ed. by Jain, Hiralal, Jain Sahityodhara samiti, Rajkot, Book-13, 1968, pp. 248, 255-258.

5.4.2 Advanced Probability Problem

In BS we find the advanced problem of probability, where various categories of the things produce various combinations of the things. Suppose number of things of different categories are n_1, n_2, n_3, n_4, n_5, n_6,…..n_m (where m are positive integers), then total number of possibilities will be :

$$n_1.n_2.n_3.n_4.n_5.n_6 \ldots . n_m$$

BS (20.5.26)[1] illustrates such problems interestingly regarding atom (*paramāṇu*) and its aggregates (*skandha*). According to Jain philosophy there are four attributes of an atom or aggregates viz. colour (*varṇa*), odour (*gandha*), taste (*rasa*), and touch (*sparśa*). The maximum number of each respectively can be 5, 2, 5, and 8. The question asked is how many colour, odour, taste and touch are possible in an atom and aggregates? And all the possibilities are elaborated in the *sūtra*-s.

Mathematically we can get the answer in various manners using the rules of permutation and combination:

a) If we consider the case of all possible five attributes then all permutations will be :

$$5 \times 2 \times 5 \times 8 = 400$$

b) Total number of all types (*eka- saṅyogī, dvi-saṅyogī, trik-saṅyogī…aṣṭa-saṅyogī* correspondingly) combinations for all will be :

$$[\binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \cdots \binom{5}{5}] \times [\binom{2}{1} + \binom{2}{2}] \times [\binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \cdots \binom{5}{5}] \times$$

$$[\binom{8}{1} + \binom{8}{2} + \binom{8}{3} + \binom{8}{4} + \cdots \binom{8}{8}]$$

$$= [(2^5 - 1) \times (2^2 - 1) \times (2^5 - 1) \times (2^8 - 1)]$$

$$= 31 \times 3 \times 31 \times 255$$

$$= 735165$$

Thus, 735165 colour, odour, taste and touch are possible in an atom and aggregates.

[1] *Bhagavaī*, Vol. V, p. 262.

5.4.3 Occupancy Problem

The occupancy problem in probability theory is the problem of randomly assigning a set of balls into a group of cells. In randomly placing *r* balls into *n* cells, what is the probability that exactly *k* cells are occupied where $k = 1, 2, 3, \ldots n$? What is the expected number of occupied cells? In other words, it is the famous problem in probability theory of determining the number of ways r balls can occupy *n* cells. This is given by the formula[1]:

$$A_{n,r} = \binom{n+r-1}{r} = \binom{n+r-1}{n-1}$$

It can be compared with the problem found in BS.

In BS (9.32.100)[2] the same concept is explained by Bhagawāna Mahāvīra. It talks about the possible ways of entry of hellish beings in hells. There are 7 hells. How many are the possible ways of one, two, three or more beings to get into 7 hells? In which hell the soul will go? The answers of these questions, as per the rule of occupancy problem are as follows:

Number of ways 1 soul can enter 7 hells = $\binom{7+1-1}{1} = \binom{7}{1} = 7$

Number of ways 2 soul can enter 7 hells = $\binom{7+2-1}{2} = \binom{8}{2} = 28$

Number of ways 3 soul can enter 7 hells = $\binom{7+3-1}{3} = \binom{9}{3} = 84$

Number of ways 4 soul can enter 7 hells $\binom{7+4-1}{4} = \binom{10}{4} = 210$

Number of ways 5 soul can enter 7 hells = $\binom{7+5-1}{5} = \binom{11}{5} = 462$

Number of ways 6 soul can enter 7 hells = $\binom{7+6-1}{6} = \binom{12}{6} = 924$

Number of ways 7 soul can enter 7 hells = $\binom{7+7-1}{7} = \binom{13}{7} = 1716$

Number of ways 8 soul can enter 7 hells = $\binom{7+8-1}{8} = \binom{14}{8} = 3003$

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[1] Feller, William, *Introduction to Probability Theory*, John Wiley and Co., Vol. I, 1968, p. 38.

[2] *Do bhaṁte! neraiyā, neraiya-pavesaṇeṇaṁ pavismāṇā kiṁ rayaṇa-ppabhāe hojjā jāva ahesattmāe hojjā? Gaṅgeyā! rayaṇappabhāe vā hojjā jāva ahesattmāe vā hojjā. ahavā ege rayaṇappabhāe ege sakkarappabhāe hojjā, ahavā ege rayaṇappabhāe ege vāluyappabhāe hojjā jāva ege rayaṇappabhāe ege ahesattmāe hojjā. ahavā ege sakkarappbhāe ege vāluyappbhāe hojjā jāva ahavā ege sakkarappbhāe ege ahesattamāe hojjā. ahavā ege vāluyappabhāe ege paṅkappabhāe hojjā, evaṁ jāva ahavā ege vāluyappabhāe ege ahesattmāe hojjā. evaṁ ekkekā puḍhavī caḍḍeyavvā jāva ahavā ege tamāe ege ahesattmāe hojjā.*

–*Bhagavaī,* Vol. III, pp. 228-255.

Number of ways 9 soul can enter 7 hells = $\binom{7+9-1}{9} = \binom{15}{9} = 5005$

Number of ways 10 soul can enter 7 hells = $\binom{7+10-1}{10} = \binom{16}{10} = 8008$

It is really laudable that the BS has all these results upto 10 souls entering 7 hells by listing them physically. Further it mentions about *saṅkhyāta* and *asaṅkhyāta* souls too. This can be treated as an outstanding achievement by the Jain saint scholars. In fact, it literally lists first two calculations above i.e. 7 and 28. Thereafter, the logical approach adopted is admirable.

One soul can go in any one of the 7 hells. It means there are 7 possibilities for him. If the living beings are two then the options are more. Both can go in the same hell or in different. It gives 28 possibilities. For 3 living beings there are 84 possible combinations.

If there are 3 souls then possible ways for all three souls entering one hell are 7[= $\binom{7}{1}$]. If 2 souls go together and one separately then they can enter two hells in 2 manners: i.e. 1+2, or 2+1. And 2 out of 7 hells can be selected in 21 [=$\binom{7}{2}$] ways. Hence there are 42 [=$\binom{7}{2}$+$\binom{7}{2}$] possible ways. And 3 souls can enter 3 hells in only one manner i.e. 1+1+1. But 3 hells out of 7 can be selected as in 35 [=$\binom{7}{3}$] ways. In this way, collectively these are 7+21+35 = 84 ways.

Though in BS only number is given, not the formula. They have not used the formula but this result exactly matches with the result obtained through calculations mentioned above. BS has calculated the value up to 10 but mentioned the problem up to numerable, innumerable and infinite numbers. Here BS calculates notionally the number of ways innumerable, and infinite souls enter 1 to 7 hells by taking symbolically innumerable as 11 and infinite as 12.

Commentator Abhayadeva of 11th C. A.D., in the same manner calculates the distribution for 1,2,3,--------,10,S(=11), and A(=12) souls entering 12 heavens by adopting the novel formula[1] :

$$A_{n,r+1} = \binom{n+r+1-1}{r+1} = \binom{n+r}{r+1}$$

$$= \binom{n+r-1}{r} \frac{n+r}{r+1}$$

$$= A_{n,r} \cdot \frac{n+r}{r+1}$$

[1] Shah, R.S., *Mathematical Ideas in Bhagavatī Sūtra*, Gaṇita Bharati, Vol. 30, 2008.

Hence once $A_{n,r}$ is known A $n,r+1$ can be calculated recursively.

$A_{12,7}$, i.e number of ways 7 souls entering 12 heavens :

$$A_{12,7} = \binom{12+7-1}{7} = \binom{18}{7}$$

$$= 31824.$$

Then, by using the formula recursively :

$$A_{12,8} = 31824 \times \frac{19}{8} = 75582$$

$$A_{12,9} = 75582 \times \frac{20}{9} = 167960$$

$$\text{and } A_{12,10} = 167960 \times \frac{21}{10} = 1352078.$$

This procedure adopted by Abhayadeva may be his innovation, or may be that he had carried forward the method already known for centuries.

Furthermore, in the same manner BS mentions the ways of entering the souls in all four realms: animals (one sensed to five sensed beings), humans, hellish beings and gods also. There are four types of gods basically mansion-dwelling (*bhavanapati*), forest-dwelling (*vyantara*), astral (*jyotiṣka*) and celestial (*vaimānika*). The possibilities of the souls entering 1, 2, … 12 heavens are also mentioned.

In fact, Abhayadeva Sūrī even tackled some problems based on conditional probabilities.[1] For example, after considering the number of combinations of 4 passions (anger, conceit, deceit and greed) he goes on to find out the number of interactions of 4 passions (singularly and plurally) with the soul, when afflicted with 1, 2, 3, or all the 4 passions. After determining this number as 80, BS poses the problem that if anger interacts plurally only what will the total number of interactions and determines this number as 27.

Śataka 20 is full of combinatorics with regard to touch, taste, odour, colour, configuration etc.

Similarly, we find many philosophical explanations through permutations and combinations.

5.5 Partition Theory

The Indian genius Rāmanujan along with his mentors Hardy and Littlewood was credited with development of partition function in modern number theory towards the earlier part of

[1] Shah R.S. *Lure of Large Numbers.*

20^{th} century although the versatile genius of Euler (18^{th} C. A.D.) was first to tackle this. This is an important function of modern number theory with application in statistical mechanics, atomic physics and quantum mechanics.

In number theory and combinatorics, a partition of a positive integer *n*, also called an integer partition, is number of ways of writing *n* as a sum of positive integers which has many applications in mathematics and science.

For example, 4 can be partitioned in five distinct ways :

4

3 + 1

2 + 2

2 + 1 + 1

1 + 1 + 1 + 1

This is denoted by P(4) = 5

5.5.1 Ferrers diagram

There are two common diagrammatic methods to represent partitions: Ferrers diagrams, named after Norman Macleod 5Ferrers, and as Young diagrams, named after the British mathematician Alfred Young. The Ferrers diagrams for the 5 partitions of the number 4 are listed below:

4	3+1	2+2	2+1+1	1+1+1+1

BS discusses this concept in explaining the disintegration of *skandha* (molecules), which can further be disintegrated in atoms and or molecule. This disintegration process they have given upto ten, molecules containing ten atoms.

In *śataka* 12 (4.69-80)[1] the possible combinations of fission of molecules are explained. In how many ways an aggregate can be disintegrated? An aggregate having two atoms can be disintegrated in one way that is one atom plus one atom; an aggregate having 3 atoms can be disintegrated in two ways as presented below and an aggregate of 5 atoms can be disintegrated in 7 ways as presented here :

Aggregates	**Possible combinations**	**Way of partitions**	**Representation**
Having 2 atoms	1	1 + 1	P'(2) = 1
3 atoms	2	1 + 2	P'(3) = 2
		1 + 1 + 1	
4 atoms	4	1 + 3	P'(4) = 4
		2 + 2	
		1 + 1 + 2	
		1 + 1 + 1 + 1	
5 atoms	6	4 + 1	P'(5) = 6
		3 + 2	
		3 + 1 + 1	
		2 + 2 + 1	
		2 + 1 + 1 + 1	
		1 + 1 + 1 + 1 + 1	

An aggregate having numerable (*saṅkhyeya*), innumerable (*asaṅkhyeya*) and *ananta* atoms can be disintegrated in *saṅkhyeya*, *asaṅkhyeya* and *ananta* ways.

The difference we find is, in partition theory the number itself is always counted. Therefore, for number 4 the number of partitions is 5. In BS the numbers of partitions is one less excluding the number itself. A molecule with 4 atoms can be disintegrated in four different ways: 3 +1 or 2+2 or 2+1+1 or 1+1+1+1. Therefore, according to BS, P'(4) = 4.

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[1] *Goyamā! duppaesie khaṅdhe bhavai. se bhijjamāṇe duhā kajjai-egayao paramāṇupoggale, egayo paramāṇu-poggale bhavai.*

tiṇṇi bhaṁte! paramāṇupoggalā egayo sāhaṇṇaṁti, sāhaṇittā kiṁ bhavai? Goyamā! tippaesie khaṅdhe. se bhijjamāṇe duhā vi tihā vi kajjai –duhā kajjamāṇe egayao paramāṇupoggale egayao duppaesie khandhe bhavai. tihā kajjamāṇe tiṇṇi paramāṇu-poggalā bhavaṅti.

–Bhagavaī, Vol. IV, pp. 29-42.

BS calculates this correctly for aggregates up to 10 *paramāṇu*-s except that for last aggregate of 10 atoms its value is 40 instead of 41. This function grows very fast for P(10) = 42; P(50) = 204, 226; P(100) = 190, 569, 292.

Naturally, it cannot be expected of them to go beyond aggregate of maximum of 10 atoms. The error in P(10) can be ascribed to writers of manuscripts.

In general it can be said that, if P(n) =m, then function in BS will be P'(n)=m-1

5.6 Exponentiation and Binomial Co-efficient

Many real world phenomena can be modelled by functions that describe how things grow or decay as time passes. Examples of such phenomena include the studies of populations, bacteria, radioactive substances, electricity, temperatures and credit payments, to mention a few.

In Jain *Āgama* we find instances of constant growth and decay of many phenomena such as bondage of *karma* particles, life-span, degree of knowledge, intensity of various qualities such as colour, taste, etc. This exponential growth or decay is described in the following manner. In the process of such consideration they stumbled upon binomial co-efficient. While considering the destruction they considered the unit of *karma* as growing by a quantity *k* (*kāṇḍaka*) per *samaya* or bondage of *karma* gave rise to binomial coefficients and it is derived in SK up to 5^{th} power.

Let's say that at the beginning of the first *samaya* the karmic bondage is 1 unit and then at the end of the first *samaya* it becomes (1+k). Thereafter, at the end of each succeeding *samaya* it grows by (1+k) times.

At the beginning of 1 *samaya* = 1

At the end of 1 *samaya* it has grown to= (1+**1**k)

At the end of 2 s = (1+k) + k(1+k) = **1** + **2**k +**1** k^2= $(1 + k)^2$

At the end of 3s = 1 + 2k + k^2+ k(1 + 2k + k^2) = **1** + **3**k + **3**k^2 +**1**k^3= $(1 + k)^3$

At the end of 4s= (1 + 3k + 3k^2 + k^3) + k(1 + 3k + **3**k^2 + k^3)

= **1** + **4**k + **6**k^2 + **4**k^3 +**1**k^4 = $(1 + k)^4$

At the end of 5s = $(1 + 4k + 6k^2 + 4k^3 + k^4) + k(1 + 4k + 6k^2 + 4k^3 + k^4)$

$= \mathbf{1} + 5k + \mathbf{10}k^2 + \mathbf{10}k^3 + 5k^4 + \mathbf{1}k^5 = (1 + k)^5$

The numbers in bold letters are nothing but binomial coefficients.

The process explained is so general that, they would have easily proceeded, further giving rise to so called Pascal's triangle which was actually realised by Virahāṅka (7^{th} C. A.D.), Gopāla and Hemcandra (14^{th} C. A.D.) well before Pascal (16^{th} C. A.D.).

5.7 Maxima and Minima (*alpabahutva*)

alpabahutva is also a unique feature found in Jain canonical texts. It is a comparative analysis of both quantitative and qualitative aspects. In almost all *āgamic* literature like BS, ADS, JDP, JJ, PS, SK etc. many aspects are dealt with maximum and minimum value and relative comparison. Such relative comparison is done from various perspectives like spatial, temporal, and quantitative. The topics discussed for comparison are mostly pertaining to number, height, life-span of living beings from subtle to gross and about - *pudgala* and other substances also.

In explaining this, let us adopt the notation: S denotes *saṅkhyāta*, A denotes *asaṅkhyāta* and I denotes *ananta* and superscript $^+$ or (1+v) denotes *viśeṣādhika* ($0<v<1$) .

a) PS has plenty of examples especially chapter three to explain the long series of maxima and minima with respect to life-span, height, *leśyā*, *kaṣāya*, knowledge, sense-organs etc. in descending or ascending order and few are being quoted here as follows:

   i) **Relative magnitude of *leśyā*:** According to the Jain theory of karma, *leśyā* is the coloring of the soul or state of the soul on account of its association with the karmic matter. The colour of *leśyā* varies from person to person depending on the psychic states and mental activities causing an action. The soul reflects the qualities of colour, taste, smell and touch of associated karmic matter, although it is usually the colour that is referred to when discussing the *leśyā*-s. As the *leśyā* is, so the personality is. The colour of *leśyā* determines attitude, nature behavior and overall personality. US elaborates the theory of *leśyā* in chapter

34. The second *uddeśaka* of chapter 17 (PS, *sūtra* 1170)[1] deals with this concept where *leśyā*-s are discussed in various living beings like hellish beings, deities and all. For example, the number of souls endowed with *śukla leśyā* (S_L) are minimum; endowed with *padma leśyā* (P_L) are *saṅkhyāta* (S) times of S_L; endowed with *tejo leśyā* (T_L) are *saṅkhyāta* times of P_L; devoid of *leśyā* (Z_L) are *ananta* (I) times of T_L; endowed with *kāpota leśyā* (K_L) are *ananta* times of Z_L; endowed with *nīla leśyā* (N_L) are little more of K_L and with *kṛṣṇa leśyā* K_{NL} are little more of N_L.

In notation it can be expressed as,

$$S_L < P_L < T_L < Z_L < K_L < N_L < K_{NL}$$

or, $P_L = S \times S_L$

$T_L = S \times P_L = S^2 \times S_L$

$Z_L = I \times T_L = I \times S^2 \times S_L$

$K_L = I \times Z_L = I^2 \times S^2 \times S_L$

$N_L = K_L^+ = I^2 \times S^2 \times S_L \times (1+v)$

$K_{NL} = N_L^+ = I^2 \times S^2 \times S_L \times (1+v)^2$

where, S = *saṅkhyāta*; A = *asaṅkhyāta*; I =*ananta*; *v* is a fractional value ($0<v<1$)

ii) **Relative magnitude of *pudgala* in various directions** [PS (3.327)][2]

Pudgala-s in upper direction (U, *urdhwa diśā*) are least in number of *pudgala*

$D = U^+$ (D is downward direction)

$NE (=SW) = A \times D$ (NE is north-east direction, SW = south-west)

$SE (=NW) = NE^+ = A \times D \times (1+v)$ (SE =south-east)

$E = A \times SE = A^2 \times D \times (1+v)$ (E =east)

$W = E^+ = A^2 \times D \times (1+v)^2$ (W = west)

$S = W^+ = A^2 \times D \times (1+v)^3$ (S =south)

$N = S^+ = A^2 \times D \times (1+v)^4$ (N = north)

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[1] *Prajñāpanā Sūtra*, Vol. II, p. 265.

[2] *Ibid*, Vol. I, p. 280.

This concept not only shows their comparison ability but it also reflects their knowledge of the number of *pudgala*-s in each direction to make the comparisons.

iii) **Relative space-points in all 6 substances:** There are six substances namely *dharmāstikāya* (D, medium of motion), *adharmāstikāya* (A_d, medium of rest), *ākāśāstikāya* (A_k, space), *kāla* (T, time), *jīvāstikāya* (J, living beings), and *pudgalāstikāya* (P, matter). Space-points of *dharmāstikāya* are the minimum which are equivalent to that of A_d. Therefore, as per PS (3.271)[1] :

$D = A_d$

$J = I \times D$

$P = I \times J = I^2 \times D$

$T = I \times P = I^3 \times D$

$A_k = I \times T = I^4 \times D$

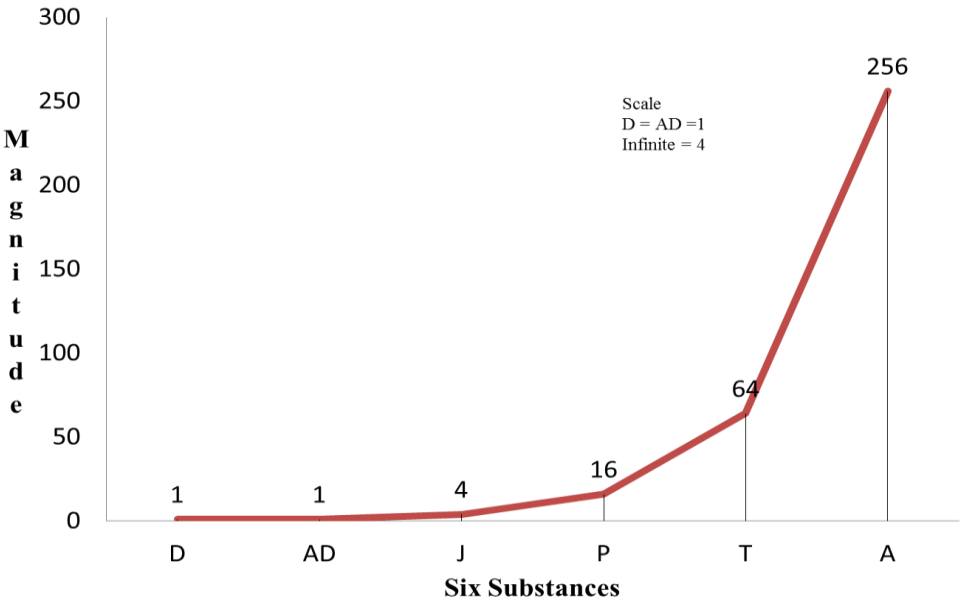

Chart: Relative magnitude of space points of 6 substances

In the above chart we can see the exponential growth in the number of space-points in the substances. On the *x*-axis are the substances and on *y*-axis is the magnitude. Base value is taken as 1. Infinite times is notionally taken as 4 times.

[1] *Prajñāpanā Sūtra*, Vol. I, p. 253.

Number of space points of J is *ananta* times of D. Therefore, it is 4 in the chart. Space points of P is I^2 times of J. Therefore it is 16. Thereafter the magnitude grows fast and symbolically becomes 64 and 256 for the succeeding substances.

iv) One is simply awe-struck to find that the Jain saint scholars have meditated so deeply as to the relative size of the number of 98 categories of souls in various realms as mentioned below. This largest series of maxima and minima is found in PS (3.336).[1] They have even specified as to what category is numerable times, innumerable times or infinite times or slightly more than the previous category. Commentator Abhayadeva have mentioned *viśeṣādhika* (slightly more) means any number which is less than the two times of the previous number i.e. (1+v) where 0<v<1

Relative Magnitude of 98 Life-forms		
S. No.	**Life form**	**Relative magnitude**
1.	Placental (*garbhaja*)	$S\times10^{14}$ (*saṅkhyāta koṭā koṭī*)
2.	Female human (*mānuṣī*)	$S^2\times10^{14}$
3.	Gross developed fire-bodied (*bādara tejaskāyika paryāptaka*)	$A\ S^2\times10^{14}$
4.	*anuttaraupapātika* god (*deva*)	$A^2\ S^2\times10^{14}$
5.	Upper (*ūparī*) *graiveyaka* god (*deva*)	$A^2\ S^3\times10^{14}$
6.	Middle (*madhyama*) *graiveyaka* god (*deva*)	$A^2\ S^4\times10^{14}$
7.	Lower (*nicale*) *graiveyaka* god (*deva*)	$A^2\ S^5\times10^{14}$
8.	Gods of 12th heaven (*acyutakalpa-deva*)	$A^2\ S^6\times10^{14}$
9.	Gods of 11th heaven (*āraṇakalpa ke deva*)	$A^2\ S^7\times10^{14}$
10.	Gods of 10th heaven (*prāṇatakalpa ke deva*)	$A^2\ S^8\times10^{14}$
11.	Gods of 9th heaven (*ānatakalpa ke deva*)	$A^2\ S^9\times10^{14}$
12.	Hellish beings of 7th hell (*saptama pṛthvī ke nairayika*)	$A^3\ S^9\times10^{14}$
13.	Hellish beings of 6th hell (*tamah-prabhā pṛthvī ke nairayika*)	$A^4\ S^9\times10^{14}$
14.	Gods of 8th heaven (*sahasrārakalpa ke deva*)	$A^5\ S^9\times10^{14}$
15.	Gods of 7th heaven (*mahāśukrakalpa ke deva*)	$A^6\ S^9\times10^{14}$
16.	Hellish beings of 5th hell (*dhūmaprabhā pṛthvī ke nairayika*)	$A^7\ S^9\times10^{14}$
17.	Gods of 6th heaven (*lāntakakalpa ke deva*)	$A^8\ S^9\times10^{14}$
18.	Hellish beings of 4th hell (*paṅkaprabhā pṛthvī ke nairayika*)	$A^9\ S^9\times10^{14}$

[1] *Prajñāpanā Sūtra*, Vol. I, pp. 286-287.

19.	Gods of 5th heaven (*brahmalokakalpa ke deva*)	$A^{10}\ S^{9}\times10^{14}$
20.	Hellish beings of 3rd hell (*bālūkāprabhā pṛthvī ke nairayika*)	$A^{11}\ S^{9}\times10^{14}$
21.	Gods of 4th heaven (*māhendrakalpa ke deva*)	$A^{12}\ S^{9}\times10^{14}$
22.	Gods of 3rd heaven (*sanatkumārakalpa ke deva*)	$A^{13}\ S^{9}\times10^{14}$
23.	Hellish beings of 2nd hell (*śarkarāprabhā pṛthvī ke nairayika*)	$A^{14}\ S^{9}\times10^{14}$
24.	Agglutinative humans (*sammūrcchima manuṣya*)	$A^{15}\ S^{9}\times10^{14}$
25.	Gods of 2nd heaven (*īśānakalpa ke deva*)	$A^{16}\ S^{9}\times10^{14}$
26.	Goddesses of 2nd heaven (*īśānakalpa ke deviyāṅ*)	$A^{16}\ S^{10}\times10^{14}$
27.	Gods of 1st heaven (*saudharmakalpa ke deva*)	$A^{16}\ S^{11}\times10^{14}$
28.	Goddesses of 1st heaven (*saudharmakalpa ke deviyāṅ*)	$A^{16}\ S^{12}\times10^{14}$
29.	Mansion-dwelling gods (*bhavanvāsī deva*)	$A^{17}\ S^{12}\times10^{14}$
30.	Mansion-dwelling goddesses(*bhavanvāsī devīyā*)	$A^{17}\ S^{13}\times10^{14}$
31.	Hellish beings of 1st hell (*ratnaprabhā pṛthvī ke nairayika*)	$A^{18}\ S^{13}\times10^{14}$
32.	5-sensed flying male animals (*khecara-pañcendriya-tiryañcayonika-puruṣa*)	$A^{19}\ S^{13}\times10^{14}$
33.	5-sensed flying female animals (*khecara-pañcendriya-tiryañcayonika-striyāṅ*)	$A^{19}\ S^{14}\times10^{14}$
34.	5-sensed terrestrial male animals (*sthalacara- pañcendriya-tiryañcayonika-puruṣa*)	$A^{19}\ S^{15}\times10^{14}$
35.	5-sensed terrestrial female animals (*sthalacara-pañcendriya-tiryañcayonika- striyāṅ*)	$A^{19}\ S^{16}\times10^{14}$
36.	5-sensed aquatic male animals (*jalacara- pañcendriya-tiryañcayonika-puruṣa*)	$A^{19}\ S^{17}\times10^{14}$
37.	5-sensed aquatic female animals (*jalacara- pañcendriya-tiryañcayonika- striyāṅ*)	$A^{19}\ S^{18}\times10^{14}$
38.	Forest-dwelling gods (*vāṇavyantara deva*)	$A^{19}\ S^{19}\times10^{14}$
39.	Forest-dwelling goddesses (*vāṇavyantar striyāṅ*)	$A^{19}\ S^{20}\times10^{14}$
40.	Astral gods (*jyotiṣka-deva*)	$A^{19}\ S^{21}\times10^{14}$
41.	Astral goddesses (*jyotiṣka-deviyaṅ*)	$A^{19}\ S^{22}\times10^{14}$
42.	5-sensed flying eunuch animals (*khecara-pañcendriya-tiryañcayonika-napuṅsaka*)	$A^{19}\ S^{23}\times10^{14}$
43.	5-sensed terrestrial eunuch animals (*sthalacara-pañcendriya-tiryañcayonika- napuṅsaka*)	$A^{19}\ S^{24}\times10^{14}$
44.	5-sensed aquatic eunuch animals (*jalacara- pañcendriya-tiryañcayonika- napuṅsaka*)	$A^{19}\ S^{25}\times10^{14}$
45.	4-sensed developed beings (*caturindriya-paryāptaka*)	$A^{19}\ S^{26}\times10^{14}$
46.	5-sensed developed beings (*pañcendriya-paryāptaka*)	$A^{19}\ S^{26}\ (1+v)\times10^{14}$
47.	2-sensed developed beings (*dvīndriya- paryāptaka*)	$A^{19}\ S^{26}\ (1+v)^{2}\times10^{14}$

48.	3-sensed developed beings (*trīndriya- paryāptaka*)	$A^{19}\ S^{26}\ (1+v)^{3}\times10^{14}$
49.	5-sensed undeveloped beings (*pañcendriya aparyāptaka*)	$A^{20}\ S^{26}\ (1+v)^{3}\times10^{14}$
50.	4-sensed undeveloped beings (*caturindriya aparyāptaka*)	$A^{20}\ S^{26}\ (1+v)^{4}\times10^{14}$
51.	3-sensed undeveloped beings (*trīndriya aparyāptaka*)	$A^{20}\ S^{26}\ (1+v)^{5}\times10^{14}$
52.	2-sensed undeveloped beings (*dvīndriya aparyāptaka*)	$A^{20}\ S^{26}\ (1+v)^{6}\times10^{14}$
53.	Gross plants with one soul in one body developed beings (*pratyekaśarīra bādara vanaspatikāyika-paryāptaka*)	$A^{21}\ S^{26}\ (1+v)^{6}\times10^{14}$
54.	Gross *nigoda* developed (*bādara-nigoda-śarīra-paryāptaka*)	$A^{22}\ S^{26}\ (1+v)^{6}\times10^{14}$
55.	Gross earth-bodied developed beings (*bādara-pṛthvī-kāyika-paryāptaka*)	$A^{23}\ S^{26}\ (1+v)^{6}\times10^{14}$
56.	Gross water-bodied developed beings (*bādara-apkāyika-paryāptaka*)	$A^{24}\ S^{26}\ (1+v)^{6}\times10^{14}$
57.	Gross air-bodied developed beings (*bādara-vāyukāyika-paryāptaka*)	$A^{25}\ S^{26}\ (1+v)^{6}\times10^{14}$
58.	Gross fire-bodied undeveloped beings (*bādara-tejaskāyika-aparyāptaka*)	$A^{26}\ S^{26}\ (1+v)^{6}\times10^{14}$
59.	Gross beings with one soul in one body undeveloped (*pratyekaśarīra bādara vanaspatikāyika-aparyāptaka*)	$A^{27}\ S^{26}\ (1+v)^{6}\times10^{14}$
60.	Gross *nigoda* undeveloped beings (*bādara-nigoda-śarīra-aparyāptaka*)	$A^{28}\ S^{26}\ (1+v)^{6}\times10^{14}$
61.	Gross earth-bodied undeveloped beings (*bādara pṛthvīkāyika-aparyāptaka*)	$A^{29}\ S^{26}\ (1+v)^{6}\times10^{14}$
62.	Gross water-bodied undeveloped beings (*bādara-apkāyika-aparyāptaka*)	$A^{30}\ S^{26}\ (1+v)^{6}\times10^{14}$
63.	Gross air-bodied undeveloped beings (*bādara-vāyukāyika-aparyāptaka*)	$A^{31}\ S^{26}\ (1+v)^{6}\times10^{14}$
64.	Fine fire-bodied undeveloped beings (*sūkṣma tejaskāyika-aparyāptaka*)	$A^{32}\ S^{26}\ (1+v)^{6}\times10^{14}$
65.	Fine earth-bodied undeveloped beings (*sūkṣma pṛthvīkāyika-aparyāptaka*)	$A^{32}\ S^{26}\ (1+v)^{7}\times10^{14}$
66.	Fine water-bodied undeveloped beings (*sūkṣma apkāyika-aparyāptaka*)	$A^{32}\ S^{26}\ (1+v)^{8}\times10^{14}$
67.	Fine air-bodied undeveloped beings (*sūkṣma vāyukāyika-aparyāptaka*)	$A^{32}\ S^{26}\ (1+v)^{9}\times10^{14}$
68.	Fine fire-bodied developed beings (*sūkṣma taijaskāyika-paryāptaka*)	$A^{32}\ S^{27}\ (1+v)^{9}\times10^{14}$
69.	Fine earth-bodied developed beings (*sūkṣma pṛthvīkāyika-paryāptaka*)	$A^{32}\ S^{27}\ (1+v)^{10}\times10^{14}$
70.	Fine water-bodied developed beings (*sūkṣma apkāyika-paryāptaka*)	$A^{32}\ S^{27}\ (1+v)^{11}\times10^{14}$

71.	Fine air-bodied developed beings (*sūkṣma vāyukāyika-paryāptaka*)	$A^{32}\ S^{27}\ (1+v)^{12}\times10^{14}$
72.	Fine undeveloped *nigoda* (*sūkṣma nigoda-śarīra-aparyāptaka*)	$A^{33}\ S^{27}\ (1+v)^{12}\times10^{14}$
73.	Fine developed *nigoda* (*sūkṣma nigoda-śarīra-paryāptaka*)	$A^{33}\ S^{28}\ (1+v)^{12}\times10^{14}$
74.	Never-liberating Souls (*abhavasiddhika*)	$I\ A^{33}\ S^{28}\ (1+v)^{12}\times10^{14}$
75.	Soul degraded from Right faith (*pratipratita*)	$I^{2}\ A^{33}\ S^{28}\ (1+v)^{12}\times10^{14}$
76.	Liberated souls(*siddha*)	$I^{3}\ A^{33}\ S^{28}\ (1+v)^{12}\times10^{14}$
77.	Gross developed plants (*bādara-vanaspatikāyika-paryāptaka*)	$I^{4}\ A^{33}\ S^{28}\ (1+v)^{12}\times10^{14}$
78.	Gross developed beings (*bādara -paryāptaka*)	$I^{4}\ A^{33}\ S^{28}\ (1+v)^{13}\times10^{14}$
79.	Gross undeveloped plants (*bādara-vanaspatikāyika - aparyāptaka*)	$I^{4}\ A^{34}\ S^{28}\ (1+v)^{13}\times10^{14}$
80.	Gross undeveloped beings(*bādara-aparyāptaka*)	$I^{4}\ A^{34}\ S^{28}\ (1+v)^{14}\times10^{14}$
81.	Gross beings (*bādara*)	$I^{4}\ A^{34}\ S^{28}\ (1+v)^{15}\times10^{14}$
82.	Fine undeveloped plants (*sūkṣma-vanaspatikāyika -aparyāptaka*)	$I^{4}\ A^{35}\ S^{28}\ (1+v)^{15}\times10^{14}$
83.	Fine undeveloped beings (*sūkṣma-aparyāptaka*)	$I^{4}\ A^{35}\ S^{28}\ (1+v)^{16}\times10^{14}$
84.	Fine developed plants (*sūkṣma-vanaspatikāyika -paryāptaka*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{16}\times10^{14}$
85.	Fine developed beings (*sūkṣma-paryāptaka*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{17}\times10^{14}$
86.	Fine beings (*sūkṣma*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{18}\times10^{14}$
87.	Soul having potentiality of liberation(*bhavasiddhika*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{19}\times10^{14}$
88.	Smallest form of living beings (*nigoda jīva*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{20}\times10^{14}$
89.	Plants souls (*vanaspati jīva*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{21}\times10^{14}$
90.	one-sensed beings (*ekendriya jīva*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{22}\times10^{14}$
91.	Animals (*tiryañcayonika*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{23}\times10^{14}$
92.	Souls with perverted faith (*mithyādṛṣṭi-jīva*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{24}\times10^{14}$
93.	Non-restraint souls (*avirata jīva*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{25}\times10^{14}$
94.	Souls with passions (*sakaṣāyī jīva*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{26}\times10^{14}$
95.	Non-omniscient souls(*chadmastha jīva*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{27}\times10^{14}$
96.	Souls with physical, mental and vocal activity (*sayogī jīva*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{28}\times10^{14}$
97.	Mundane souls (*saṅsārastha jīva*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{29}\times10^{14}$
98.	All living beings (*sarvajīva*)	$I^{4}\ A^{35}\ S^{29}\ (1+v)^{30}\times10^{14}$

Table 5.3: Maxima-Minima of the Souls

The table begins with placental lives, which is clearly stated as falling in numerable quantities i.e. $S\times10^{14}$ which is a few *koḍākoḍī*-s. The last figure in the table is $I^{4}A^{35}S^{29}(1+v)^{30}\times10^{14}$. Since, I is a variable quantity. Hence, when we say I^{4}, is actually

product of I_1, I_2, I_3, and I_4, where each of these I is a different quantity but in the class of infinity.

Similarly, A^{35} may be product of 35 different innumerable quantities and S^{29} may be product of 29 different numerable quantities. We have already mentioned, v may be any fraction. So that $0<v<1$.

Aforementioned table shows that the Jain saint scholars have thought so deeply and minutely about the number of beings in all the existing realms of life-forms. It's not just their imagination, but must be based on some sound logic.

Of course, all *jīva* will have the highest figure, but if we analyze the places of *jīva* assigned we can understand the underlying reality. It is said that *loka* is full of *nigoda* and it holds 88^{th} place in this series. How enormous the number of *nigoda* is! Simultaneously, we see that liberated souls (*siddha*) are more than each of developed and undeveloped *nigoda* but less than all *nigoda*. How it is that *siddha* are so high in number. The reason is *siddha* are counted from time immemorial.

5.8 Rules of Indices

In algebra rule of indices are used as,

The first rule: $a^m \times a^n = a^{m+n}$

The second rule: $(a^m)^n = a^{mn}$

(where m and n are integer, fraction, positive or negative number)

Along with the operations on large integers, and large fractions Jain saint scholars were aware of rules of indices. PS and ADS have numerous examples of rules of indices of modern mathematics.

i) As we have already seen in chapter 3 while describing the human population, ADS describe the results as multiplication of "5^{th} successive square of $2 = 2^{2^5} = 2^{32}$" and "6^{th} successive square of $2 = 2^{2^6} = 2^{64}$"; which gives the result as

$(2^{32}) \times (2^{64}) = 2^{(32+64)} = 2^{96}$ (which follows both the rules mentioned above).

ii) Not only this, but there are many references in ADS and other *āgamic* works like PS that they were aware of these operations for fractional indices also. E.g., ADS, *sūtra* 503 while describing the number of transformational (*vaikrayika*) bodies of gods in heavens they have given the rules as multiplication of space-points in the second and third square roots of *aṅgula* and further mentions that it is equal to space-points in cube of the third successive square root of *aṅgula bīyavaggamūlaṁ taiyavaggamūlapaḍḍupaṇṇam, ahava ṇaṁ aṅgula taiyavaggamūlaghaṇappamāṇamettāo*[1]*...*

i.e. $$\sqrt{\sqrt{a_{\circ}}} \times \sqrt{\sqrt{\sqrt{a_{\circ}}}} = \left(\sqrt{\sqrt{\sqrt{a_{\circ}}}}\right)^3$$

i.e. $$a_{\circ}^{1/4} \times a_{\circ}^{1/8} = a_{\circ}^{3/8}$$

$$= (a_{\circ}^{1/8})^3 \text{ (cube of the third square root)}$$

This illustration shows the familiarity with rules of exponentiation in respect of fractional indices also.

In PS (12.911b) while describing the number of transformational (*vaikrayiak*) bodies of hellish beings in hell they have stated that the multiplication of space-points in the first and the second square roots of *aṅgula* is equal to space-points in cube of the second square root of *aṅgula* (*aṅgulapaḍhamavaggamūlam bīyavaggamūlam… aṅgulabitiyavaggamūlaghaṇappamāṇamettāo…*) i.e. multiplication of space-points in the first square root and second square root of an *aṅgula*

$$\sqrt{a_{\circ}} \times \sqrt{\sqrt{a_{\circ}}} = \left(\sqrt{\sqrt{a_{\circ}}}\right)^3$$

i.e. $$a_{\circ}^{1/2} \times a_{\circ}^{1/4} = a_{\circ}^{3/4}$$

Furthermore, it states that the resultant number is equal to space-points in cube of the second square root of *aṅgula* = $(\sqrt{\sqrt{a_{\circ}}})^3 = (a_{\circ}^{1/4})^3$

iii) *sūtra* 10.482 of ADS[2] states that corporeal bodies of two-sensed beings are innumerable in number. From stand-point of space, this number is exhausted by innumerable world-lines lying in innumerable-th part of world-plane. The breadth of

[1] *Anuogadārāiṁ*, p. 276.

[2] *Ibid*, p. 272.

these world lines is innumerable *koḍākoḍi* (10^{14}) *yojana*-s which is equal to innumerable times the square-root of those world lines.

This is again equivalent to innumerable square roots of space-points in those world-lines. Which can be expressed as,

$$\sqrt{a_{\circ}} = (a_{\circ})^{1/2}$$

$$\sqrt{\sqrt{a_{\circ}}} = (a_{\circ})^{1/4}$$

$$\sqrt{\sqrt{\sqrt{a_{\circ}}}} = (a_{\circ})^{1/8}$$

$$\vdots$$

$$\vdots$$

$$\left(\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{a_{\circ}}}\ldots\ldots\sqrt{a_{\circ}}}}}}\right)^{A} = a_{\circ}^{1/2^{A}}$$

This shows the familiarity and expertise with tackling multiple roots of roots of various quantities.

5.9 Conclusion

Jain literature is an ocean of philosophy. In this chapter we have seen that how the philosophical concepts like karma, *leśyā,* knowledge, substances etc. are described with the use of abstract mathematics. The calculations they have made and the results obtained reflect their expertise in application of the mathematical ideas to metaphysical concepts in *Āgama*.

In the process, they stumbled upon advanced concepts like partition theory, occupancy problem in probability theory, hierarchy of Cantorian infinities, combinatorial problems etc. Extraction of square roots and operations with fractions were simply mastered, it seems as of 5th C. A.D. By that time, they had fully mastered rules of indices for integral and fractional powers.

Chapter-6

Conclusion

Thales is supposed to have used geometry to solve problems such as calculating the height of pyramids based on the length of shadows, and the distance of ships from the shore. The Pythagoreans regarded numerology and geometry as fundamental to understanding the nature of the universe and therefore central to their philosophical and religious ideas. Emphasis was given to abstract reasoning and to the goal of using reason to understand the universe. Mathematics in Jaina philosophy is free from the religious and ritualistic ground and is based on purely metaphysical and spiritual aspects. Jaina saint scholars have contributed to the world of mathematics even in the so-called dark period (500 BC to 500 AD) and kept the Indian mathematics alive.

This thesis may be regarded as evolution of mathematical concepts in that period and lot of examples of the same have been given. Decimal system is the gift to the world from Jain Scholar. That prompted me to study the *Āgama* and find out the nature of the treatment.

For example, how the concept of analogy measures and transfinite numbers have undergone gradual change is explained by me in detail in chapter 4. I have stretched the period of half century further than the Vallabhī council period. Because the *ācārya* immediately following that period e.g. *ācārya* Sarvanandi, Jinabhadragaṇi, Umāsvāti, Yativṛṣabha etc. were regarded in high esteem and there mathematics was direct outcome of development of mathematics in the first few centuries of the Christian era.

Many western scholars and oriental scholars have done stupendous work on *Āgama*. But very few have done on mathematics in it specially none of western scholars have done work in the area of mathematics. Very few Indian scholars L.C. Jaina, R.C. Gupta, S.S. Lishk, Anupam Jaina, R.S. Shah, etc. have done considerable work in Jaina mathematics. They too have concentrated on sporadic topics and that too in very few works of *Āgama* and concentrating mainly on BS, ADS, JDP and SCP. But my studies revealed that many more *āgamic* works are rich in mathematical content. Therefore I had deliberately for the first time have brought into

light mathematical contents of other *āgamic_works* such as TS, SAM, US, JJ, PS, SK etc. and given a number of examples from these *āgamic works* which were somehow ignored for this purpose.

Moreover the Indian scholars have also somehow concentrated on the geographical and cosmographical and certain arithmetical feature found in *Āgama* but I found the genesis of these concepts mainly in their metaphysical, spiritual, religious and philosophical principles. Hence I made it a point wherever possible, to cite examples (*leśyā, saptabhaṅgī, pudgala, nigoda,* etc.) involving metaphysical, and philosophical concepts in my work.

For comparison purpose I brought out the work done by various contemporary civilizations in number theory in chapter 1. One unique feature found is that, although number of ancient civilizations have some unique and advanced feature for their times, yet Jaina mathematics stands out for its totally fresh outlook. This required me to study in detail the development of mathematics in various other cultures-Egypt, Babylonian, Greek, Chinese, Mayan, Vedic, Buddhist etc.

In this thesis during the intensive survey of *Āgama* in chapter 2, I have tried to establish how Jaina mathematics has totally new features and were quite advanced for their times. I may cite few examples such as *pṛthaktva, partition theory, occupancy problem, combinatorics, beginning of modular mathematics, treatment of hierarchy of infinites, and infinitesimals*.

My comparative studies of these cultures indicated clearly to me that the earliest evidence of decimal place value system with zero having place value is found in Jaina sources (chapter3). Can we therefore, dare to suggest that ***zero*** is the greatest gift of Jaina scholars to the world.

I have given then the indirect proof of the earliest evidence of use of zero for positional value from Jain sources which is earliest known so far, for this I have given (chapter 3) quotations from ADS regarding minimum number of *audārika śarīra* calculated as,

$$2^{96} = 79,228,162,514,264,337,593,543,950,336,$$

(which contains one zero, though the figure is not given in ADS, it seems that they might have some idea of something like zero.)

The comparative list of the largest countable number *śīrṣaprahelikā* found in *āgamic* and non-*āgamic* works is also given.

Decimal system was adopted by many cultures and was naturally on our fingers and toes. Even in India the Vedic and Buddhist scriptures abound in number based on decimal system. But use of zero as a number and that too with place value is regarded unanimously as the greatest gift of India to the world of mathematics.

Normally, western scholars attribute any achievement of India to Greek influence. Then how come that the nearly correct value of π as 22/7 was not adopted by Indian scholars like Brahmagupta (7^{th} C. A.D.) who too accepted $\pi = \sqrt{10}$. Not only this, but it was only in 9^{th} C. A.D. Vīrasena gave a close value of π as 355/113 = 3.14159292... against the correct value 3.14159265... correct up to 6 decimal places.

Introductory remark in *Mathematics of Dhavalā* by Dr, A. N. Singh, 'The use of big number necessitated the development of simple symbolism for writing these numbers and has been responsible for the invention of decimal place value notation. It is now established beyond doubt that the place value system of notation was invented in India about the beginning of the Christian era – the brightest periods of Jainism.

What impressed me much was the Jaina mathematicians' almost obsession for large numbers. They evolved large numbers comparable with such modern numbers as number of monster group, skews' number of type1 and 2, Graham number etc. This also lead them to the realms of innumerable and infinities.

A very complicated circuitous and recursive process is employed for arriving at the estimate of A (*asaṅkhyāta*) is given in chapter 4.

$$10^{44.30},\ 10^{10^{44.07}},\ 10^{10^{10^{44.07}}},\ 10^{10^{10^{10^{44.07}}}},\ \ldots\ldots 10^{10^{10^{10^{\cdot^{\cdot^{\cdot^{10^{44.07}}}}}}}}$$

Just for completeness I have also added a paragraph in chapter 4 in respect of post *āgamic* changes in the concepts of transfinite numbers. The *Āgama* affirm that there is no maximum of infinite, indicating that the series is endless and which is in consonance with modern theory of infinity.

Most significant part I found was how great geniuses think alike without barriers of time. Towards the end of 19th century the great mathematician Georg Cantor proposed the most beautiful mathematical theory of transfinite number i.e. infinity. This was more so to highlight that the treatment in ADS is far scientific and in accordance with modern Cantorian theory of infinite.

How mathematical ideas are employed in metaphysical concept is explained in chapter five. The procedure for extraction of square root of large number was possible by modern algorithm only. I have tried to show by concrete examples that this can be only method by which they could do so. I have illustrated it by 2 examples in my work. The Jain mathematicians were equally proficient in delivering in micro world as they were in macro world.

As the mathematics was evolved for the purpose of philosophy and metaphysics but later they reveled in it purely for mathematical purpose. This can be borne out by the treatment of transfinite number and permutations and combinations. Since this thesis concentrates only on *Number Theory*, geometrical ideas, trigonometry, and differential calculus etc. are ignored. Because of the emphasis on philosophy of *anekānta* and *syādvād* Jain scholars required to see the multitude of facets of anything and this idea gave rise to permutations and combinations.

The main charge against Indian or Jaina mathematics is that it is mainly empirical and not based upon logical analysis. I am convinced; this can't be so as I find **Saṅgrahiṇī sūtra** (formula) quoted for many things in *Āgama* (e.g. BS. Vol. v). Further, when such a logical treatment with help of permutation and combinations is given to certain topic, it can't be said that they were unaware of logical analysis.

One very unique feature of Jaina mathematics is that when they were dealing with different entities, they were very particular on their quantitative relationship (*alpabahutva*). For example they have listed and tabulated types of living beings (*daṇḍaka*-s) and they have very elaborately given the relative magnitudes from the smallest to the biggest among 98 categories In another example the relative magnitude is shown through graph in the thesis, which tells the exponential growth in magnitude of the succeeding entities (chapter 5).

They have manually worked on all the combinations. It is the famous occupancy problem of probability theory and statistical mathematics.

They have worked out how,

7 souls can enter 7 hells in 1716 ways.

8 souls can enter 7 hells in 3003 ways.

9 souls can enter 7 hells in 5005 ways.

10 souls can enter 7 hells in 8008 ways.

Abhaydeva knew the recursive mathematical formula for it.

In modern atomic Physics, there are lot of applications of partition theory for determining in how many ways a molecule is composed of 2, 3, 4, 5….atoms can be split in atoms and molecules.

In number theory if amounts to finding out as to how many ways an integer can be represented as sum of smaller integers,

1=1 (P_1=1)

2=2, 1+1 (P_2=2)

3=3, 1+2, 1+1+1(P_3=3)

4=4, 1+3, 1+2+1,1+1+1+1, 2+2(P_4=5)

.

.

P_{10} = 42, P_{50} =204226, P_{100} =190569292

In BS they have calculated the value up to 10. This value is less by one than the modern system of presentation because original number is not counted.

BS has explained if for split of *sakandha* (molecule). In how many ways it can be splited.

No Indian scholar has actually calculated the total army of Ram and Ravana and the same for the armies of Kaurava-s and Pāṇḍava-s. In this thesis the figure is given.

I had also calculated the army of first emperor of ancient India Bharat.

All said and done I was rather taken aback to find that some very crucial areas in mathematics don't find even a mention in *Āgama*. For example prime numbers which are building blocks of entire number system are conspicuous by their non-mention. There is no application of Pythagoras theorem, which is only remotely suggested by the formula for length of chord of circle. Similarly, topics in geometry are very elementary, not to talk about conic sections, which were developed by Greek thinkers about 300 BC onwards.

It can be concluded that mathematics has been part of Jain philosophy since ancient time to explain the physical and non-physical phenomenon and concepts. Without understanding mathematics the study of Jain philosophy is incomplete, which uses mathematical ideas to gain the perfect and accurate knowledge.

GLOSSARY

***Sanskrit* words**	**English**
abādhā kāla	dormant (inactive) period of karma
abhavya, abhavasiddhika	souls not having potentiality of getting liberation
abhivṛddhi	presiding god of asterism of *Uttarabhaddavaya*
acalātmaka	Largest countable number in Digambara tradition
ācārya	head of the religious order
acitta mahāskandha	Inanimate mega-aggregate
acyutakalpa deva	celestial gods of 12^{th} heavens
aḍaḍa	$(8400000)^6 \approx 3.51 \times 10^{41}$ years.
aḍaḍāṅga	$(8400000)^5 \approx 4.18 \times 10^{34}$ years.
addhā	practical time
aḍhāi'	two and half
adharmāstikāya	medium of rest
adhikāra	chapter
adhyavasāya	subtle states of consciousness
āditya	sun
āgamic	related to *āgama*
agrāyaṇī	name of one of the *pūrva*-s
āhāraka śarīra	assimilative body
ahiṁsā	non-violence
ahorātra	day-night = 30 *muhutta* (= 24 hours)
ajaghanya-anutkṛṣṭa	intermediate
ajaghanyānutkṛṣṭa anantānanta	intermediate higher order infinity
ajaghanyānutkṛṣṭa asaṅkhyātāsaṅkhyāta	intermediate higher order innumerable
ajaghanyānutkṛṣṭa paritānanta	intermediate lower order infinity
ajaghanyānutkṛṣṭa paritāsaṅkhyāta	intermediate lower order innumerable
ajaghanyānutkṛṣṭa saṅkhyāta	intermediate numerable
ajaghanyānutkṛṣṭa yuktāsaṅkhyāta	intermediate middle order innumerable

Ajitanātha	the 2nd *tīrthaṅkara* in Jainism
ajīva	inanimate
Ājīvaka	one of the sects in the era of Bhagawāna Mahāvīra
ākāśa	space
akṣauhiṇi	type of an army explained in Mahābhārata
aloka	non-cosmos
alpabahutva	maxima-minima
alpabahutva dvāra	maxima-minima section
amadhya	which has no middle
aṇāgataddhā	future period
ananta	infinite
anantadvāra	infinite section
anānupūrvī	residual orders (all permutations except serial and reverse orders)
anardha	cannot be bisected further
ānatakalpa ke deva	gods of 9th heaven
anavasthā	variable
anekānta	doctrine of multi-fold aspects
aṅga	Jaina scriptures (containing the preaching of *tīrthaṅkara*)
aṅgabāhya	auxiliary canonical scriptures inherited by scholar Jaina ācārya-s (*sthavira*-s)
aṅgapraviṣṭa	primary canonical scriptures (12 *aṅga*-s)
aṅgula	a small linear measure of
aṅkalivi	language of numbers
aṇṇamaṇṇabbhāso	number raised to its own power
antara kāla	interregnum of a living being between the two births
antarāya karma	hindrance producing karma
antarmuhūrta	the time between 2 samaya to 48 minutes less one samaya
anubhāga	intensity of karma
anuttaraupapātika deva	gods of the highest heaven
aparyāpta	undeveloped
aparyāpta bādara vanaspati	gross undeveloped plant
apradeśa	which does not occupy any space

ārā	epoch of the time-cycle
āraṇakalpa ke deva	gods of 11th heaven
ardha-māgadhī	*prākṛta* language (spoken in the half part of Magadha)
arthanipura	$(8400000)^{18}$ years
arthanipurāṅga	$(8400000)^{17}$ years
Aruṇodaya	name of an ocean
Āryabhaṭṭa	an Indian mathematician
āsāliyā	a type of worm
asaṅkhejjāsaṅkhejja	higher order innumerable
asātā vedanīya	pain producing karma
aṣṭa-saṅyogī	combination of eight
asti	is (exists)
aśūnya kāla	active time-period when the living beings come and go from any realm
Asurendra Camar	one of two Indra-s of *asurakumāras*, one of 10 types of mansion-dwelling gods (other Indra is Bāli)
ātmā	soul
audārika śarīra	physical body
aupamika kāla	analogy-measure of time
autāṅga	$(84\times10^5)^{19}$ years
avadhi jñāna	clairvoyance
avaktavya	inexpressible
āvalikā	smallest practical unit of time composed of innumerable samaya-s
avamāna	some linear scale to measure the land etc.
avasarpiṇi (*osappiṇi*)	regressive cycle
avava	$(84\times10^5)^8$ years
avavāṅga	$(84\times10^5)^7$ years
avibhāga praticcheda	ultimate units
avirata jīva	non-restrained souls
avirati	non-restraint
ayana	solstice = 3 seasons (= 180 day-nights)
āyata	rectangle
ayuta	$(8400000)^{20}$ years

bādara aparyāpta	gross undeveloped soul
bādara aparyāpta vanaspati	gross undeveloped plants
bādara apkāyika-aparyāptaka	gross water-bodied undeveloped beings
bādara apkāyika-paryāptaka	gross water-bodied developed beings
bādara jīva	gross living beings
bādara nigoda-śarīra-aparyāptaka	gross *nigoda* undeveloped beings
bādara nigoda-śarīra-paryāptaka	gross *nigoda* developed beings
bādara paryāptaka	gross developed soul
bādara pṛthvīkāyika-aparyāptaka	gross earth-bodied undeveloped beings
bādara pṛthvī-kāyika-paryāptaka	gross earth-bodied developed beings
bādara tejaskāyika- aparyāptaka	gross fire-bodied undeveloped beings
bādara tejaskāyika paryāptaka	gross fire-bodied developed beings
bādara vanaspatikāyika -aparyāptaka	gross undeveloped plants
bādara vanaspatikāyika-paryāptaka	gross developed plants
bādara vāyukāyika- aparyāptaka	gross air-bodied undeveloped plants
bādara vāyukāyika- paryāptaka	gross air-bodied developed plants
baddha audārika śarira	bound corporeal body
baladeva	conqueror of 3 continents
bālāgra (*bālagga*)	hair-tips
bālūkāprabhā pṛthvī ke nairayika	Hellish beings of 3rd hell
Bhagawāna Mahāvīra	The 24th *tīrthaṅkara*
bhaṅga samutkīrtana	permutation -combination
bhavanpati (*bhavanvāsī deva)*	mansion-dwelling gods
bhavanvāsī devīyā	mansion-dwelling goddesses
bhāvappamāṇe	measurement of quantities
bhavasiddhika, bhavī	soul having potentiality of liberation
bhuja-parisarpa	reptiles
biyamalapaya	number with 16 digits
Bodhisatva	Bhagawāna Buddha
brahmalokakalpa ke deva	Gods of 5th heaven
Brāhmī	daughter of king Bharata and an ancient Indian numeration
cakravartī	conqueror of 6 continents
Camarcañca	Capital of the king of the 1st mansion-dwelling (demi-) gods.

candra māsa	moon-month
caraṇa-karaṇānuyoga	ethical Jain scriptures
caritra	conduct
caturasra (*cauraṁsā*)	square
caturdaśa pūrvadhara	*ācārya*-s having knowledge of 14 *pūrva*-s of twelfth *aṅga*
caturindriya aparyāptaka	4-sensed undeveloped beings
caturindriya-paryāptaka	4-sensed developed beings
cauyamalapaya	number with 32 digits
chadmastha jīva	non-omniscient souls
cheda sūtra	texts relating to the conduct and behaviour of monks and nuns
cūlikā	appendix (supporting text of the canonical scriptures)
cūlikā	$(84\times10^5)^{26}$ years
cūlikāṅga	$(84\times10^5)^{25}$
cūrṇi	commentary
daṇḍa (*dhanuṣa, yuga, nālikā, akṣa or musal*)	a measure equal to 96 *aṅgula*-s (literally meaning a bow)
darśanāvaraṇīya karma	intuition-obstructing karma
dasa (*dasaṁ*)	ten
dasa koḍisayāiṁ	10^{10} (1000 crore)
dasa koṭi (*dasa koḍio*)	10^8 (10 crore)
dasa lakṣa (*dasa sayasahassāiṁ*)	10 lakh (10^6)
dasa sahassa (*dasasahassāiṁ*)	10 thousand
dasapuhatta	10 *pṛthaktva*
davvappamāṇe	measurement of the substance
Devakuru	located at the south of Mahāvideha
devkuru-uttrakuru manushya bālāgra	hair-tips of the humans of *devakuru uttrakuru* land
dhanuṣa	see *daṇḍa*
dharmakathānuyoga	scriptures containing religious parables
dharmāstikāya	medium of motion
Dhātakīkhanḍa	2nd island in the middle cosmos
dhūmaprabhā pṛthvī ke nairayika	hellish beings of 5th hell
Digambara	one of the tradition of Jainism
divaddhaṁ	two less half

diwasa	day
dravya	substance
dravyānuyoga	metaphysics
duaddha	two and half
dusamā	5^{th} (out of 6) epochs of descending cycle (osappiṇī) and 2^{nd} (out of 6) epoch of ascending cycle (ussappiṇī), of duration 21000 years. (At present we are supposed to be in this epoch which started in c. 523 B.C.
dusama dusamā	last (6^{th}) epoch of descending cycle (osappiṇī) and 1^{st} (out of 6) epoch of ascending cycle (ussappiṇī) of time with duration 21000 years
dusama susamā	4^{th} (out of 6) epochs of descendig cycle (osappiṇī) and 3rd (out of 6) epochs of ascending cycle of (ussappiṇī), of duration 10^{14} sagarovama-s less 42000 years
dvādaśāṅga (*gaṇipiṭaka*)	12 aṅga-s
dvāpara yugma	set of such numbers where remainder is 2 after dividing the multiple of 4 by the positive integer.
dvāparayugma kalyoja	set of such numbers when the remainder is 9 after dividing from16
dvāparayugma- kṛtayugma	set of such numbers when the remainder is 8 after dividing from16
dvāparayugma- tryoja	set of such numbers when the remainder is 11 after dividing from16
dvāparayugma-dvāpara yugma	set of such numbers when the remainder is 10 after dividing from16
dvik saṅyogī	combination of 2 things out of many
dvīndriya aparyāptaka	4-sensed undeveloped beings
dvīndriya paryāptaka	4-sensed developed beings
eka (*ego*)	one
eka saṅyogī	picking up one out any number of things
ekendriya jīva	one-sensed (touch) beings
gaccha	number of terms
gaṇadhara	chief disciples of *tīrthaṅkara*
gaṇanā	mental arithmetic

gandha	odour
Gandhamādana	great *vakkhara* (breast-bag like) mountain shaped like elephant-tusk (almost quarter-circle) to north-west of Mt. Mandara forming western border of Uttarakura
Gaṅgādīva	island in the centre of pond *Gaṅgappavayakuṇḍa*, circular in shape with diameter 8 y and 2 kośa above the waters of the pond
Gaṅgāppavāyakuṇḍa	round pond, diameter 60 y and 10 y deep, in which cascades down great river *Gaṅga* from a height slightly over 100 y
gaṇima	numerals to count the numbers of objects
gaṇita (*gaṇiya*)	mathematics
gaṇitamāna	measures of counting
gaṇitānuyoga	mathematics scripture
gaṇiyalivi	language of arithmetic
garbhaja	placental
gāthā	prose
Gautama	first *gaṇadhara*
Gautamadvīpa	an island in *Lavaṇasamudra*
ghana	cube
gośālaka	a follower of *Ājīvaka* sect who became disciple of Bhagawāna Mahāvīra for few years
grantha-s	scriptures
guṇa	attributes
gunasthāna	stages of spiritual development
haimavaṅta	mountain in the north of *Jambūdvīpa* island
haimavata-	2nd (from south) of 7 continents of JD, running east-west and touching *Lavaṇa* sea at both ends
hairaṇyavata	6th (from south) and 2nd (from north) of 7 continents of JD, running east-west and touching *Lavaṇa* sea at both ends,
harappa	The Indus Valley Civilization
harivarṣa	3rd (from south) of 7 continents of JD, running east-west and touching *Lavaṇa* Sea at both ends
hasta prahelita	$(84)^{30} \times 10^{85}$

hātha	length one hand
hāthīgumphā	inscription of *Khāravel*
hīyamāna	decreasing
hūhūka	$(84\times10^5)^{10}$
hūhūkāṅga	$(84\times10^5)^9$
īśānakalpa ke deva	gods of 2nd heaven
īśānakalpa ke deviyāṅ	goddesses of 2nd heaven
iṣatprāgbhārā pṛthvī	abode of emancipated (liberated) souls at the apex of *loka* (universe)
jagaśreṇī	innumerable world-lines
jaghanya	minimum
jaghanya anantānanta	minimum higher order infinity
jaghanya asaṅkhyātāsaṅkhyāta	minimum higher order innumerable
jaghanya paritānanta	minimum lower order infinite
jaghanya paritāsaṅkhyāta	minimum lower order innumerable
jaghanya saṅkhyāta	minimum numerable
jaghanya yuktānanta	minimum middle order infinity
jaghanya yuktāsaṅkhyāta	minimum middle order innumerable
jaina	One of the oldest religion of India
jalacara pañcendriya-tiryañcayonika- napuṅsaka	5-sensed aquatic eunuch animals
jalacara pañcendriya-tiryañcayonika- striyāṅ	5-sensed aquatic female animals
jalacara pañcendriya-tiryañcayonika-puruṣa	5-sensed aquatic male animals
Jambudvīpa	The innermost first island in middle cosmos
jīva	living-being
jīvāstikāya	class of all the souls
jñāna	knowledge
jñānavarṇīya karma	knowledge obscuring karma
jyotiṣa	astronomy
jyotiṣka deva	astral gods
jyotiṣka deviyaṅ	astral goddesses
kāla	time

kāla cakra	time-cycle
kāladvāra	time section
kālamāna	scale of time
kālappamāṇe	measurement of time
kālasavanna	fraction
kālodadhi	2nd ocean in middle cosmos
kalpa	heaven, abode of celestial (vaimānika) gods
kalyoja	set of numbers where remainder is after dividing a positive integer by the multiple of 4
kalyoja dvāparayugma	set of numbers where remainder is 6 after dividing a positive integer by the multiple of 16
kalyoja- kalyoja	set of numbers where remainder is 5 after dividing a positive integer by the multiple of 16
kalyoja kṛtayugma	set of numbers where remainder is 4 after dividing a positive integer by the multiple of 4
kalyoja tryoja	set of numbers where remainder is 7 after dividing a positive integer by the multiple of 4
kāṇḍaka	a quantity representing the rate of growth of karma
kāpota leśyā	grey (pigeon) psychic colour
karma	material aggregates attracted to the soul caused by the activity of the soul
kāya	physical
kevala jñāna	omniscience knowledge
kevalī	omniscient
kevalī samudghāta	omniscient inflation
khaṇḍa	section/part
Kharoṣṭhī	an ancient script used in ancient *gaṇadhara*
khecara	flying living beings
khecara pañcendriya-tiryañcayonika-napuṅsaka	5-sensed flying eunuch animals
khecara pañcendriya-tiryañcayonika-puruṣa	5-sensed flying male animals

khecara pañcendriya-tiryañcayonika-striyāṅ	5-sensed flying female animals
koḍākoḍākoḍākoḍī	10^{21}
koḍākoḍākoḍī	10^{14}
koḍi pūrva	7056×10^{17}
koṭi (*koḍi*)	10^{7}
koṭi-pṛthaktva (*koḍipuhatta*)	10^{7}-separator as a number between 1×10^{7} and 9×10^{7}
koṭiśataṁ (*koḍisayaṁ*)	10^{9}
kṛṣṇa	black
kṛṣṇarāji	dark black the earth-bodied beings located in the upper cosmos with extremely gravity force
kṛta yugma	set of such numbers where remainder is 4 or 0 after dividing a positive integer by the multiple of 4.
kṛtayugma dvāpara yugma	set of numbers where remainder is 4 after dividing a positive integer by the multiple of 16
kṛtayugma kalyoja	set of numbers where remainder is 1 after dividing a positive integer by the multiple of 16
kṛtayugma kṛtayugma	set of numbers where remainder is 0 after dividing a positive integer by the multiple of 16
kṛtayugma tryoja	set of numbers where remainder is 3 after dividing a positive integer by the multiple of 16
kṣayopaśama	destruction-cum-suppression of karma
kṣetra pramāṇa (*khettappamāṇe*)	measurement of the area
kṣetradwāra	space section
kṣhetra (*khetta*)	spatial
kṣullaka dvāpara yugma	set of such numbers where remainder is 2 after dividing a positive integer by the multiple of 4.
kṣullaka kalyoja	set of such numbers where remainder is 1 after dividing a positive integer by the multiple of 4.
kṣullaka kṛtayugma	set of such numbers where remainder is 4 or 0 after dividing a positive integer by the multiple of 4.
kṣullaka tryoja	set of such numbers where remainder is 3 after dividing a positive integer by the multiple of 4.
kṣullaka yugma (*khuḍḍa jumma*)	set of the class of small numbers

kukṣi	womb
kulakara Progenitor	Progenitor
lāntakakalpa ke deva	gods of 6th heaven
Lavaṇa samudra	the first ocean in middle cosmos next to JD
lekha (*lehaṁ*)	alphabets
loka	cosmos
lokānuprekṣā.	to meditate on the nature of universe
Madhya loka	middle cosmos
madhyama graiveyaka deva	Middle *graiveyaka* god
Mahāhimavāna	2nd continental mountain range running east-west touching *Lavaṇa* sea at both ends and 200 y high forming northern boundary of continent of *Hemavaya*.
mahāśalākā kuṇḍa	3rd measure pit imagined to get higher order of numerable
mahāśukrakalpa ke deva	Gods of 7th heaven
Mahāvideha	central, 4th from south and biggest of 7 continents of JD, north of Mt. *Niṣadha* and south of Mt. *Nīla*;
māhendrakalpa ke deva	Gods of 4th heaven
mana	mental
māna	scaled receptacles were used to measure the grains or liquid
manah paryava jñāna	mind-reading knowledge
mānuṣī	female human
māsa	month
mithyā dṛṣṭi	soul with perverted faith
mohaniya karma	deluding karma
Mt. Meru	Colossus of a mountain at the center of JD (and universe), rising 99000 *y* above ground with diameter there 10000 *y*, around which the entire astral world of human world revolves.
Mt. Vaitāḍhya	4 mountains, cylindrical in shape, bisecting along with great rivers the continents of *Hemavaya*, *Harivasa*, *Rammagavasa* and *Heraṇyaavaya*
mudrā	finger arithmetic

muhūrta (*muhutta*)	unit of time, 48 minutes
mūrtipūjaka	idol-worshiper
nakṣatra	asterism
nalina	$(8400000)^{16}$ years
nalināṅga	$(84\times10^5)^{15}$
nāsti	does not exist
naya	viewpoints
nayuta	$(84\times10^5)^{24}$
nayutāṅga	$(84\times10^5)^{23}$
nicale graiveyaka deva	lower *graiveyaka* god
nigoda jīva	the primordial form of living beings
niśwāsa	exhalation
nyāya	logic
pada	words
pāda	foot
padma	$(84\times10^5)^{14}$
padmāṅga	$(84\times10^5)^{13}$
pakṣa	fortnight = 15 day-nights.
palyopama (*paliovame*)	a measure of time not expressible numerically and can be described by analogy. By certain reckoning as described in the text $\approx 1.69 \times 10^{41}$ or 3.31×10^{38} years.
panak	fungus
pañcaka-saṅyogī	combination of 5 things
pañcendriya aparyāptaka	5-sensed undeveloped beings
pañcendriya-paryāptaka	5-sensed developed beings
paṅkaprabhā pṛthvī ke nairayika	hellish beings of 4th hell
paramāṇu	atom, the smallest indivisible unit of the matter
Parikarma	arithmetic
parimāṇa dwāra	measurement section
parimaṇḍala	ring
parittānanta	lower order infinity
parittāsaṅkhejja	lower order innumerable
paryāpta	developed
paryāpta asañjī pañcendriya	developed five sensed beings without mind

paryāpta asañjī-pañcendriya-tiryagyonika	five-sensed developed animal-species devoid of mind
paryāya	modes
paścānupūrvī	reverse ordering
prābhṛta	chapter
pradeśa	the smallest indivisible unit of the matter joined with its molecule or aggregate
pramāṇa	measure
pramāṇa aṅgula	a measure equal to 400 *utsedhāṅgula*-s (500 *utsedhāṅgula*-s as per Digambara tradition
pramatta saṅyata	6th guṇasthāna, the stage of complete self-discipline of a monk, although sometimes brought into wavering through negligence.
prāṇatakalpa ke deva	gods of 10th heaven
pratimāna	highly precise weighing tools to weigh the precious things like gold et
pratipatti	chapters
pratipratita	soul degraded from right faith
pratiśalākā kuṇḍa	second measure-pit imagined for analogy measure of maximum innumerable
pratyeka sarīrī	organisms one soul in one body
pratyekaśarīra bādara vanaspatikāyika-aparyāptaka	gross plants with one soul in one body undeveloped beings
pratyekaśarīra bādara vanaspatikāyika-paryāptaka	gross plants with one soul in one body beings
prayuta	$(84\times10^5)^{23}$
prayutāṅga	$(84\times10^5)^{22}$
pudgalaparāvartana poggalapariyaṭṭe	an analogy measure of time equal to innumerable *osappiṇīs* and *ussappiṇīs*
pudgalāstikāya	
pūrva	1) = 8400000 *purvāṅga* = $(8400000)^2$ years $=70560000000000 \approx 7.05\times10^{13}$ years. 2) There are 14 *pūrva*-s in 12th *aṅga Dṛṣṭivāda* of Jaina canon.
pūrva videha-apara videha	east and west *Videha*

pūrvāṅga	(84×10^5)
pūrvānupūrvī	serial ordering
Puśkaradvipa	The third island in middle cosmos
rajju	
ramyakvarṣa	one of 8 provinces to west of province of *Ramyak* of Southern *Pūrvavideha*
rasa	taste
rāśi	Set theory
ratnaprabhā pṛthvī ke nairayika	hellish beings of 1st hell
rekhā-gaṇita	mensuration
ṛju gati	straight-line motion taken by soul while going for the next birth
Ṛṣabha	The first *tīrthaṅkara*
ṛtu	season
ṛtu-māsa	seasonal month
rūpa (*rūvaṁ rūpa*)	geometry
sabhā	assembly
sādhanā	spiritual practices
sādhu	monk
sādhvī	nuns
sāgara	ocean
sahasrārakalpa ke deva	gods of 8th heaven
sahassa sahassaṁ	thousand-thousand (10^6)
sajñī	living being endowed with mind
śaka calendar	about 78 years behind the christian calendar
sakaṣāyī jīva	living beings endowed with passion
śākya	Gautama Buddha
śalākā kuṇḍa	the first measure pit imagined for maximum numerable number
śāli	grain
samaya	the smallest indivisible unit of time
sammūrchim	human born through agglutination
saṁvatsara	year
sanatkumārakalpa ke deva	gods of 3rd heaven

saṅkhā	number
saṅkhyāta	numerable, countable
sansārī, saṅsārastha jīva	Mundane souls
saṅsthāna kāla	the life-duration of a living being in that realm
sapradeśatva	spatial characteristics
saptabhaṅgī	seven-fold statements
saptama pṛthvī ke nairayika	hellish beings of the 7th hell
śarkarāprabhā pṛthvī ke nairayika	hellish beings of 2nd hell
sarṣapa	mustard
sarva	all
sarvārthasiddhi	one of the highest heavens
śata, (*saya, sayaṁ*)	hundred
śataka	chapter
ṣaṭsthāna vṛddhi	exponential growth
ṣaṭsthānapatita	six-ways decay
saudharmakalpa ke deva	gods of 1st heaven
saudharmakalpa ke deviyāṅ	goddesses of 1st heaven
savvaddhā	all-time
sayogī jīva	soul endowed with physical, mental and vocal activities
seḍhi	progression; line, world-line
siddha	liberated soul
siddhaśīlā'	The permanent abode of the liberated soul
śīrṣaprahelikā	the largest countable number $(84\times10^5)^{28}$ (as per Māthurī council)
śīrṣaprahelikāṅga	$(84\times10^5)^{27}$
skandha	aggregate
sparśa	touch
sparśanā	contact
sparśanadvāra	contact section
śramaṇa	one of the oldest Indian religion
śrāvaka	lay followers
śrāvikā	lay woman
sthalacara pañcendriya-tiryañcayonika- napuṅsaka	5-sensed terrestrial eunuch animals

sthalacara pañcendriya-tiryañcayonika- striyāṅ	5-sensed terrestrial female animals
sthalacara pañcendriya-tiryañcayonika-puruṣa	5-sensed terrestrial male animals
Sthānakavāsī	one of non-idol worshipping sects of Jain Śvetāmbara tradition
sthiti	duration
Subhadrā	chief queen of emperor Bharata
dravya	substance
sucī aṅgula	an aṅgula-long line of space points
śukla leśyā	white psychic colour
sūkṣma	fine, subtle
sūkṣma addhā palyopama	subtle temporal-pit-measure
sukṣma addhā-sāgarovama	subtle temporal-sea-measure
sukṣma aparyāpta	subtle undeveloped souls
sukṣma aparyāpta vanaspati	subtle undeveloped plant
sukṣma aparyāpta vanaspati	subtle undeveloped plants
sūkṣma aparyāptaka	subtle undeveloped souls
sūkṣma apkāyika-aparyāptaka	fine water-bodied undeveloped beings
sūkṣma apkāyika-paryāptaka	fine water-bodied developed beings
sukṣma jīva	subtle souls
sūkṣma khetta palyopama	subtle spatial-pit-measure
sūkṣma nigoda-śarīra-aparyāptaka	fine undeveloped *nigoda*
sūkṣma nigoda-śarīra-paryāptaka	fine developed *nigoda*
sukṣma paryāpta jīva	subtle developed soul
sūkṣma pṛthvīkāyika-aparyāptaka	subtle earth-bodied undeveloped soul
sūkṣma pṛthvīkāyika-paryāptaka	subtle earth-bodied developed soul
sūkṣma tejaskāyika-aparyāptaka	subtle fire-bodied undeveloped soul
sūkṣma tejaskāyika-paryāptaka	subtle fire-bodied developed soul
sūkṣma uddhāra palyopama	subtle-linear pit-measure
sūkṣma vanaspatikāyika -aparyāptaka	subtle undeveloped plants
sūkṣma vanaspatikāyika -paryāptaka	subtle developed plants
sūkṣma vāyukāyika- aparyāptaka	subtle air-bodied undeveloped plants
sūkṣma vāyukāyika-paryāptaka	subtle air-bodied developed plants

Sundarī	daughter of the emperor Bharata
śūnya kāla	null period
śūnya'	zero
sūrya māsa	sun-month
susamā	2nd epoch of descending cycle (*osappiṇī*) and 5th epoch of ascending cycle (*ussappiṇī*) of time spanning over 3 x 10^{14} *sāgarovama*-s
susama dusamā	3rd (out of 6) epoch of descending cycle (*osappiṇī*) and 4th epoch of ascending cycle (*ussappiṇī*) of time spanning over 2 x 10^{14} *sāgarovama-s*
susama susamā	1st (out of 6) epoch of descending cycle (*osappiṇī*) and 6th (last) epoch of ascending cycle (*ussappiṇī*) of time spanning over a period of 4 x 1014 *sāgarovama*-s
sūtra	verse
swādhyāya	scriptural study
Swayambhūramaṇa	the last ocean or island in middle cosmos
Śwetāmbara	One of the tradition of Jainism
syādvāda	theory of conditional dialect
syāt	from some perspective
taijasa	fiery-body
tamah prabhā pṛthvī ke nairayika	hellish beings of 6th hell
tamaskāya	water-bodied dark area in the upper cosmos
tejo leśyā	red psychic colour
terāpnathī	one of the non-idol worshipping sect of Śvetāṁbara tradition
tīrtha-kāla	the religious tradition
tīrthaṅkara	preceptor, who establishes the four-fold order
tiryañca	animal kingdom
tītaddhā	past period
tithi	day reckoned by phases of Moon in 15 days (according to Hindi calendar)
tiyamalapaya	number with 24 digits
trayasra (*taṁsā*)	triangle
trīndriya aparyāptaka	3-sensed undeveloped beings

trīndriya paryāptaka	3-sensed developed beings
truṭita	$(84\times10^5)^4$
truṭitāṅga	$(84\times10^5)^3$
tryoja	set of numbers where remainder is 3 after dividing the multiple of 4 by positive integer
tryoja dvāpara yugma	set of numbers where remainder is 14 after dividing the multiple of 16 by positive integer
tryoja kalyoja	set of numbers where remainder is 13 after dividing the multiple of 16 by positive integer
tryoja kṛtayugma	set of numbers where remainder is 12 after dividing the multiple of 16 by positive integer
tryoja tryoja	set of numbers where remainder is 15 after dividing the multiple of 16 by positive integer
ucchwāsa	inhaling
uddeśaka	section of the chapter
uddhāra	exhaustion
unmāna	some weighing tools to weigh the solid goods
upamā māna (*uvamiye*)	analogy measure
upāṅga	secondary canonical literatures authored by scholar ācārya-s
ūparī graiveyaka deva	gods of higher *graiveyaka*
ura parisarpa	reptiles
urdhwa diśā	upper direction
utkṛṣṭa anantānanta	maximum higher order infinity
utkṛṣṭa asaṅkhyātāsaṅkhyāta	maximum higher order innumerable
utkṛṣṭa paritānanta	maximum lower order infinity
utkṛṣṭa paritāsaṅkhyāta	maximum lower order innumerable
utkṛṣṭa saṅkhyāta	maximum numerable
utkṛṣṭa yuktānanta	maximum middle order infinity
utkṛṣṭa yuktāsaṅkhyāta	maximum middle order innumerable
utpala	$(84\times10^5)^{12}$
utpalāṅga	$(84\times10^5)^{11}$
utsarpiṇi (*ussappiṇi*)	progressive cycle

utsedha aṅgula	measure-finger
uttara prakṛti	sub-classification
Uttarakuru	northern and one of 4 subcontinents of *Mahavideha*, north of Mt. Meru,
vācanā	councils
vaikrya	transformational
valayavṛtta	circle
vāṇamantara deva	forest-dwelling gods
vanaspati jīva	living beings of plant-kingdom
vāṇavyantar striyāṅ	forest-dwelling goddesses
varga	square
vargaṇā	a group of *paramāṇu*-s of similar attributes
varga-varga	square of square
vargita saṁvargita	successive square
varṇa	colour
vāsudeva	younger brother of *baladeva*
Veda	scriptures of Vedic traditions
vedanīya pain	pleasure producing karma
vedikā	rampart
vedikānta	rampart
Vejayanta	name of southern gate of rampart around JD
Vijaya	name of eastern gate of rampart around JD
vikalpa	combination
viraha kāla	null period
vṛddhi	increase
vrīhi	rice
vṛtti	commentary
vyāvhārika	practical
vyāvhārika addhā palyopama	practical temporal-pit-measure
vyāvhārika khetta palyopama	practical spatial-pit-measure
vyāvhārika uddhāra palyopama:	practical exhaustion-pit-measure
yamalapaya	number with 8 digits

yāvat-tāvat	Simple algebraic equation
yoga	physical, mental and verbal activity
yojana	a measure of length different in different regions, but in Jaina tradition = 768000 *aṅgula*-s ≈ 14.63 km (approx.) and a variation of same 400 or 500 times of this.
yoni	species
yuga	pentad = 5 years
yuktāsaṅkhyāta (*juttāsaṅkhejja*)	middle order innumerable
Āgama	Jain canonical literature

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